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ERRORS OF CALCULATIONS IN M/M/1/FIFO/N/F MODEL WITH LIMITED DURATION OF SHIFT

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ABSTRACT

The popular models developed under of queueing theory describe how systems function within an infinitely long time interval. Yet, a real work is executed within the limited time intervals of work-shifts. The study was performed on the basis of one example involving excavations made by one excavator and the transportation of soils excavated using dumps tracks driving within "a closed circle". An Numerical Queueing Model (*NQM*; Polish abbreviation *nmk*) was developed, and, on its basis, it was found that the discrepancies among the results obtained were significant when the start and end conditions of work-shifts were omitted/disregarded. For example, relative errors referring to the mean number of units in a queue could amount to several dozen percentage points. In addition, the variations in those discrepancies were analyzed from the point of view of the magnitude of work-shift time intervals.

KEYWORDS

Queueing Theory; Work-Shift; Construction.

1. INTRODUCTION

In the past, the deterministic calculus was applied to describe a service system developed to imitate, for example, the work of excavator and dumps tracks used to remove excavated soils. In analytical models, there were used special coefficients to represent: mean values of the single operation durations, effects of the non-anticipated disturbances occurring during the realization of a given work, and utilization rate of the work-shift time periods; those coefficients were usually 'expertly' identified and selected from the specific ranges of values. Undoubtedly, uncomplicated models and simple calculations were the advantage of this approach [6]. At the same time, there was one serious disadvantage: those models did not involve dependencies that would show mutual interactions among units functioning within one set of units.

When employing the calculus of probability, it is possible to study time losses resulting from the cooperation of several units. In this case, the models developed on the basis of the Theory of Queue *(TQ)* are usually applied to analyze how a set of units functions; such models have pre-determined flow charts and pre-determined process characteristics (profiles) [1, 2, 3, 4]. Those models make it possible to study both the operational time of work and the times of technological intervals/breaks generated by the mutual wait time. However, there is one big problem: discrepancies existing between the calculation results obtained and the performance effects produced.

The assumption that the systems continuously function within an infinitely long time interval is a significant simplification incorporated into the models applied *TQ*. And yet, civil engineering works are started and finished every day, i.e. they are carried out within regular time intervals. In this paper, the author analyzes the magnitude of calculation errors resulting from the fact that the *TQ* model under discussion disregards the condition requiring that the work be carried out within limited time periods of working shifts.

On the other hand, it is obvious that, in addition to the operational work, there are also other works performed during one single work-shift, such as technical servicing of the equipment; also, there are indispensable breaks, and certain time losses [7]. The existing *TQ* models do not consider them, although they generate discrepancies between the calculation results and real performance effects produced. In this paper, the author does not refer to those discrepancies and the errors accompanying them just because the scope of this paper does not permit it.

In order to represent and imitate the execution processes under the conditions appearing close to the authentic determinants, the author developed a 'Numerical Queuing Model' *(NQM)*. With this Model, it is possible to describe two issues using the distributions shifted towards the origin of the coordinates. The first issue is functioning of the queuing system (among other things, during a limited time-period, i.e. from the moment the work starts at the beginning of work-shift until the work ends at the end of work-shift) including components of the work-shift time period. The second issue described consists of the characteristics (profiles) of the service processes and the cycle. Furthermore, the author assessed the discrepancies found between the results obtained under the application of his *NQM* Model with the real time duration of work-shift considered, and the results obtained using the *TQ* Model. For the purpose of this study, an process consisting of making excavations and removing the soils excavated was an example of the work executed. In this example, the magnitudes of calculation errors were identified, i.e. calculation errors occurring when computing (using a standard *TQ* Model) the probabilities of stoppages of service object & clients (i.e. truck arrived to remove excavated soils), and the mean numbers of queuing units in the situation when the limited work-time during one working shift was not considered.

2. M/M/1/FIFO/N/F MODEL

According to [2, 5, 8], a *M/M/*1*/FIFO/N/F* Model can be applied to describe the work of a set consisting of an excavator and of *N* vehicles removing excavated soils. Each truck works in a closed cycle, i.e. instantaneously upon the unloading, it returns to the excavation site to take a next batch of soils for removal. This Model represents a queue service system where: the cycle time (measured from the moment when the client leaves the system to the next moment when the client returns to the system) is consistent with the exponential distribution with a λ parameter; also, the service time is characterized by the exponential distribution, which, however, has a μ parameter. This system has one service route with a queue formed according to the *FIFO* procedure (clients are serviced in the order of their arrival) and with *N* units working within the *F* closed cycle.

