APPLICATION HEIGHT CONTROL SYSTEM OF A TRACTOR
CARRYING UP WEAK SLOPE AT DRIVING STATE

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ABSTRACT

When a tractor is carrying up a scraper under transportation of soil on a weak slope, the application height of the effective driving force of the tractor could be controlled to obtain the maximum drawbar pull. Several tractive performances of a 50 kN flexible rubber tracked vehicle trailing up a given weak sandy slope terrain of Cone Index 95 kPa have been simulated. As the results, it is clarified that the optimum application height of the drawbar pull to take the maximum optimum effective driving force of the tractor carrying up the weak sandy slope terrain of 0, \( \pi/18 \), \( \pi/9 \), and \( \pi/6 \) rad results in about 50, 60, 20 and -40 cm for the vehicle eccentricity 0, 0.01, respectively. Then, the automated control system of the optimum application height should be considered in accordance with the tractive effort jointed to the scraper.

1. INTRODUCTION

At earth moving site in highway construction, the position of applied forces of the drawbar pull to a tractor trailing up a scraper on a weak slope terrain influences predominantly the relations between driving force, effective driving force, sinkage, vehicle trim angle, eccentricity of resultant normal force, and slip ratio. Also, the flexibility of a rubber track belt affects the shape of contact pressure distribution and it is dominant not only over the thrust but over the amount of slip sinkage at the rear sprocket.

The purpose of this study is to verify the traffic performances of a flexible tracked vehicle carrying up a weak sandy slope terrain at driving state for establishing the fundamental application height control system of the tractor.

Here, several tractive performances of 50 kN weight, 320 cm track length and 25 cm track width tractor trailing up a scraper on the weak sandy slope terrain of 95 kPa Cone Index and the given terrain-track system constants have been analysed by use of the simulation program which predicts the method of the optimum effective driving force determination from the thrust developed on the interface between track belt and slope terrain and the locomotion resistance and the inclined vehicle weight etc. Afterwards, several relations between the application height of the effective driving force on the tractor and tractive performance are verified for various slope angles and vehicle eccentricities. Then some positioning system of the attachment of tractor to the optimum application height could be presented by use of some drawbar pull or torque sensor and rut depth sensor, etc.

2. TERRAIN-TRACK SYSTEM CONSTANTS

Fig. 1 shows the flexible track model plate of which the grouser
height $H$ is 6.5 cm. the pitch $G_p$ is 14.6 cm and the grousers shape is an equilateral trapezoid of $\alpha = \pi / 6$ rad. contact length $L=2$ cm. The dimensions of track model is the length $D=73$ cm, the width $B=25$ cm. And the track belt is made of natural rubber of spring hardness 62. The weak terrain is composed of a loose sandy soil of which the bulk density is 1.44 Mg/m$^3$. the water content is 2.38 %, the relative density is 44.0 %, the average grain size is 0.78 mm, the coefficient of uniformity is 12.0, and the Cone Index is 95.1 kPa.

To predict the interaction between track plate and terrain, next experimental equations could be presented:

\[
p = k_1 s_o^{n_1} \quad \text{(1)} \\
p = k_1 s_v^{n_1} - k_2 (s_v - s_o)^{n_2} \quad \text{(2)}
\]

for track plate loading and reloading test, where $p$ (kPa) is the contact pressure, $s_o$ (cm) is the amount of static sinkage and $s_v$ (cm) is the static sinkage at the beginning of reloading state.

\[
\tau = (m_c + m_f p)(1 - \exp(-a j)) \quad \text{(3)}
\]

for track plate traction test, where $\tau$ (kPa) is the shear resistance and $j$ (cm) is the amount of slippage of grousers.

\[
s_s = c_0 p^{c_1} j_s^{c_2} \quad \text{(4)}
\]

for track plate slip sinkage test, where $s_s$ (cm) is the amount of slip sinkage and $j_s$ (cm) is the amount of slippage of soil.

Those ten terrain-track system constants are shown in Table 1.

