An Analytic Model Combining Monte Carlo Simulation and PSO in Estimating Project Completion Probability of Project Durations and Costs

Henyi Jen[1*] and Yu-chiun Chen[2]

[1,2]Department of Industrial Engineering and Management, Yuanze Univerisity, 135 Yuan Tung Road, Jhongli, 320, Taiwan; PH: +886-3-4638800; Email: henyi@saturn.yzu.edu.tw

Abstract

Project uncertainties are always the reason of project delay or budget overrun. Especially in tight schedule or project crashing, it is hard to balance both project duration and costs. Past research focused on the optimal schedule and costs, without knowledge of its on-time, within-budget completion risk. This research provides an analytical model by first using PSO heuristic algorithm to find the minimum project costs under time constraint. Monte Carlo Simulation is then implemented to build a completion probability table of time/cost combinations. The time and cost from PSO method is compared with the probability matrix. An analysis is provided as a demonstration.

Keywords: Time-cost trade-off problem, PSO, Monte Carlo Simulation, Probability Matrix

1. Introduction

In project management, time and cost are two control factors. Project should be delivered in time. However, project execution is often impacted by the uncertainties and thus may delay the committed project deadline. This delay not only taints the reputation of project manager, but in some cases, the project owner may claim liquidation damages from project performer. High penalty costs and loss of revenue would be ensued. Project performer usually concerns costs more than time in taking projects; nonetheless, only when both time and costs are concerned simultaneously and equally, can a project be taken with minimal chance of loss of profit.

The problem to find the optimal schedule and cost combination of a project is NP-hard. Most scholars approach this type of problems by using heuristics to find better solutions. This study approaches this problem from risk perspective by using the project completion probability of specified time and costs to see if it is feasible for a project performer to take a project. A combined Monte Carlo simulation with Particle Swarm Optimization (PSO) heuristic will be used in the study to evaluate the completion probability of a project under various time and costs combinations.

2. Previous Study

Project activity cost varies with its duration and resource inputs. On the other hand, different resource inputs and costs will vary the activity duration as well. This is categorized as Time-Cost Trade-Off Problem (TCTP). Kelly (1961) assumes the relationship between activity time and cost to be linear to reduce the difficulty in solving the TCTP. Meyer and Shaffer (1963), Patterson and Huber (1974) suggest to use mixed integer programming technique to handle the TCTPs that have discrete and linear relationship between time and costs.

Project execution is full of many uncertain factors. Examples include unforeseen bad weather that causes the interruption of the project progress; the project owner may request scope change and the change order due to unknown job site conditions. Even the labor and the equipment have inherent variability. The materials may be delivered irregularly or not sufficient materials when needed. The simplest method to process the uncertainties is to use Monte Carlo simulation (Diaz and Hadipriono, 1993). Other approaches also include probabilistic TCTP or the combination of both deterministic and probabilistic TCTP (Laslo, 2003; Ke and Liu 2005) and heuristic methods (Daisy et al. 2004, Yang 2006). Yang (2006) introduces PSO heuristic in the project crashing analysis. Total minimized project costs can be found efficiently under specific time window should the activities be crashed in three forms: discrete, piecewise-linear, and non-linear.

3. Research Methodology

The successful implementation of PSO heuristic into project crashing analysis with various types of activity encourages the authors to further the study into the analysis of optimal project time and costs by performing the combined Monte Carlo simulation and PSO heuristic method. This combined method is executed according to the following three steps.

3.1 Step 1: Use PSO to find the minimum project direct costs with specific time constraint

In the project, for any activity Ai, the direct activity cost is Ci, the objective function is to minimize the total costs of the summation of all activities' costs.

$$\text{Min } \Sigma Ci \quad (1)$$

subject to:

$$ESi+ti-ESj \leqq 0 \quad (2)$$

$$D=\max\{ESi+ti\} \leqq \overline{D} \quad (3)$$

$$ESi,\ ti>0 \quad (4)$$

$$Ci=fi\ (ti) \quad (5)$$

Equation (2) states that the earliest start time of activity Ai plus its time duration cannot exceed the earliest start time of its following activities. Equation (3) limits the time of last finished activity to be less than or equal to the specified date $\overline{D}$. Equation (4) states the earliest start time and the time duration for every activity should be greater than zero. Equation (5) indicates that the activity cost is the function of time. As previously mentioned, the time and cost relationship is complex and the study uses the definitions from Yang (2006), namely, the three various time and cost relations—discrete, piecewise-linear and non-linear functional form to reflect the different impact of time to the activity costs.

