

## A MODEL FOR AUTOMATIC CONTROL OF VIBRATORY PLATE COMPACTOR

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### Abstract

Application of automatic control into compaction equipment is one of the most complex problem compared with other machines used in civil engineering works. This is due to the difficulties in getting proper feedback, consisting of a transfer to the compactor and information on the state of the soil being just under compaction.

The paper presents an approach to this difficult and practically important problem. The discussion contains evaluation of several different responses of the soil which have been used so far in compaction analysis. Additionally, possibility of applying information on mechanical behaviour of compactor without involving mathematical model of soil is considered.

### 1. INTRODUCTION

A large amount of compaction processes during construction works is completed by dynamic compactors. For smaller areas plate vibratory compactors are in use. In road construction mostly vibratory rollers are applied. Regardless of the type of dynamic compactor, its efficiency depends strongly upon several parameters. Among them, the most important are centrifugal forces of the excitation system and related to them angular velocities. If these parameters could be controlled during the compaction then efficiency of the process might be improved. It is then the aim of the paper to propose a model of automatic control of compaction. We are dealing in this case with one degree of freedom (DOF) plate compactor. However in the following discussion it is not essential for the system to have only one DOF and for instance a roller can be used. The essence of the discussion is not on the mechanical behaviour of the compactor itself. Dynamics of a system consisting of rigid bodies connected with springs and dampers is a rather very well known branch of mechanics. Simpler machine allows to concentrate ones attention on the behaviour of the compactor/solid system. Knowing more about this behaviour we are closer to design an automatic controlled compactor with higher efficiency and more energy efficient.

### 2. SOIL - COMPACTOR INTERACTION

The soil under investigation was chosen to be a dry sand, the simplest of possible material to be compacted. It seems that even dealing with the simplest type of soil, to

have only one model is not enough. The behaviour of the soil depends first of all on type of compaction.

We recognize two main types, the first one represents the permanent contact of compactor with soil and the second one is seen as a process performed through impact of machine against soil. Probably it would be not appropriate to assume the same model in both cases.

The simplest soil model is related to the compaction process when the machine is not losing contact with sand. Below we are limiting our consideration to this case only.

Among the most often applied and the best known sand models, there are certainly the generalized Kelvin - Voigt and Maxwell models of solid body and dense fluid (Figs.1 and 2).

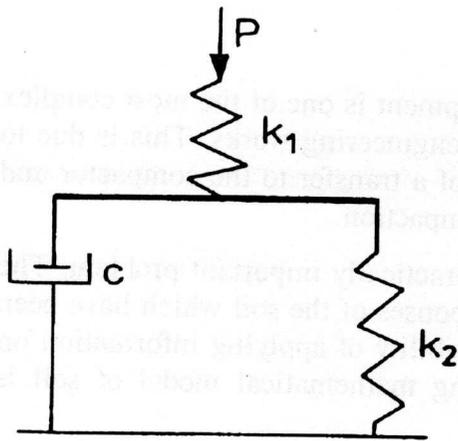


Fig. 1

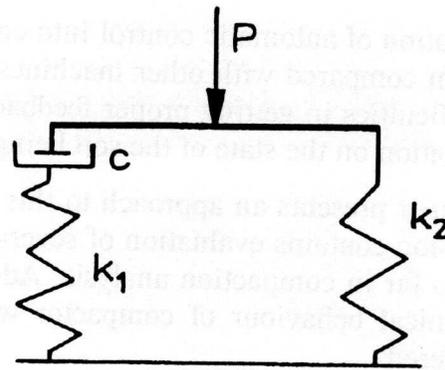


Fig. 2

However the response of sand during the down motion of the compactor is sometimes seen as different one to the model for the motion up. This is because the compaction process consists mainly of plastic deformation of the soil. It is due re-arrangement of sand grains which are filling voids and tend to maximize packing. It might be useful then, following [4] to recognize two models. The Maxwell type model for motion down and for unloading during the motion up, the Kelvin- Voigt generalized model (Fig.3).

It should be clearly pointed out that the mechanical soil model may be useful in automatic control of compaction only in a very limited range. This is because the compaction process is happening relatively fast. There is not then practically any possibility of measuring properties of the sand in real time during its motion.

The process which can be relatively easily observed, is the motion of the compactor. We can for instance put an accelerometer on the machine and get constant information of its acceleration. This in turn, might serve to control the angular velocity of the motor driving the rotating mass-type excitation. This approach will be discussed in detail below.

