A Trajectory Following Control for a Miniature Wheel Loader

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Abstract

To implement an autonomy upon a wheel loader is emphasized in this research work. Such an autonomy will contribute in assisting human operator of the wheel loader. The present authors have built a miniature wheel loader (MWL) whose name is "Yamazumi", on which a trajectory following control scheme has been being developed. In this report, a kinematics analysis and a proposal of a trajectory following scheme are presented. Behavior of MWL after simple test running is also illustrated.

1 Introduction

A wheel loader (WL) is one of the heavy machines at the construction site. There is a large shovel in front of WL to load gravel onto a dump truck from a pile of gravel. The wheel loader has the articulated structure with front and rear bodies which are connected at a articulated (center) joint. Each body has a pair of wheels. It has such a special kinematics that distance from the center joint to each wheel axle in rear and front bodies is the same. Normally, all the wheels are actuated by an engine, and the center joint is also actuated for WL to be steered. This type of steering is called as an articulated steering.

Emphasis of this research work is to put an autonomy upon a wheel loader. That is, the final goal is to accomplish autonomous behavior of the wheel loader. As a first step to develop an autonomy on the wheel loader, the present authors have built a miniature wheel loader (MWL) named "Yamazumi" whose scale is 1/10 of a real wheel loader (Figure 1). They have also implemented a trajectory following control scheme on "Yamazumi". This MWL has the same actuator configuration with the real one, *i.e.* one DC motor is assigned to steer MWL and the other DC motor is assigned to drive all the wheels. Driving power for all the wheels is distributed via propeller shaft and differential gears.

A kinematics analysis and a trajectory following control scheme for the miniature wheel loader is presented in the following sections.

13th ISARC

2 Kinematics analysis of the wheel loader

Figure 2 illustrates a fundamental kinematic configurations of the wheel loader (WL). Let us put $x_f - y_f$ coordinate system on the front body of WL whose origin is placed at intersection of the wheel axle and center axis of the WL. Let us also put $x_r - y_r$ coordinate system on the rear body. If we assume that there is no lateral slippage of WL, tangent velocity vector of the front (rear) body $v_{f(r)}$ points to the direction of $x_{f(r)}$ axis. Let $\omega_{f(r)}$ be angular velocity at the origin of the front (rear) body coordinate system $x_f - y_f$ ($x_r - y_r$). Now, velocity vector at the center joint in the front body coordinate system fv_c can be represented as a linear combination of a vector which points to $-y_f$ direction with magnitude $l\omega_f$ and a vector v_f as illustrated in Figure 2(b). Similarly, velocity vector at the center joint in the represented as a linear combination of a vector with magnitude $l\omega_r$ and a vector v_r as illustrated in Figure 2(c). Vectors fv_c and rv_c must be identical, since the front and rear body are connected at the center joint. Next equation can be derived from this constraint as follows:

$${}^{f}\boldsymbol{v}_{c} = \begin{pmatrix} v_{f} \\ -l\omega_{f} \end{pmatrix} = \begin{pmatrix} \cos\sigma & \sin\sigma \\ -\sin\sigma & \cos\sigma \end{pmatrix} {}^{r}\boldsymbol{v}_{c} = \begin{pmatrix} \cos\sigma & \sin\sigma \\ -\sin\sigma & \cos\sigma \end{pmatrix} \begin{pmatrix} v_{r} \\ l\omega_{r} \end{pmatrix}.$$
(1)

Therefore, ω_r and v_f can be derived from (1) as follows:

$$\omega_r = \frac{-\omega_{st} + v_r \sin \sigma}{l(1 + \cos \sigma)} \tag{2}$$

$$v_f = v_r - l\omega_{st} \frac{\sin\sigma}{1 + \cos\sigma}.$$
(3)



Figure 1: A miniature wheel loader "Yamazumi"

-700-

Let us notice the relation:

$$\omega_{st} \equiv \dot{\sigma} = \omega_f - \omega_r. \tag{4}$$

(1)

and assume that steering angle σ and steering angular velocity ω_{st} are obtained from the measurement. In this case, v_f , v_r , ω_f and ω_r can be derived from (2), (3) and (4) if at least one of v_f , v_r , ω_f and ω_r can be measured.

Even when σ and $\dot{\sigma}$ can not be measured, however if we could assume that ω_f and ω_r are measured, $\dot{\sigma}$ is derived from (4) and its integral produces σ . Therefore, v_f and v_r can be derived from (2) and (3).