The relationships characterizing the *M*/*M*/1/*FIFO*/*N*/*F* Model are described in detail in the respective literature [2, 5, 8].

The probability that the service object has its stoppage is:

$$p_o = \left(\sum_{s=0}^{N} \left(\frac{\lambda}{\mu} \right)^s \frac{N!}{(N-s)!} \right)^{-1} . \qquad (1)$$

The probability that there is a stoppage of clients, which queue, is:

$$p_{os} = \sum_{s=2}^{N} p_s \frac{s-1}{N} \qquad \text{for} \qquad p_s = p_o \left(\frac{\lambda}{\mu} \right)^s \frac{N!}{(N-s)!} ,$$

$$0 < s \le N , \qquad (2)$$

The mean number of the clients in the queue is:

$$q = \sum_{s=2}^{N} p_o \left(\frac{\lambda}{\mu} \right)^s \frac{(s-1)N!}{(N-s)!} . \qquad (3)$$

3. NUMERICAL QUEUING MODEL

The numerical queuing model can be applied to research the operation of *X*/*Y*/1/*FIFO*/*N*/*F* systems using simulation methods. The *NQM* Model is used to calculate the characteristics similar to those covered by the *TQ* Models, among other things, the frequencies of stoppages: p_o^* - stoppage of service object, p_{os}^* - stoppage of clients, and q^* - mean number of clients in the queue.

Owing to the specific structure of the *NQM* Model under discussion, it is possible to reproduce/imitate many detailed guidelines and postulations usually accompanying the execution. It is also possible to apply either standard or any other distributions to describe the cycle and the service durations. The inspection of how the systems function can involve the research into the work-shift durations with the work starting and ending conditions identified, as well as the verification of the principles and guidelines referring to the accompanying processes during the work realization.

The method of calculations under the *NQM* Model agrees with the procedure as described below. First, there are determined potential moments for the events to occur, but an authentic, real event does occur as soon as the realization conditions permit its occurrence. For example, upon the arrival of a client, its service may start, but, in fact, this service starts as soon as all the crucial conditions are fulfilled, among other things, when the service of all preceding clients in the queue (i.e. the clients, which arrived earlier) is finished. Another example: any service cannot be carried out if the moment of potential start or finish of the service takes place after the working shift has finished.

Below, only fragments are shown of the calculation algorithm of the *NQM* Model because this algorithm is too large to be presented in whole.

The conditions for starting:

$$t \in R, \ t = t_o, \tag{4}$$

$$t_{zm} = t_o + \tau^{zm}, \tag{4a}$$

$$t^{zw} = t_o, \tag{4b}$$

$$i = 1, \dots, N, \ \ I \ni i, \tag{4c}$$

$$\bigwedge_{i \in I} \left| \begin{array}{ll} t_i^{ps} = t_o, & (4d) \\ T^{ps} \ni t_i^{ps}, & (4e) \end{array} \right.$$

Values of the characteristics:

$$\bigwedge_{t \geq t_{zm}} \left| \begin{array}{ll} p_o^* = \tau_k / \tau_{zm}, & (5) \\ p_k^* = k/c, & (5a) \\ p_{os}^* = p_k^* \tau_{pr} / \tau_{zm}, & (5b) \\ w^* = \tau_s / k, & (5c) \\ q^* = p_{os}^* w^* \mu. & (5d) \end{array} \right.$$

The calculation algorithm of the *NQM* Model can be interpreted as follows below.

With regard to the conditions of start to happen, it is assumed that t is a current clock time moment and its value is from the R set of real numbers, i.e. $t \in R$. The t moment corresponds with the successive moments when particular events occur during the processes under realization. At the beginning, the clock time corresponds with a t_o moment, which is the moment for the system to start functioning, it means: $t = t_o$ (4). While measuring the time from a t_o moment, after the whole τ^{zm} period when the system functioned (and this period matched one work-shift duration), the moment when the system finishes its functioning is t_{zm}, thus, $t_{zm} = t_o + \tau^{zm}$, (4a).