PSO heuristic is in play to handle the complex relation stated in equation (5), the solving procedure is as below:

1. Generate Initial Random Solution

Let xij be the position of j particle of activity Ai, where
i =1,2,…,N, and N is total number of activities,
j =1,2,…,M, and M is the number of particles

xij can be generated from uniform distribution function in the range between 0 and 1. In PSO term, the xij is encoded. Each xij stands for the time duration for Ai, or is decoded. For example, if xij=0.7, and if the upper and lower bounds for xij are 20 and 10 days respectively, xij is decoded as 10+0.7*(20-10), or 17 days. The positions of PSO particles can be decoded to the time durations of all activities. The activity costs can be calculated from equation (5). When the time and duration of each activity are decided, the project time can be derived using Critical Path Method (CPM) and its respective total costs can be summed up. This project total cost will become the initial solution.

2. Define Fitness Function

The total project cost is defined as the fitness function of PSO heuristic. Please note the cost outlay is only effective when the project duration should be less than or equal to $\overline{D}$, as stated in equation (3)

3. Update the velocity vector

PSO heuristic adjusts xij values to be more suitable through the update of velocity vectors. Following is the velocity vector and the updating mechanism.

$$Vid\_new=\omega * Vid\_old+\varphi i * (Pid-xid)+ \varphi g * (Pgd-xid) \quad (6)$$

$$Xid\_new=Xid\_old+Vid\_new \quad (7)$$

$$Vid\_old=Vid\_new \quad (8)$$

$$Xid\_old=Xid\_new \quad (9)$$

Here:

Vid_new: Velocity vector for i particle in d dimension (new)

Vid_old：Velocity vector for i particle in d dimension (old)

Xid_new：Position vector for i particle in d dimension (new)

Xid_old：Position vector for i particle in d dimension (old)

ω:inertia weight, φi and φg: parameters of weight, it is usually within [0,2]

Pid：Best fitness value of individual particle

Pgd：Best fitness among all particles

In the beginning, xij and the velocity vectors are generated randomly. Equation (6) is composed of three parts: the inertia of particle's previous behavior, as indicated by inertia weight ω; the cognitive consistency part, as indicated by fine-tuning the position (solution) based on the current position and the best fitness value of the particle; the social influence part, as indicated by the imitating behavior of the particles. Φi equals to c1*rand() and φg equals to c2*rand(), where c1 and c2 are accelerating constants; rand() stands for the random number generated from the uniform distribution between [0,1]. Equations (7), (8) and (9) update the position and velocity vectors.

Since the generated velocity vectors may be too large to control, a maximum velocity Vmax is defined to limit the range of the velocity vectors. In addition, the xij value should be kept between 0 and 1; therefore, the value of Vmax cannot be too large to reduce the solving power of the algorithm and the slower convergence.

4. Check the stop criterion

In order for the algorithm to better approximate the optimal value, the number of iterations plays a crucial role. Here the number of iterations is the stop criterion and more iteration is preferred; however, the efficiency may be compromised. Various iteration choices can be experimented to compare the convergence speed.

5. Find the optimal solution

The position (solution) calculated through PSO algorithm which satisfies both the stop criterion and constraints is converted to project direct cost and is regarded optimal.

3.2 Step 2: Calculate the completion probability through Monte Carlo simulation

Monte Carlo simulation is a widely used simple method based on the theory of large number. It is very suitable in finding the estimates of some non-linear, non-analytic problems. Although PSO algorithm can help in finding the optimal costs within specific time constraint, however, the optimal cost may hardly be achieved since it is pushing the envelope to the limit with little tolerance to the uncertainties. Thus Monte Carlo simulation is introduced in order to handle the uncertainties. Following are the applied steps using the Monte Carlo simulation.

For each time constraint:

- Set the total number of simulations N, the counter N' =0
- Set the time constraint
- Input the functions of time and cost for each project activity
- Generate the time and cost for each activity
- Calculate the total project duration
- If project duration ≦ time constraint, N' =N'+1, record the project direct costs
- Calculate Pc=N' / N, which is the completion probability for the specific time constraint

3.3 Step 3: Tabulate the probabilities for various time/cost combinations

A completion probability table is prepared from the values of the minimal project direct cost through PSO in step 1 and the ones with the simulation results from Monte Carlo simulation in step 2.

4. A Numerical Case

An underground sewage pipeline project constructed in Taiwan is used as a numerical case study. Figure 1 shows the project network.