To keep track of the position of WL, a representative point P_o on WL must be assigned. The present authors placed such a point P_o at an intersection point of trajectory arc and radius of the trajectory arc passing through the center joint as illustrated in Figure 3. When v_f , v_r , ω_f and ω_r are obtained, angular velocity ω_o at a point P_o is derived from (5):

$$\omega_o = \frac{1}{2}(\omega_f + \omega_r). \tag{5}$$



Figure 2: MWL kinematics

Tangent velocity \boldsymbol{v}_o at a point P_o is derived from (6):

$$\boldsymbol{v}_o = {}^f \boldsymbol{v}_c \cos \frac{\sigma}{2} = {}^r \boldsymbol{v}_c \cos \frac{\sigma}{2}.$$
 (6)

Where,

$$|{}^{f}\boldsymbol{v}_{c}| = |{}^{r}\boldsymbol{v}_{c}| = (v_{f}^{2} + (l\omega_{f})^{2})^{\frac{1}{2}} = (v_{r}^{2} + (l\omega_{r})^{2})^{\frac{1}{2}}.$$
(7)

Therefore, the position of WL $P_o = (x_o, y_o, \theta_o)$ at time t can be represented as follows:

$$\theta_o = \theta_{o\phi} + \int_0^t \omega_o(\tau) d\dot{\tau}$$
(8)

$$x_o = x_{o\phi} + \int_0^t v_o(\tau) \cos \theta_o(\tau) d\tau$$
(9)

$$y_o = y_{o\phi} + \int_0^t v_o(\tau) \sin \theta_o(\tau) d\tau.$$
(10)

Where $(x_{o\phi}, y_{o\phi}\theta_{o\phi})$ is an initial position of WL. (8), (9) and (10) can be recognized as an odometry of WL.

3 Trajectory following control scheme of MWL

Here, we consider the control of miniature wheel loader (MWL) "Yamazumi" when the desired trajectory is given. The fundamental idea of the trajectory following control



Figure 3: Representative point on WL



Figure 4: Error coordinate system

scheme for MWL is that reference steering angular velocity ω_{st}^{ref} and reference velocity v^{ref} to be followed by feedback controller for each actuator are determined by some "rules" which take account of a "position error" between the estimated position (x_o, y_o, θ_o) based on odometry and desired trajectory. The position error is defined as follows (Figure 4):

- 1. Find the nearest point (x_{trj}, y_{trj}) on the desired trajectory from (x_o, y_o) .
- 2. Let ξ axis be a tangent line of the desired trajectory at (x_{trj}, y_{trj}) .
- 3. Let η axis be perpendicular to ξ axis and pass through (x_{trj}, y_{trj}) .
- 4. Let θ_{trj} be a direction of ξ axis.
- 5. Then, the position error (ξ, η, ϕ) is obtained by (11):

$$\begin{pmatrix} \xi \\ \eta \\ \phi \end{pmatrix} = \begin{pmatrix} \cos \theta_{trj} & \sin \theta_{trj} & 0 \\ -\sin \theta_{trj} & \cos \theta_{trj} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_o - x_{trj} \\ y_o - y_{trj} \\ \theta_o - \theta_{trj} \end{pmatrix}$$
(11)

Let κ_{trj} be a curvature of the desired trajectory at (x_{trj}, y_{trj}) . When MWL is circling with curvature κ_{trj} , steering angle σ_{trj} must fulfill the relation:

$$\kappa_{trj} = \frac{\tan \frac{\sigma_{trj}}{2}}{l}.$$
(12)

13th ISARC

Therefore, desired steering angle σ_{trj} is obtained by:

$$\sigma_{tri} = 2 \tan^{-1}(l\kappa_{tri}). \tag{13}$$

The outline of the "rules" to specify reference velocity v^{ref} and steering angular velocity ω_{st}^{ref} are described as follows:

1. Reference velocity of MWL v^{ref} is obtained from (14):

$$v^{ref} = v_{trj} - k_{\xi}\xi, \tag{14}$$

where coefficient k_{ξ} is a gain to follow the desired position on the desired trajectory, and v_{trj} is a desired speed of MWL.

2. ω_{st}^{ref} is obtained from (15):

$$\frac{d}{dt}\omega_{st}^{ref} = k_{\sigma}(\sigma_{trj} - \sigma) - k_{\eta}\eta - k_{\phi}\phi - k_{\omega_{st}}\omega_{st}, \qquad (15)$$

where, the coefficients k_{σ} , k_{ϕ} and $k_{\omega_{st}}$ are chosen to stabilize the equation (15) under the assumption of ω_{st} is identical with ω_{st}^{ref} .

 v^{ref} and ω_{st}^{ref} are treated as reference inputs for driving and steering DC motors. Therefore, the present authors have designed a trajectory following controller as illustrated in Figure 5.