### Table 1 Terrain-track system constants

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Constant Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static pressure sinkage test</td>
<td>$k_1 = 8.526$</td>
</tr>
<tr>
<td></td>
<td>$k_2 = 32.63$</td>
</tr>
<tr>
<td></td>
<td>$n_1 = 0.866$</td>
</tr>
<tr>
<td></td>
<td>$n_2 = 0.632$</td>
</tr>
<tr>
<td>Shear deformation test</td>
<td>$m_c = 0$ kPa</td>
</tr>
<tr>
<td></td>
<td>$m_f = 0.769$</td>
</tr>
<tr>
<td></td>
<td>$a = 0.244$ 1/cm</td>
</tr>
<tr>
<td>Slip sinkage test</td>
<td>$c_0 = 1.588$</td>
</tr>
<tr>
<td></td>
<td>$c_1 = 0.075$</td>
</tr>
<tr>
<td></td>
<td>$c_2 = 0.240$</td>
</tr>
</tbody>
</table>

#### 3. Force and Energy Balances

Fig. 2 shows the vehicle dimensions of tractor and several forces acting on the two track belts carrying up the weak terrain of slope angle $\beta$ at driving state. From the force balances, the effective driving force $T_4$ is given as

\[
T_4 = \frac{T_3}{\cos \theta_t} - \frac{W \sin (\theta_t + \beta)}{\cos \theta_t} - T_2 \quad \text{(5)}
\]

where $T_3$ is the thrust acting along the contact part of track belt. $T_2$ is the land
locomotion resistance acting on the front part of undercarriage at depth z. W is the vehicle weight and $\theta_{\text{trim}}$ is the trim angle of the vehicle.

The energy balances between effective input energy $E_1$ supplied by the driving torque $Q = T_1 R$, and output energy which is the sum of the compaction energy $E_2$ to make a rut under the tractor, the slippage energy $E_3$ developed under the track belt, the effective driving force energy $E_4$ and the potential energy $E_5$ are given as follows:

$$E_1 = E_2 + E_3 + E_4 + E_5$$

where

$$E_1 = T_3 V'$$
$$E_2 = T_2 V'(1 - i_d)$$
$$E_3 = T_3 (1 - \frac{1 - i_d}{\cos \theta_{\text{trim}}}) V' + W V'(1 - i_d) \tan \theta_{\text{trim}} \cos \beta$$
$$E_4 = T_4 V'(1 - i_d)$$
$$E_5 = W V'(1 - i_d) \sin \beta$$

$V$ is the vehicle speed in the direction of slope terrain surface and $V'$ is the rotation speed of track belt.

4. FLOW CHART

Fig. 3 shows the flow chart to calculate the tractive performances of the flexible rubber tracked tractor carrying up the weak slope terrain at driving state. First of all, the vehicle dimensions of the tractor as shown in Table 2 and the terrain-track system constants should be read as input data.

At rest, the relation between contact pressure $p_0(X)$ and amount of sinkage $s_0(X)$ could be calculated from the constants $k_1, n_1$ and $k_2, n_2$. At driving state for a given slip ratio $i_d$, the resultant normal force $F_n$ and the distribution of contact pressure $p_1(X)$ and amount of sinkage $s_1(X)$ of the flexible track belt are repeatedly calculated until the eccentricity $e_1$ is determined, in dependence on the three kinds of flow system. The flexibility of track belt depends on the initial track tension $H_0$, the ground reaction $p_1(X)$ and shear resistance $\tau_1(X)$, so that the distribution of track tension $T_0(X)$ is given as the summation of $H_0$ and the thrust which is calculated as

<table>
<thead>
<tr>
<th>Table 2 Vehicle dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle weight</td>
</tr>
<tr>
<td>Width of track belt</td>
</tr>
<tr>
<td>Contact length of track belt</td>
</tr>
<tr>
<td>Mean contact pressure</td>
</tr>
<tr>
<td>Radius of front idler</td>
</tr>
<tr>
<td>Radius of rear sprocket</td>
</tr>
<tr>
<td>Radius of track roller</td>
</tr>
<tr>
<td>Height of grouser</td>
</tr>
<tr>
<td>Grouser pitch</td>
</tr>
<tr>
<td>Interval of track roller</td>
</tr>
<tr>
<td>Eccentricity of gravity center of vehicle</td>
</tr>
<tr>
<td>Height of gravity center of vehicle</td>
</tr>
<tr>
<td>Distance between central axis of vehicle and point acting effective driving force</td>
</tr>
<tr>
<td>Height of point acting effective driving force</td>
</tr>
<tr>
<td>Initial track tension</td>
</tr>
<tr>
<td>Rotation speed of track belt at driving state</td>
</tr>
</tbody>
</table>
the integration of $\tau_i(X)$.