The assumption was made that, at the t_o moment, no processes were carried out, and the service of reports was finished. So, at the moment of starting the calculations, t^{zw} - is the moment of finishing the service of reports: $t^{zw} = t_o$, (4b).

The Model studied replicates the following system: one service object and N clients within one closed cycle. Each clients has an identification number i assigned to it. Consequently, the next reports, $i = 1,\ldots, N$ form a N –element set denoted as I ; this set is characterized by the fact that $I \ni i$, (4c).

In the most frequently analyzed cases, when the arrivals of clients prior to the moment of starting a work-shift are disregarded (omitted), it is assumed for all the i - reports, $i \in I$, that the t_i^{ps} - moments of the first arrivals of reports correspond with the moment of starting; accordingly, $t_i^{ps} = t_o$, $i \in I$ (4d). An N - element set called T^{ps} is compiled of all the t_i^{ps} moments that are the moments when all the clients arrived, this N - element set T^{ps} fulfils the condition $T^{ps} \ni t_i^{ps}$, (4e).

In reality (i.e. during the real execution of the service procedures), the first truck usually arrive much earlier than the moment t_o , which is the moment of starting a new work-shift because those trucks want to get a good position in the queue being formed. In order to consider such cases, the following alternative calculations are usually done:

$$\bigwedge_{i \in I} \left| \begin{array}{l} \tau^{\lambda s} = x(F_{\lambda s}^{-1}(los{:}\langle 0,\ 1\rangle)), \\ t_i^{ps} = t_o - \tau^{\lambda s}, \\ T^{ps} \ni t_i^{ps}. \end{array} \right.$$

Here, the $\tau^{\lambda s}$ time, measured from the moment when a client (a given vehicle) arrived, to the t_o - moment when the system started to function, is defined as the value of random variable of the function of the inversion cumulative distribution of times when the first clients arrived in the system, $x(F_{\lambda s}^{-1}(los{:}\langle 0,\ 1\rangle))$. Thus, the t_i^{ps} moment of the first arrival of the i - report is: $t_i^{ps} = t_o - \tau^{\lambda s}$ (the minus sign means that some units/vehicles arrived prior to the moment when the next work-shift started).

During the simulation of the work of this system, a repeatable step is carried out as long as the t moment of the clock time precedes the t_{zm} moment when the given work-shift is finished, i.e. when $t < t_{zm}$.

The values of characteristics are calculated upon the accomplishment of the simulation, provided the clock time comes up to the moment when the work-shift ends: $t \geq t_{zm}$.

The p_o^* frequency of stoppages of the service object is defined as a quotient of two factors: τ_k - cumulated durations of the equipment stoppages divided by τ_{zm} - duration of the system functioning, therefore: $p_o^* = \tau_k / \tau_{zm}$, (5).

The value of p_k^* = frequency of service provided to clients in the queue including their stoppage is calculated as the quotient of two factors: k - a number of waiting events divided by ‘ c ’ – a number of service cycles, thus, $p_k^* = k/c$, (5a).

The value of p_{os}^* - frequency of queue occurrences is defined as the product of two factors: p_k^* - frequency of services rendered including stoppages in the queue multiplied by the cumulated occupancy rates (operational work) of the service object, which is $\tau_{pr} = \tau_{zm} - \tau_k$, divided by τ_{zm} time when the system functioned. So, the following relationship was created: $p_{os}^* = p_k^* \tau_{pr} / \tau_{zm}$, (5b).

The mean stoppage time of clients in the queue, w^* , equals the quotient of τ_s - the cumulated time of their stoppages divided by k - the number of waiting events, thus, $w^* = \tau_s / k$, (5c).

And the mean number of clients in the queue, q^* , is determined as a product of multiplying three elements: p_{os}^* - queue occurrence frequency rate, w^* - mean stoppage time in the queue, and μ - mean service rate, thus $q^* = p_{os}^* w^* \mu$, (5d).

4. BOUNDARY CONDITIONS FOR THE *TQ* AND *NQM* MODELS

The analytical model, developed under the theory of queue and denoted as *M/M/*1*/FIFO/N/F*, describes the functioning of the system provided the following conditions are fulfilled [1]:

- analysis begins at a moment when the processes have been stabilized and the period of their realization is long enough, for example, their realization period started at a moment that is approaching from - ∞, and
- a very long time period is studied, and this period ends at a moment approaching +∞.