4 Implementation and experiments

The present authors have implemented the trajectory following controller on the miniature wheel loader which is illustrated in Figure 1. As is mentioned in Section 2, at least one of the parameters v_f , v_r , ω_f or ω_r must be measured in order to obtain all of v_f , v_r , ω_f and ω_r under the assumption that steering angle σ and steering angular velocity ω_{st} can be measured. However, there is no sensing devise to measure v_f , v_r , ω_f nor ω_r directly at present. Therefore, the present authors temporarily derived the position of MWL $P_o = (x_o, y_o, \theta_o)$ at time t from following formulas by using the steering angle σ_{mwl} and driving speed v_{mwl} which are measured by the shaft encoders of those motors:

$$\theta'_{o} = \theta_{o\phi} + \int_{0}^{t} \frac{v_{mwl}(\tau)}{l} \tan \frac{\sigma(\tau)}{2} d\tau \qquad (16)$$

$$x'_{o} = x_{o\phi} + \int_{0}^{t} v_{mwl}(\tau) \cos \theta_{o}(\tau) d\tau$$
(17)

$$y'_o = y_{o\phi} + \int_0^t v_{mwl}(\tau) \sin \theta_o(\tau) d\tau.$$
(18)



Figure 5: Controller of MWL

 (x'_o, y'_o, θ'_o) is used instead of (x_o, y_o, θ_o) in (11) in order to obtain error coordinate values (ξ, η, ϕ) in this implementation.

A torque constant of DC motor for steering MWL is 43.8 (mNm/A) and a gear ratio γ_{st} is 0.92 revolution of steering motor per steering angle in degree. A torque constant of DC motor for driving MWL is 52.5 (mNm/A) and a gear ratio γ_{dr} is 170 revolutions of driving motor per revolution of a wheel. The reference current for each motor is determined by traditional PI control:

$$i_{dr}^{ref} = k_{drp}(\omega_{dr}^{ref} - \omega_{dr}) + k_{dri} \int_0^t (\omega_{dr}^{ref} - \omega_{dr}) dt,$$
(19)

$$i_{st}^{ref} = k_{stp}(\omega_{st}^{ref} - \omega_{st}) + k_{dri} \int_0^t (\omega_{st}^{ref} - \omega_{st}) dt$$
(20)

where ω_{dr}^{ref} is obtained by:

$$\omega_{dr}^{ref} = \frac{\gamma_{dr}}{R} v^{ref}, \tag{21}$$

R is a radius of the wheel. In this implementation, $k_{drp} = 320 \ (mA \ sec/deg.), \ k_{dri} = 25 \ (mA/deg.), \ k_{stp} = 1250 \ (mA \ sec/deg.) \ and \ k_{sti} = 100 \ (mA/deg.).$

Other parameters were set as follows: $k_{\xi} = 0 \ (sec^{-1}), \ k_{\eta} = 2.0 \ (deg./mm \ sec^{-2}), \ k_{\phi} = 1200.0 \ (deg./rad \ sec^{2}), \ k_{\omega_{st}} \ (sec^{-1}), \ k_{\sigma} = 15.0 \ (sec^{-2}).$

Figures 6, 7 and 8 illustrate the behaviors of test running of MWL In Figure 6, the desired trajectory to be followed by MWL is given by a line which is y = 0(mm) from x = 0(mm) to x = 600(mm), and then y = -300(mm). Figure 7 illustrates a case that the desired trajectory is given by a line which is y = -300(mm). The last Figure 8 illustrates a case that the desired trajectory is $y = x \tan \frac{\pi}{6}$. In all these three Figures, initial position of MWL is at the origin, the initial speed is 0(mm/sec) and the desired speed of MWL v_{trj} is 100(mm/sec). In Figures 6, we can observe that the trajectory based on odometry converged on the line which is expected to be followed. However, real trajectory has a error from the trajectory of odometry. It may be because equations (17), (18) and (16) are used instead of (9) (10) and (8). After sensors to measure ω_f or ω_l are equipped, the real trajectory must be compared with the trajectory based on odometry (9), (10) and (8).

5 Conclusions

In this paper, kinematics analysis for the wheel loader and a trajectory following scheme are presented. Real trajectories of miniature wheel loader "Yamazumi" after simple test runnings are also illustrated. Based on the implementation of MWL by the present authors, the behaviors and performances of MWL which is controlled by proposed control scheme must be investigated more precisely against the several kinds of trajectories. To specify the arbitrary curved trajectory to be followed by MWL, some command system









-707-



Figure 8: Test running [case 3]

to give such a trajectory must be developed. These problems will be addressed in the future.

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