Several experiments made with sand compaction result in some common observations which may be utilized in automatic control of discussed process. The first important

observation is that the compaction depth depends upon the magnitude of the compactor amplitude. Sawicki et al [7,8] present the relation between the irreversible strain  $\epsilon^p$ , number of cycles  $N$  and shear strain amplitude  $\gamma$  (Fig. 4). From these results a conclusion may be drawn that compaction effect depends on the magnitude of amplitude of the compactor.

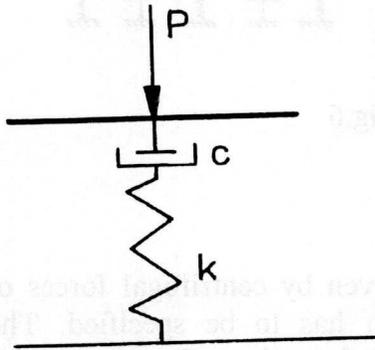


Fig.3

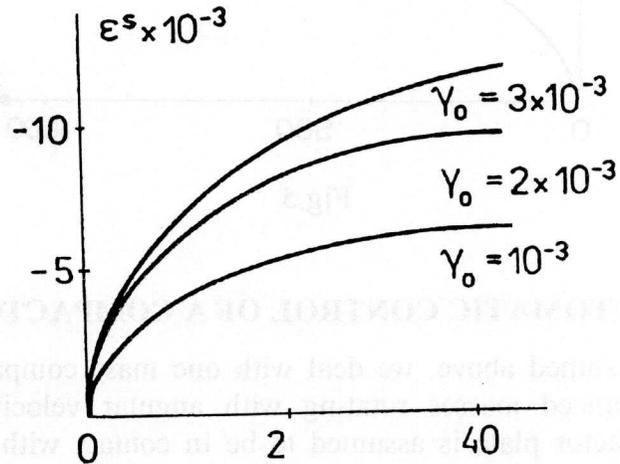


Fig.4

The same authors present the constitutive equation for the compaction law as:

$$\Phi = -\frac{1 - n_0}{n_0} \epsilon_p \quad (1)$$

where  $n_0$  and  $n$  are initial porosity and porosity respectively. From experimental data the compaction function may be approximated as follows:

$$\Phi = C_1 \ln(1 + C_2 z) \quad (2)$$

where  $C_1$  and  $C_2$  are constants to be determined from experiment and  $z$  is a variable defined as:

$$z = \frac{1}{4} \gamma_0^{-2} N \quad (3)$$

where  $N$  is the number of loading cycles Fig.5.

The compaction efficiency was observed experimentally by F. Degraere [3] and depends on vibration frequency close to resonance. It means that it may be seen that the best compaction depends on the largest value of the vibration amplitude, similar to the observations made by Sawicki et al [8]. From the above we conclude that automatic control should assure the largest possible amplitude of the compactor during the whole of the compaction process. Let us apply this conclusion into our model.

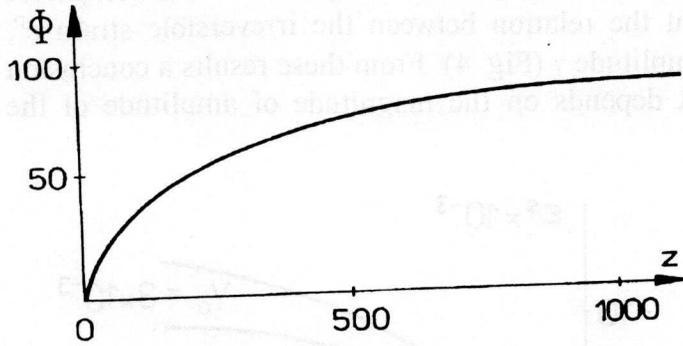


Fig.5

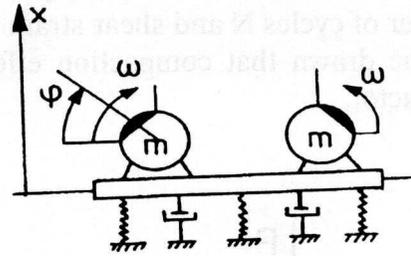


Fig.6

### 3. AUTOMATIC CONTROL OF A COMPACTOR

As assumed above, we deal with one mass compactor driven by centrifugal forces of unbalanced masses rotating with angular velocity which has to be specified. The compactor plate is assumed to be in contact with the soil throughout the compaction process.