Moreover, this model engages exponential distributions, and this is why it fulfills the conditions of stationarity, uniformity, and autonomity of both the service processes and the cycle.

Consequently, the *TQ* model represents an ideal, randomly conducted realization of processes, and the analytical relationships deduced incorporate, very precisely, all the assumptions made. Those relationships give exact, unique results because they have been mathematically deduced. This is why those relationships constitute the basis of assessing discrepancies relating to the real/authentic realization.

In the *NQM* model, it is possible to calculate characteristic values with the execution determinants taken into consideration. Owing to this fact, the results obtained while imitating the real/authentic execution conditions can be compared with the results achieved for the ideal progress of the processes, pursuant to the analytical *TQ* relationships.

It was assumed that Δx absolute error is a difference between the x^* value obtained from the numerical calculations with the *NQM* Model and the respective x values, calculated analytically according to the *TQ* Model [4], thus:

$$\Delta x = x^* - x \,. \tag{6}$$

The relative error, denoted as δx, is :

$$\delta x = \frac{\Delta x}{x} \,. \tag{7}$$

5. INITIAL DATA

During the analysis carried on, data obtained at the construction site of „Galeria Krakowska Mall" were applied. Those data referred to a broad-spatial earthwork (cut) excavated. The dimensions of this earthwork were: its cubic volume: 184,000 m^3, its average depth: 7.5 m. The plan section of the earthwork excavated was almost a 340 m x 105 m rectangle. The excavated soils amounting to about 69% of the total excavated material were transported outside the construction site, to a permanent dump situated 12.5 km from the site [5].

The results referred to in this paper were received from the research into the work performed by a set consisting of one excavator having a bucket of 1.8 m^3 and 11 self-dumping trucks transporting excavated soils to the dump and operating in a closed cycle. The research was conducted from 24 November 2004 to 17 December 2004.

According to the data compiled during the empirical analysis, the 'cycle' time of the transporting vehicles was characterized by a mean value of this cycle time $\bar{t} = 0.8229$ h, therefore, a parameter λ=1.2152 was attributed to the exponential distribution. Similarly, for the service durations found during the research conducted, the mean value of the service time was $\bar{t} = 0.0631$ h, thus, a parameter μ = 15.8366 was for the exponential distribution.

In order to asses the error connected with the fact that the *TQ* Model did not take into consideration a limited work-shift duration period, the results obtained from this Model were compared with the results obtained when the *NQM* Model was used since the latter Model imitated the real/authentic conditions for starts and ends of the work. Hence, in the two Models, the same systems were assumed, which matched the whole set employed at the site and comprising one service machine and 11 units (trucks) functioning (working) in a closed cycle. Moreover, the same assumption was made in the two Models, namely: the processes of service and cycle were compatible with the exponential distributions having the frequencies as shown above, i.e. $\lambda = 1.2152$ and $\mu = 15.8366$.

6. CALCULATION RESULTS

In the *TQ* Model, the characteristics of the *M/M/*1*/FIFO/*11*/F* system were calculated on the basis of the relationships (1), (2), and (3). Additionally, those characteristics were determined with respect to the conditions as mentioned above: the analysis started at a moment when the realization of processes has been progressing for a period that was long and the research has been conducted for a period long enough, too, i.e. until the moment to approach +∞. Only then the probability of stoppage of the service equipment could be $p_o = 0.2886$, the probability of the stoppage of the reports: $p_{os} = 0.4678$, and the mean number of queuing vehicles: $q = 1.0172$.

In the *NQM* Model that imitated the real/authentic execution conditions, it was assumed that at $t_o = 0$ the moment of beginning the work-shift, all the units were in the system. They were ordered according to their arrival times (indices $i = 1,2,\ldots,N$), and the service equipment (machine) started to function at a t_o moment, and began with rendering the service to the first report (i.e. to the truck to arrive the first) denoted as $i = 1$. Next, the service object carried on its function and served the subsequent reports waiting, i.e. those trucks which arrived after they had finished each of their cycles. The whole process progressed until the particular work-shift ended at a t_{zm} moment.