The final values of $T_1 = T_3$, $T_4$ and $e_i$, the distributions of amount of slippage $j(X)$, normal pressure $p_1'(X)$, shear resistance $t_1'(X)$, amount of sinkage $s_i(X)$ and track belt tension $T_0(X)$, the energies $E_1$, $E_2$, $E_3$, $E_4$ and $E_5$, and the tractive efficiency of power $E_t$ can be determined for each slip ratio $i_d$. And the optimum effective driving force $T_4_{opt}$ at the optimum slip ratio $i_{d_{opt}}$ or the maximum effective driving force $T_4_{max}$ at the slip ratio $i_m$ can be determined.

At rest

$$|s_0(X) - s_{0j}(X)| \leq \varepsilon$$

At driving state

$$e_i < -1/6$$

$$-1/6 \leq e_i \leq 1/6$$

$$e_i > 1/6$$

$$P, P', p_{fri}, p_{i}(X)$$

$$S_{f01}, S_{r01}, S_{01}(X), \theta_{t01}$$

$$|s_{01}(X) - s_{01j}(X)| \leq \varepsilon$$

$$S_{f01} > H > S_{r01} \geq 0$$

$$S_{f01} > H > S_{r01} \geq H$$

$$S_{f1}, S_{r1}, S_{i}(X), \theta_{t1}$$

$$S_{f01}, S_{r01}, S_{01}(X), \theta_{t01}$$

$$S_{f1}, S_{r1}, S_{i}(X), \theta_{t1}$$

$$T_3 \begin{bmatrix} T_{mb} & T_{ms} \\ T_{rb} & T_{rs} \end{bmatrix}$$

$$T_1, T_2, T_4, P, S_{f1}, S_{r1}, \theta_{t1}, e_i$$

$$|T_3 - T_{3j}| \leq \varepsilon$$

$$T_0(X), E_1, E_2, E_3, E_4, E_5, E_t, j(X), p_{1}'(X), \tau_1'(X), s_i(X)$$

$$T_{4_{opt}}, i_{d_{opt}}, T_{4_{max}}, i_m$$

STOP

END

Fig. 3 Flow chart to calculate the tractive performances of a flexible tracked tractor carrying up weak slope terrain
5. TRACTIVE PERFORMANCES ON SLOPE

The tractive performances of the given rubber tracked tractor have been simulated for the loose sandy soil of slope angle \( \beta = \pi / 9 \) rad at driving state. The rotation speed of the track belt \( V \) is 100 cm/s and the initial track tension \( H_0 \) is 19.6 kN. As shown in Fig. 4, the driving force \( T_1 \) increases rapidly with \( i_d \) at lower slip ratio and does gradually at higher slip ratio, but the effective driving force \( T_4 \) decreases gradually with \( i_d \) after taking the maximum value \( T_{4\text{max}} = 20.1 \) kN at \( i_d = 23 \% \) due to the increasing locomotion resistance and drops suddenly over \( i_d = 90 \% \). The optimum effective driving force \( T_{4\text{opt}} \) is 19.2 kN and the effective driving force energy \( E_4 \) reaches the maximum value 1711 kNcm/s at \( i_{d\text{opt}} = 11 \% \), and the tractive efficiency of power \( E_t \) is 43.1 \%. Fig. 5 shows that the amount of sinkage \( s_{r1} \) increases monotonously with \( i_d \) and does rapidly over \( i_d = 90 \% \), and \( s_{r1} \) is always larger than \( s_{f1} \) due to the increasing amount of slip sinkage. Fig. 6 shows that the trim angle of vehicle \( \theta_{r1} \) increases gradually with \( i_d \), and the eccentricity of resultant normal force \( e_1 \) decreases gradually after taking the maximum value 0.0384 at \( i_d = 6 \% \) and drops suddenly over \( i_d = 90 \% \). Fig. 7 shows that \( E_1 \) increases rapidly with \( i_d \) at lower slip ratio and does gradually at higher slip ratio. \( E_2 \) and \( E_4 \) has a peak value at some slip ratio and reaches zero at \( i_d = 100 \% \) respectively. \( E_3 \) increases almost linearly with the increment of \( i_d \). and \( E_5 \) decreases linearly.
with \( i_d \) toward zero at \( i_d = 100 \% \). Fig. 8 shows that the tractive efficiency of power \( E_t \) decreases gradually with \( i_d \) after taking the maximum value 43.5\% at \( i_d = 9 \% \) and reaches zero at \( i_d = 99 \% \). Fig. 9 shows the contact pressure distribution under the flexible track belt at \( i_d = 10 - 90 \% \), in which the normal stress \( p_1'(X) \) and the shear resistance \( \tau_1'(X) \) have peak values under the track rollers. The amplitude of deflection of track belt varies from 0.33 mm to 2.39 mm at \( i_d = 10 \% \). They show some sinusoidal distribution, of which amplitude at front part of track belt is comparatively larger than that at the rear part due to the increasing thrust. Fig. 10 shows the distributions of track belt tension \( T_0(X) \) around the track belt at \( i_d = 10 - 90 \% \). \( T_0(X) \) increases almost linearly with the distance from the bottom dead center of front idler \( X \) and reaches the maximum value 35.5 kN at \( X = 0 \) for \( i_d = 10 \% \).

