Application of queuing theory in construction industry

Vjacheslav Usmanov [1]*, Čeněk Jarský [1]

[1] *Department of Construction Technology, FCE, CTU Prague, Czech Republic*
** Corresponding author (usmanov@seznam.cz)*

Purpose Each production process in construction is closely connected with the question of costs and deadlines. In every project an investor or customer, as well as the construction company, has to meet the planned completion date and the estimated costs associated with the construction. In practice, determining the duration of construction at minimum costs is still not based on the reliable calculation, and in the planning of costs, the connection between terms and financial costs is rarely taken into account. **Method** The queues theory examines systems with operating channels, where the process of queues formation takes place and subsequent servicing of the customers by servicing centers. The main objective of the queues theory is to determine the laws under which the system works, and further to create the most accurate mathematical model that takes into account various stochastic influences on the process. The entire construction process can be examined from the point of view of a customer who is waiting in the queue and is interested primarily in the waiting time, as well as from the point of view of servicing centers. A waiting element decides if you join the queue, or to go to another system entirely. In terms of servicing centers, the priority is to determine the occupancy of the channel and the probability of failure, including the time of repair. A servicing center should also reliably identify the time per customer service, taking into account the current construction task. **Results & Discussion** The present study demonstrates that it is possible to simulate the complex process of construction, containing hundreds of individual construction processes, mathematically and technically, with a number of simplifications, and then perform various calculations and changes for effective and long-term planning of construction. The mathematical simulation should show that some variants of machines combinations fail to accomplish the task under the given conditions, some will not be optimal in terms of costs or other parameters, other variants will be optimal in the view of costs required to fulfill the construction task. The simulation software allows a look at the results in graphical form or to export data to other programs. Application of the queues theory allows the introduction into the system waiting time the servicing elements and to approximate the mathematical model to a real working tasks on site.

***Keywords**: queues theory, optimal choice, machines combination, construction, simulation*

INTRODUCTION

Each production process in construction is closely connected with the question of costs and deadlines. An investor or customer, as well as construction companies themselves in every project have to meet the planned completion date and the estimated costs associated with the construction. In practice, determining the duration of construction at minimum cost is still not founded on a reliable calculation, and in the planning of costs, the connection between terms and financial costs is rarely taken into account.

At present, some programs are available for the analysis of building production and for scheduling with the help of network graphs, timetables and their optimization. A further supporting facility to optimize the prices is so called internal corporate guidelines; construction companies produce them depending on their own experience and statistical values. Some of these documents are then used to optimize the use of machinery and to create building plans. Other similar information instruments, especially for the optimization of construction processes, do not yet exist, or they are produced only in a small and evidently insufficient volume. In particular, the sphere of the optimal choice of machinery is still in its early stages of development, and a software, which would allow an easy optimal choice of machinery in terms of minimizing labour content and costs, fuel consumption, time of construction, environmental impact, etc., including relevant data, does not exist on the internet basis.

THE QUEUING THEORY

The current practice does not allow construction companies to perform detailed time-consuming calculations of optimization during formulation of their offers. An offer is usually focused on the contract price, which has to match the situation in the construction market. Optimization steps are therefore made only after the contract is signed. This study is devoted to the creation of a technical and mathematic model and to searching the methods leading to the optimization of construction processes (minimization of labor and costs, fuel consumption, time of construction, environmental impact, etc.) by means of special simulation software.

The queuing theory examines systems with operating channels, where the process of queues formation takes place and subsequently the servicing of the customers by servicing centers. The main objective of the queuing theory is to determine the regularities under which the system works, and further to create the most accurate mathematical model that takes into account various stochastic influences on the process. The entire construction process can be examined from the point of view of a customer who is waiting in the queue and is interested primarily in the waiting time, as well as from the point of view of servicing centers. A waiting element decides in what queue to be included or whether to go to another system entirely. In terms of servicing centers, the priority is to determine the occupancy of the channel and the probability of failure, including the time of repair. A servicing center should also reliably identify the time of customer servicing, taking into account the current construction task.

As a result of the application of the queuing theory, a mathematical model should provide the data regarding the optimal design of servicing centers and, at the same time, determine the number of customers taking into account the optimization parameters. The parameters, under which the construction process will be optimized, may be the following: time, number of failures, fuel consumption, financial costs, environmental impact etc.

The queuing theory appeared in the early 20th century. Fundamentals of the theory were developed by Danish mathematician Agner Krarup Erlang (1878-1929), who examined the development of call centres. According to D.G. Kendall[6], any system of the queuing theory can be classified according to the following combination of letters and numbers:

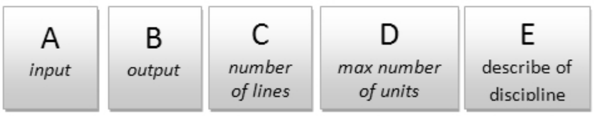

Fig.1. Classification of the queuing theory

where A – describes the input stream of elements, B – describes the probability distribution during service time, C – describes the number of service lines, D – specifies the maximum number of elements in the system, E – describes the queue discipline (finite, infinite, FIFO, LIFO, etc.).

The parameters "A" and "B": in place "A" and "B" may be presented by the following symbols:

- M – for exponential distribution,
- D – for constants (deterministic intervals),
- KK – for Erlang distribution of k-type,
- G – any distribution.

Basic structure of the queuing system is illustrated in Figure 2.

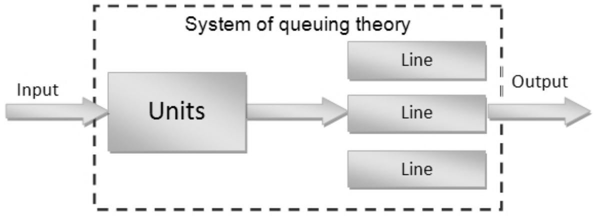

Fig. 2. Basic structure of the queuing system

A closed queuing system is shown in Figure 3.