The characteristics of the *NQM* Model were calculated on the basis of 10,000 simulation cycles with the system's limited work periods incorporated. For the purpose of assessing the effect of the work-shift duration period (its magnitude) on the values of individual characteristics, the calculations were conducted for the following time intervals: τ^{zm} = 8, 10, 20, 30,…, and 100 hours. In Tab. 1, all the calculation results are listed: p_o^* - service object's stoppage frequencies; p_{os}^* - stoppage frequencies of the clients queuing; q^* - mean number of the clients in the queue. In the same Tab. 1, there are shown discrepancies between the results calculated according to the relationships 6 and 7 and the results calculated empirically pursuant to the relationship under the *TQ*. They are absolute 'Δ' and relative 'δ' errors referring to the following characteristics, respectively: Δ_{po} and δ_{po} - errors in the service object's stoppage frequencies; - Δ_{ps} and δ_{ps} errors in the stoppage frequencies of the clients; Δ_q and δ_q - errors in the mean number of units in the queue. Fig. 1 shows how the relative errors vary.

The analysis of both the individual values of the p_o^*, p_{os}^* and q^* characteristics in Tab. 1, obtained through the *NQM* Model applied (that imitated the execution conditions) and the p_o, p_{os}, and q values calculated pursuant to the *TQ* Model, allows for the statement that there are very big differences between the corresponding results. The discrepancies are smaller only when the work realization time periods are longer.

With regard to the relative errors, the situation is comparable to the absolute errors; for the longer and longer time intervals, from 8 hrs to 100 hrs, when the system is in operation, the absolute errors are characterized by the respectively, monotonically decreasing (absolute) values. For those changing time intervals of systems in operation (when the systems function), the relative errors in the service object's stoppage frequencies are negative (-) and range from -11.26 % to –0.89 %, whereas the positive (+) relative errors are those in the stoppage frequencies of the clients, their values range from 22.05 % to 1.64 %. The errors in the mean number of the units in the queue amount from 49.88 % to 2.59 %.

It should be noted that the errors of each type show really high values, especially with regard to 8 hrs and 10 hrs work-shifts, which appear typical for any building/construction sites and for the execution of works.

Table 1. Values of the characteristics according to the *NQM* Model that imitates the execution conditions, and absolute & relative errors owing to the fact that the limited work-shift duration period was disregarded.

Duration period of one work-shift τ^{zm} h	Stoppage Frequencies						Mean Number		
	of the service object			of the clients in the queue			of the clients in the queue		
	p_o^*	Errors		p_{os}^*	Errors		q^*	Errors	
		Δ_{po}	δ_{po}		Δ_{ps}	δ_{ps}		Δ_q	δ_q
8	0,2561	-0,0324	-0,1126	0,571	0,1032	0,2205	1,5246	0,5074	0,4988
10	0,2617	-0,0269	-0,0931	0,5494	0,0816	0,1745	1,4185	0,4012	0,3945
20	0,2753	-0,0133	-0,0461	0,5076	0,0398	0,0851	1,2049	0,1876	0,1845
30	0,2807	-0,0079	-0,0272	0,4929	0,0251	0,0536	1,1301	0,1129	0,1110
40	0,2832	-0,0055	-0,0189	0,4861	0,0183	0,0391	1,0995	0,0823	0,0809
50	0,2846	-0,004	-0,0138	0,4818	0,014	0,0298	1,0769	0,0597	0,0587
60	0,2849	-0,0037	-0,013	0,48	0,0122	0,0261	1,0671	0,0498	0,049
70	0,2849	-0,0037	-0,0127	0,4786	0,0108	0,0230	1,0587	0,0415	0,0408
80	0,2855	-0,0031	-0,0107	0,4776	0,0098	0,021	1,0515	0,0342	0,0336
90	0,2856	-0,003	-0,0105	0,4766	0,0088	0,0187	1,0472	0,0299	0,0294
100	0,2858	-0,0028	-0,0097	0,476	0,0082	0,0174	1,0435	0,0263	0,0259