6. APPLICATION HEIGHT

Several relations between the optimum effective driving force \( T_{opt} \), the total amount of sinkage of rear sprocket \( s_r \), the eccentricity of resultant normal force \( e_r \), the trim angle of vehicle \( \theta_t \), the tractive efficiency of

![Fig. 7 Relations between energy \( E_1, E_2, E_3, E_4, E_5 \) and slip ratio \( i_d \) (\( \beta = \pi / 9 \) rad)](image)

![Fig. 8 Relation between tractive efficiency of power \( E_t \) and slip ratio \( i_d \) (\( \beta = \pi / 9 \) rad)](image)

![Fig. 9 Distributions of contact pressure (\( \beta = \pi / 9 \) rad)](image)
power $E_i$ and the height of point acting effective driving force $h_d$ have been calculated for the loose sandy soil of slope angle $\beta = \pi/9$ rad at driving state and for the same vehicle dimensions except the eccentricity of gravity center of vehicle $e = 0.05$, 0.00 and -0.05 and $h_d = -100$ to 100 cm at every 20 cm. Fig. 11 shows the relations between $T_{4opt}$ and $h_d$ for the given eccentricities. $T_{4opt}$ takes a maximum value at $h_d = -20$ cm for $e = 0.05$, at $h_d = 20$ cm for $e = 0.00$, and at $h_d = 60$ cm for $e = -0.05$. For a given depth of application of excavating force, the larger the value of eccentricity $e$ the larger the excavating force $T_{4opt}$ that could be obtained. In these cases, the maximum $T_{4opt}$ are less than the half of the vehicle weight $W$. Fig. 12 shows the relations between $s_{r1}'$ and $h_d$ for the given eccentricities: $s_{r1}'$ takes a minimum value at $h_d = -20$ cm for $e = 0.05$. at $h_d = 20$ cm for $e = 0.00$, and at $h_d = 60$ cm for $e = -0.05$. For $h_d$ larger than the height at minimum value of sinkage, $s_{r1}'$ increases gradually with the increment of $h_d$. However, for $h_d$ less than that, $s_{r1}'$ increases remarkably.

Fig. 10 Distributions of track belt tension $T_0$ around track belt ($\beta = \pi/9$ rad)

Fig. 11 Optimum effective driving force $T_{4opt}$ and height of application force $h_d$ for three kinds of eccentricity $e$ ($\beta = \pi/9$ rad)

Fig. 12 Total amount of sinkage of rear sprocket $s_{r1}'$ and height of application force $h_d$ for three kinds of eccentricity $e$ ($\beta = \pi/9$ rad)