Let us start with the equation of motion for the compactor presented on he Fig.6 and described by:

$$M\ddot{x} + C\dot{x} + kx = 2me\omega^2 \sin \omega t \tag{4}$$

where,

$M$  - compactor mass

$\alpha$  - vertical displacement of the compactor

$k$  - "spring" constant of sand

$C$  - damping coefficient of sand

$m$  - eccentric masses in rotating mass-type excitation

$e$  - eccentricity of eccentric masses

$\omega$  - angular velocity of excitation

The solution of the Eq.(4) which may be found in any book on vibration is

$$x(t) = \frac{2me\omega^2}{k\sqrt{[1 - (\omega/\omega_n)^2]^2 + (2\xi\omega/\omega_n)^2}} \sin[\omega t - \tan^{-1} \frac{2\xi\omega/\omega_n}{1 - (\omega/\omega_n)^2}] \tag{5}$$

where :  $\omega_n = \sqrt{\frac{k}{M}}$  and  $\xi = c/2\sqrt{kM}$ .

The above solution for compactor displacement  $x$  is obtained under assumption of constant angular velocity. This has to be kept in mind in further discussion. From the equation (5) a diagram of the amplitude  $A$  of displacement  $x$  against ratio  $\omega/\omega_n$  may be drawn. An example of such a diagram, for relatively small damping coefficient  $\zeta$  being in the range of values for sand type of soil, is presented on Fig. 7.

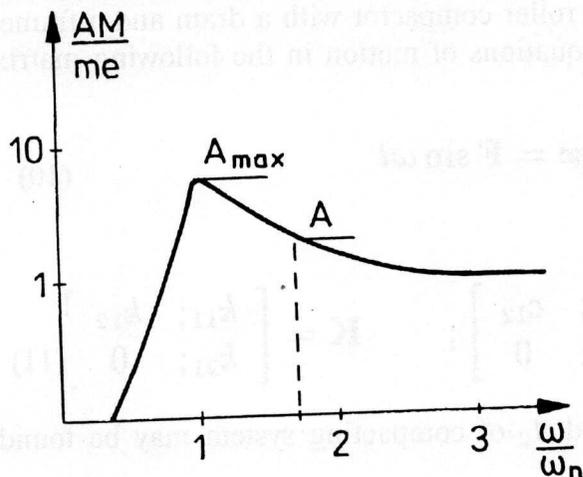


Fig.7

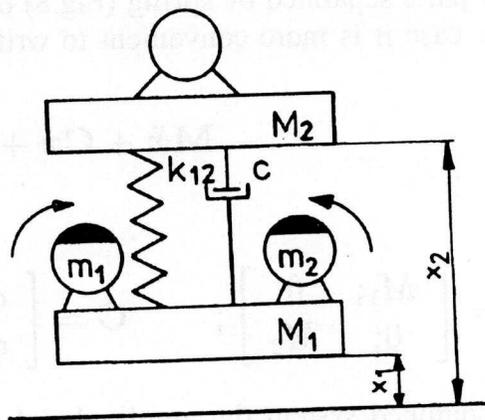


Fig.8

Assume that appropriate transducer allowing to measure the amplitude  $A$  is mounted on the compactor. Let us assume that at a given instance of time the measured amplitude of motion is  $A_1$  Fig.7 and given by relation

$$A_1 = \frac{2me \omega_1^2}{M \omega_n^2} \left[ \left(1 - \frac{\omega_1^2}{\omega_n^2}\right)^2 + \left(2\zeta \frac{\omega_1}{\omega_n}\right)^2 \right]^{-1/2}. \quad (6)$$

Knowing the amplitude, the "spring" constant  $k_1$  of the sand can be found assuming that  $\zeta$  is small enough to be neglected in this case.

$$k_1 \cong \left( \frac{2me}{A_1} + M \right) \omega_1^2. \quad (7)$$

In order to obtain the largest possible amplitude which is the best for compaction, we should change the angular velocity  $\omega_1$  to new one  $\omega_2$  which would bring the motion in the vicinity of resonance. In other words  $\omega_2$  may be equal to one causing resonance for undamped system

$$\omega_2^2 \cong \frac{k_1}{M} \cong \left( \frac{2me}{A_1 M} + 1 \right) \omega_1^2. \quad (8)$$

Coupling transducer with the motor assuring rotating motion of eccentric masses leads to obtaining the constant correction of angular velocity  $\omega_k$

$$\omega_k^2 = \left( \frac{2me}{A_{n-1}M} + 1 \right) \omega_{k-1}^2. \quad (9)$$

The discussed problem of one degree of freedom compactor may be easily extended to two degrees of freedom. The latter would be a model of either plate compactor composed of two parts separated by spring (Fig.8) or a roller compactor with a drum and a frame. In this case it is more convenient to write equations of motion in the following matrix form