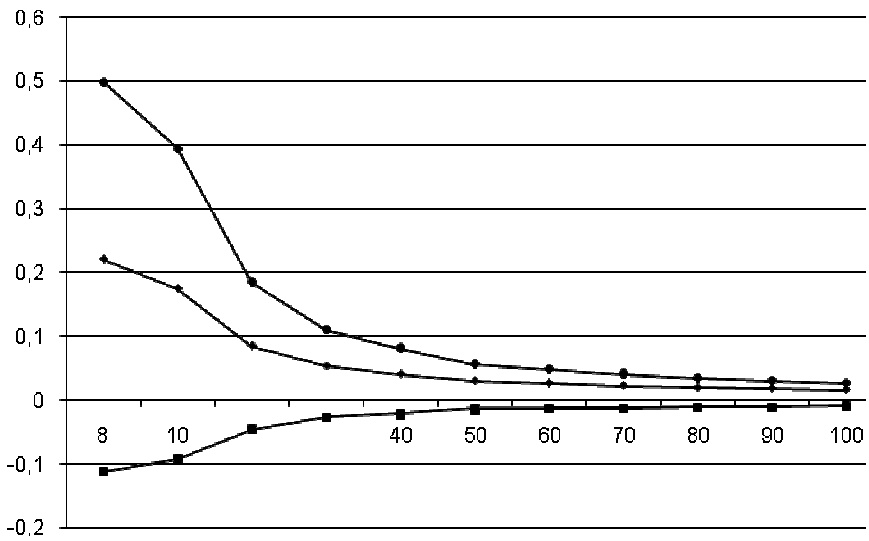

Figure 1. Relative errors in the stoppage frequencies: δpo - error in the service object; δps - error in the clients; δq - error in the mean numbers of the clients in the queue (the errors occurred because the limited work-shift duration period was omitted); τ^{zm} - time interval when the whole system functions

7. MECHANISM GENERATING DISCREPANCIES AMONG THE RESULTS

A limited time of work-shift duration impacts the service system at the beginning and at the end of the work. During the execution, and also in the *NQM* Model, *N* – trucks waiting are continuously served from the $t_o = 0$ moment, when the work-shift started. This continuous service with no breaks at all proceeds from the beginning of the work-shift and continues during a period of $\tau_c = N\tau_\mu$ (τ_μ - service duration). During the initial phase of the rest of this work-shift, the effects of this continuous service occur until the processes are stabilized. For the *N* - services rendered without any break, trucks leave their positions more intensively, especially after the first cycle, and this fact intensifies the arrivals of units compared to the stabilized system. Consequently, the continuous service at the beginning of the work-shift and the higher intensity of arrivals thereafter, until the processes become stabilized, cause a decrease in the stoppage of service equipment, and, at the same time, an increase in the waiting time for the units in the queue.

Yet, there are different effects when the work ends at a t_{zm}- moment of the work-shift end. The situations develop that at a $t^{ps} < t_{zm}$ moment, a client arrives and its potential service lasts until the t^{vs} moment that is after the end of the work-shift. Since the potential moment of the service to end is after the end of the work-shift, i.e. $t^{vs} > t_{zm}$, then, the real/authentic service is not started nor rendered; so, this client is lost. A similar situation develops when one 'client' queues and will be not served before the end of the work-shift. The effects of the lost clients are contrary to the effects of the queue of waiting units at the beginning of the work. They contribute to the increased stoppages of the service object and to the reduced waiting time of the units (trucks) queuing. Of course, the degree of this effect is significantly lower.

Owing to the contrary effect of the start conditions on the end, the total effect matches their resultant effects. Since the effect of the start conditions is stronger, the consequence is that the stoppage time of the service object is reduced and the clients waiting time increases pursuant to the results of the analysis presented.

The existence of the queue at the start and losses of the clients at the end of the work-shift practically exert their impact during the comparable time periods, and this statement is true for both the 8 hr work-shifts and the longer work-shifts under analysis. Hence, the ratio between the level of this constant impact and the bigger and bigger time periods of work-shifts decreases similar to the relative errors in calculations as shown above.

8. SUMMARY

When the *M/M/1/FIFO/N/F* Model is applied to analyze the work of service systems with clients functioning in a closed cycle, then, it is recommended to control the compatibility between the theoretical assumptions and the real/authentic execution conditions that are imitated. If only the conditions of the limited duration of a work-shift are disregarded, then, this exclusion results in very big calculation errors. As for the example studied, with a work-shift lasting 8 hrs, such relative calculation errors are above 10% with regard to the probability of stoppages of the service object, and 20% with regard to the probability of stoppages of clients, and with reference to the mean number of queuing units, this relative calculation error is as high as nearly 50 %.

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