Fig. 13 Eccentricity of resultant normal force $e_i$ and height of application force $h_d$ for three kinds of eccentricity $e$ ($\beta = \pi/9$ rad)
with the decrement of \( h_d \) due to the negative eccentricity \( e_i \) of the resultant normal force. In these cases, the average of minimum amounts of sinkage \( s_{ri} \) is about 1.4 cm depth. Fig. 13 shows the relations between \( e_i \) and \( h_d \) for the given eccentricities. In general, \( e_i \) increases monotonously with the increment of \( h_d \). For a given \( h_d \), the larger the eccentricity \( e \), the larger \( e_i \) that could be obtained, and every \( e_i \) is located within the middle third of the bottom track belt. Fig. 14 shows the relations between \( \theta_{t1} \) and \( h_d \) for the given eccentricities: \( \theta_{t1} \) decreases almost linearly with the decrement of \( h_d \) and reduces to some value at a large depth of excavation. Fig. 15 shows the relations between \( E_t \) and \( h_d \) for the given eccentricities: \( E_t \) takes a maximum value at the optimum application height \( h_{d_{opt}} \) for \( e = 0.05 \), at \( h_{d_{opt}} = 20 \text{ cm} \) for \( e = 0.00 \), and at \( h_{d_{opt}} = 60 \text{ cm} \) for \( e = -0.05 \). In these cases, the average of maximum \( E_t \) is about 43.6 %. These optimum heights of application force \( h_{d_{opt}} \) agree well with the heights which show the minimum sinkage \( s_{ri} \) and the maximum effective driving force \( T_{4_{opt}} \), due to the minimum land locomotion resistance.

As a result, it is clarified that \( h_{d_{opt}} \) depends on the eccentricity \( e \) of the vehicle gravity center.

7. SLOPE ANGLE

Several effects of slope angle of the loose sandy soil on the tractive performances of the given rubber tracked tractor have been simulated to make an application height control system. As an example, Fig. 16 shows the relations between effective driving force \( T_4 \) and slip ratio \( i_d \) for the vehicle eccentricity \( e = 0.00 \), at the optimum application height \( h_{d_{opt}} = 60 \text{ cm} \) for the slope angle \( \beta = 0 \text{ rad} \), at \( h_{d_{opt}} = 60 \text{ cm} \) for \( \beta = \pi /18 \text{ rad} \), and at \( h_{d_{opt}} = 20 \text{ cm} \) for \( \beta = \pi /9 \text{ rad} \), respectively. Fig. 17 shows the relations between total amount of...
sinkage of rear sprocket $s_{r,1}$ and slip ratio $i_d$ in correspondence with the same vehicle dimensions and slope angle, respectively.

In general, the optimum effective driving force $T_{4, opt}$ decreases remarkably with the increment of slope angle $\beta$ due to the increasing vehicle weight component $W \sin(\theta_1 + \beta) / \cos \theta_1$ in Eq(5), while the $s_{r,1} - i_d$ curve has almost the same relationship as shown in Fig.17 for various $\beta$ values.

The optimum application height $h_{d, opt}$ to maximize the effective driving force and the tractive power efficiency at the optimum slip ratio depends on the eccentricity of gravity center of the vehicle $e$ and the slope angle $\beta$. As shown in Fig.18, $h_{d, opt}$ decreases with the increment of $e$ and $\beta$, and then the positioning system of the attachment of tractor to $h_{d, opt}$ could be developed by use of some inclinometer to measure the slope angle $\beta$ of the running terrain.

8. CONCLUSIONS

To establish an optimum operation and robotization system of a flexible rubber tracked tractor carrying up weak slope terrain, some application height control system to obtain the maximum drawbar pull should be considered. Here, several tractive performances of a 50 kN weight tractor carrying up a loose sandy soil slope terrain have been simulated for various slope angles and vehicle eccentricities to find the optimum application height by use of the given terrain-vehicle constants. As the results, it is clarified that:

1. The optimum height of application of effective optimum drawbar pull to obtain the maximum value agrees well with the height, at which the amount of sinkage of rear sprocket takes a minimum value and the tractive efficiency of power takes a maximum value.

2. For the slope angle $\pi/9$ rad, the optimum application height results in about $-20$, $20$ and $60$ cm for the eccentricity of vehicle gravity center $0.05$, $0.00$ and $-0.05$, respectively.

3. The optimum effective driving force decreases remarkably with the increment of slope angle due to the increasing vehicle weight component toward the slope terrain surface, while the amount of sinkage slip ratio relation is almost unchangeable for the slope angle.

4. The optimum application height of the optimum effective driving force tends to decrease with the increment of eccentricity of vehicle gravity center and slope angle of terrain. For the eccentricity of vehicle gravity center $0.00$, the optimum application height for the weak sandy slope terrain of $0$, $\pi/18$, $\pi/9$ and $\pi/6$ rad results in about $60$, $60$, $20$
and - 40 cm. respectively.

(5) The automated control system of the optimum application height, i.e. the positioning system of the attachment of tractor could be developed by use of some inclinometer.

REFERENCES


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