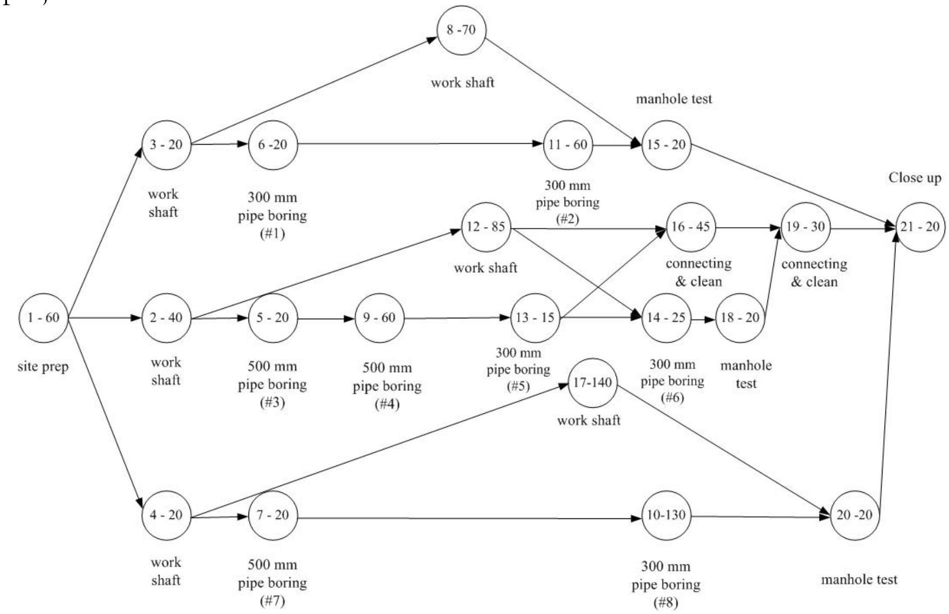

Figure 1. Network Diagram of the Underground Sewage Pipeline Project

Assume that the possible duration of this project are 220, 230, 240, 250, 260, 270, 280 and 290 days; as for the project costs, 3710, 3730, 3760, 3790, 3820, 3850, 3880, 3910, 3940, 3970 (in the unit of 10 thousand NT) are given. It is also common in project scheduling to assume the activity time distribution to be beta distribution. The time/cost relation is modeled as a quadratic function as proposed by Li et al. (1999). The quadratic functional form is that of $C(i)=aiT(i)^2+bi$, where C(i) is the cost and T(i) is the time; ai and bi are constants. The functional relationship for each activity has its own functional form is built up from field data. Finally, a complete table showing the relationship of every activity is established.

Step 1. Estimate the minimal project direct cost using PSO heuristic

Based on the assumed time durations of the project, PSO algorithm is performed with following settings: (iterations=500, number of particles=20, Vmax=5, initial inertia weight=1.5, final inertia=0.5, range of time duration: 240~290 days.) The minimal project direct costs for this time range are derived and the curve is drawn, as shown in Figure 2.

Step 2: Completion Probability by Monte Carlo Simulation

According to step 2 as mentioned in the previous section, number of simulations is set to be 10000, which can provide a precision of 4^{th} digit.

Step 3：Tabulate the project completion probability for various time/cost combinations

As shown in Table 1, the probability matrix based on Monte Carlo simulation is generated. The value in the cell indicates the completion probability under certain time/cost constraint.

4.1 Case Analysis

The completion probability, as seen in Table 1, shows the chance of successful completion under certain time/cost constraint. Often, the project performer is more concerned about the profit margin, but overlooks the risk of its inability to carry out the project under tight time constraint. It is understandable that the more resources being put in the project, the faster the project can be carried out. However, as can be seen from Table, there is a region where hardly any time/cost combination can generate higher completion probability, or higher chance of success completion of project. This probability matrix can be further divided into three regimes: the first regime with cell values very close to zero and it indicates the situation that hardly these time/cost combinations should exist since the success chance is minimal; the second regime includes the cell value between (>0) and 0.6 and it indicates the success chance varies a lot from minimal to improving. Finally, the third regime includes cell values over 0.6 and it indicates very high success chance and promising to complete the project within these time/cost combinations. Therefore in the studied project, in order to have a successful and promising result, the time should be above 280 days and the costs should be above 39,100 thousand NT.