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F} \sin \omega t \quad (10)$$

where:

$$\mathbf{M} = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}; \quad \mathbf{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & 0 \end{bmatrix}; \quad \mathbf{K} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & 0 \end{bmatrix} \quad (11)$$

For undamped system the amplitudes  $A_1$  and  $A_2$  of compacting system may be found from following relation:

$$\begin{bmatrix} (k_{11} - M_1\omega^2) & k_{12} \\ k_{21} & (-M_2\omega^2) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} 2me\omega^2 \\ 0 \end{bmatrix}. \quad (12)$$

With transducer measured amplitude  $A_1$  and knowing  $m$ ,  $e$ ,  $M_1$ ,  $M_2$  and  $k_{12}$  it is possible to evaluate from (12) approximately  $k_{11}$ , representing the elasticity of sand:

$$k_{11} = \frac{2me\omega_k^2}{A_1} + M_1\omega_k^2 - \frac{k_{12}^2}{M_2\omega_k^2}. \quad (13)$$

Knowing  $k_{11}$ , we can find angular velocity  $\omega_{k+1}$  for close to resonance circumstances, by equating to zero determinant of the matrix of equation (12)

$$\omega_{k_{11}}^2 = \frac{k_{11}M_2 \pm \sqrt{k_{11}^2M_2^2 + 4M_1M_2k_{12}^2}}{2M_1M_2} \quad (14)$$

Substituting  $k_{11}$  into (14) from (13) we get angular velocity  $\omega_{k+1}$  which brings our system to the vicinity of the resonance.

At this point it should be recalled that the above calculations were carried out under the assumption of the value  $\omega_k$  being constant. In other words, the above consideration was held under assumption that changes in angular velocity are made not continuously but at given intervals of time for which motion may be seen as constant. It is an open question to find the discussed model for automatic control of compaction for continuously varying angular velocity  $\omega$ , as several problems would arise including stability of motion, sensitivity of  $\omega$  with varying parameters etc.

#### 4. CONCLUSIONS

With an increase of the sophistication of road machinery there is a need of joining contract market with best use of new equipment. An area which requires special attention is compaction. The link between the performance of the pavement and degree of compaction has been shown to be of the basic importance. The paper has presented an approach for improving compaction by applying automatic control. The basic idea of proposed design is to control the angular velocity of exciting system depending on the state of the compacted soil. This is achieved by coupling the compactor motor with transducer and defining magnitude of amplitude of vibrating system. The paper is only the first step to the complete solution of proposed method. It is assumed that motor is rotating with constant angular velocity which then is changed to a new value, while the constant velocity is maintained. No discussion on what is happening with the system between these two stable stages is presented. However the simplicity of proposed control looks promising for further investigation.

#### REFERENCES

- [1] Y. K. Chow, D. M. Yong, K. Y. Yong, S. L. Lee, Dynamic Compaction Analysis, *J. Geot. Eng.* vol.118 No8, August 1992
- [2] D. J. D'Appolonia, E. D'Appolonia, Determination of the Maximum Density of Cohesionless Soils, *Proc. Third Asian Conf. on Soil Mechanics and Foundation Eng. Haifa 1967*, vol.1
- [3] F. Degraeve, Optimization des performance d'un compactor vibrant par ajustement automatique de la frequence, *Int. Conf. on Compaction, Paris, France, April 1980*
- [4] J. Kulejewski, A Model of Surface Compactor Interaction with Cohesionless Materials, Ph. D. Thesis, Warsaw Technical University Dept. of Civil Engineering, Warszawa 1986, (in Polish)
- [5] D. Pietsch, Simulation of Soil - Compaction with Vibratory Rollers, *8th Int. Symp. Automation and Robotics in Construction, 3-5 June 1991 Stuttgart*, vol.2, pp.489-506
- [6] F. E. Richart, Jr. J. R. Hall, Jr. R. D. Woods, *Vibrations of Soils and Foundation*, Prentice- Hall Int. Ser. in Theor. and App. Mech., 1970
- [7] A. Sawicki, An engineering model for compaction of sand under cyclic loading, *Rozpr. Inz.*, vol.35,4 pp.677-693, (1997)
- [8] A. Sawicki, W. Swidzinski, Mechanics of a sandy subsoil subjected to cyclic loadings, *Int. J. Nunzer. Anal. Method Geomech.*, vol.13, pp.511-529, (1989)
- [9] H. Thurner, A. Sandstrom, A new Derive for Instant Compaction Control, *Int. Conf. on Compaction, Paris, France, April 1980*
- [10] W. Ullah, E. T. Selig, Test of a Vibratory Plate Compactor, *Int. Conf. on Compaction, Paris, France, April 1980.*