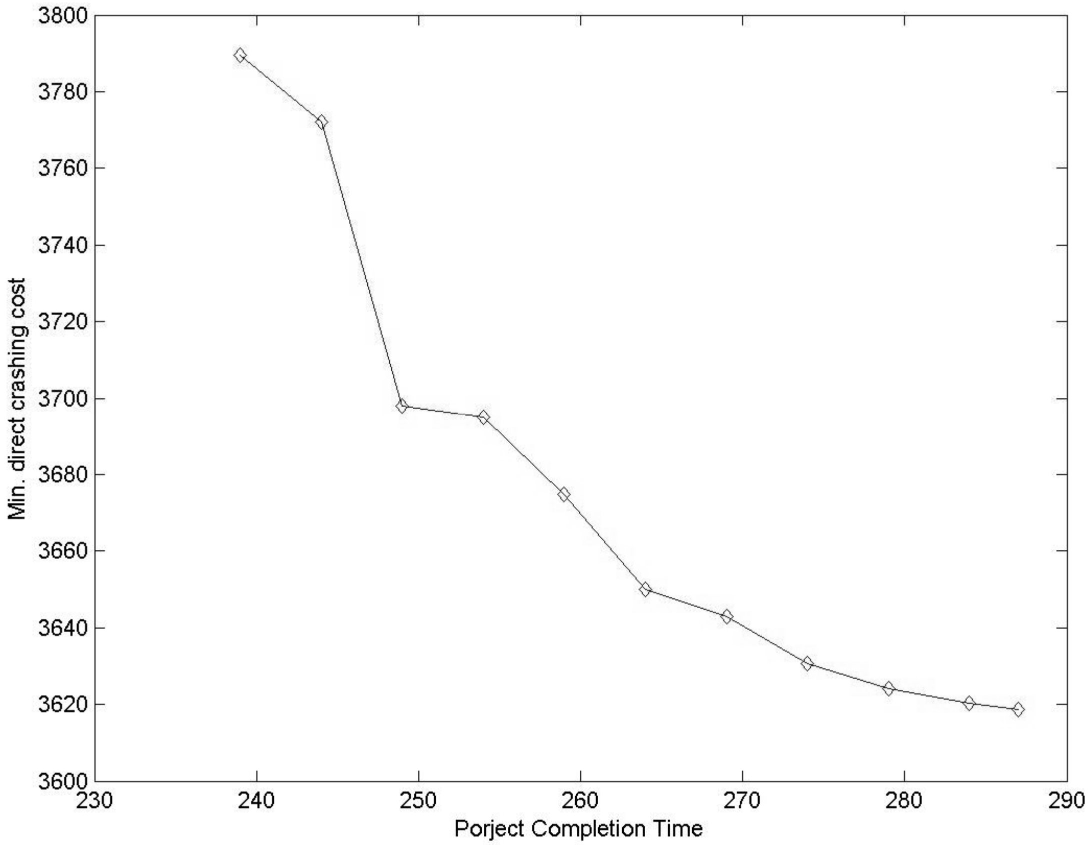

Figure 2. Activity Time and Cost Relationship (Beta Distribution)

Table 1. Completion Probability Matrix (Beta Distribution)

		Time Constraint (in days)							
		<=220	<=230	<=240	<=250	<=260	<=270	<=280	<=290
Project Direct Costs in 10,000 NT	<=3710	0	0	0	0	0	0	0	0
	<=3730	0	0	0	0	0.0002	0.0002	0.0002	0.0002
	<=3760	0	0	0	0	0.0025	0.0034	0.0034	0.0034
	<=3790	0	0	0	0.0014	0.0519	0.0669	0.0669	0.0669
	<=3820	0	0	0	0.0338	0.3425	0.3903	0.3904	0.3904
	<=3850	0	0	0	0.1557	0.7765	0.8398	0.8399	0.8399
	<=3880	0	0	0.0003	0.2348	0.9275	0.992	0.9921	0.9921
	<=3910	0	0	0.0005	0.2412	0.9354	0.9999	1	1
	<=3940	0	0	0.0005	0.2412	0.9354	0.9999	1	1
	<=3970	0	0	0.0005	0.2412	0.9354	0.9999	1	1

5. Conclusions and Recommendation

The study proposes a framework incorporating the PSO heuristic and Monte Carlo simulation to analyze the project completion probabilities among various time/cost constraints. From the numerical case study, it can be found that in the most cases, the project direct costs from PSO under time constraint are in the lower completion probability regime. Often the project direct costs derived from PSO are usually very optimistic in executing the projects.

In the study, it is assumed that the executing sequence of project activities is fixed and cannot be adjusted should there be resource constraints or fast-tracking. For project crashing problem, it is suggested that the resource conflicts and modified executing sequence can be further studied to fit into more realistic needs.

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