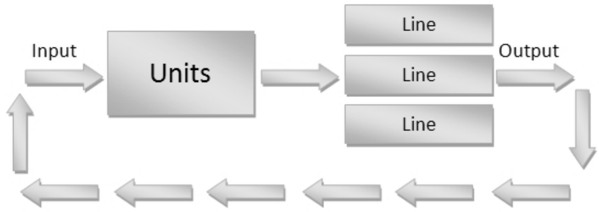

Fig.3. Closed queuing system

For the optimization of a construction process, a closed system is more convenient, where customers given a certain time after the service return back into the system and go to the queue again. Under a closed process there is understood a situation, where the source of requirements is final. The queue length is limited and the processing of customers' requirements is done according to the FIFO method (first in - first out).

As an example of the application in use there can be given an optimization of construction machinery at the stage of "earth works", where the role of servicing centers is performed by the loaders or excavators (number C) and the role of customers is played in the system by trucks or dumpers (number D). We investigate a system where D is greater than C.

For each construction process, a time unit can be determined according to the depth of view on the mathematical model of optimization, for example, minute, hour, shift, week, month etc. For a proper functioning of the mathematical model of the queuing theory application, the following range of conditions has to be met[5]:

- input of an element into the queue can occur at any moment of time;
- the number of inputs during the time interval depends on the length of the interval and the type of distribution of a servicing centre performance (e.g. uniform, power-series, falling or rising) and the scheme of a servicing machine performance is given by the parameters of the construction task; these are determined before the mathematical simulation and do not change during the mathematical modelling;
- the probability, that in the interval of the length δT occurs, more than one input converges to zero more quickly than the length of the interval δT;

- the average number of inputs per the unit of time is equal to λ.

To calculate the characteristics of the system, the following formulae have to be used. The service intensity can be determined as follows:

$$\rho = \frac{\lambda}{C \cdot \mu}, \qquad (1)$$

where: λ is a parameter of the exponential distribution, which characterizes the time spent by an element outside the operating system, for example, removal of soil to landfill and return of the truck back to the servicing centre; μ is a parameter of the exponential distribution, which characterizes the time spent by an element during service, for example, soil loading on a truck by a loader.

The probability of P_k function, that in the time interval of the length T number of elements $k= 1, ..., C$ enter the system, can be expressed as follows:

$$P_k = \frac{1}{k!} \cdot \left(\frac{\lambda}{\mu}\right)^k \cdot P_0, \qquad (2)$$

where P_0 is determined as follows:

$$P_0 = \frac{1}{\sum_{k=0}^{C} \frac{1}{k!} \cdot \left(\frac{\lambda}{\mu}\right)^k + \frac{1}{C!} \cdot \left(\frac{\lambda}{\mu}\right)^C \cdot \rho \cdot \frac{1-\rho^{D-C}}{1-\rho}}, \qquad (3)$$

After that we can easily calculate other properties of the system:

The average number of customers in service:

$$ES = \frac{\lambda}{\mu} \cdot (1 - P_D), \qquad (4)$$

The average number of customers in the queue:

$$EL = \sum_{l=1}^{D-C} l \cdot P_{C+l}, \qquad (5)$$

The average number of customers in the system:

$$EK = ES + EL, \qquad (6)$$

We can easily derive and use formulas for calculation of further parameters of the system, for example, system use, the average waiting time of an element in the queue inside and outside the service system. Different probabilities of elements' failure and idle times can be identified. After inclusion of other parameters in the mathematical model we can calculate further properties of the system, such as cost characteristics, duration of construction processes, environmental impact, fuel consumption, etc. After the introduction of all of the regularities in the mathematical model we can perform a variety of optimization tasks and propose an optimal choice of machines combination for a particular construction process according to various criteria.

SIMULATION SOFTWARE

For the mathematical modelling and simulation, special software was used[1]. The software allows an easy input of additional more precise parameters and coefficients into the model. A mathematical modelling was performed by the example of a construction phase "earth works". On the basis of the simulation results, an optimal design of machinery (excavators) for this task will be chosen from three possible options, as well as the optimal number of trucks for removal of soil. The decisive factor is the task fulfilment within a certain time with minimal financial costs. The volume of the task is constant and does not change. Other machinery, in this case the type of trucks (dumper), is predefined. The number of channels is based on the geographical conditions of the construction site, and, in our example, is given, N = 1. For simplicity, we will always choose the same kind of excavators for each channel.

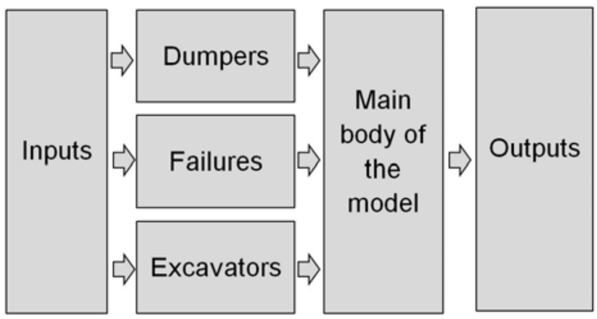

Fig. 4. Mathematical model of the system

The whole mathematical model can be divided into three parts: input, core of the model, and output. See Figure 4. The mathematical model contains several important subsystems for the calculation of machinery failure and for the determination of economic parameters of the system; see Figure 5.

Further, the input parameters of the mathematical model will be described:

- construction process: excavation, construction pit, figure 1;
- volume of the task: 6 000 m^3, workability class: 3, loosening coefficient K_n = 1.25, the total volume of soil Q_{exc} = 6 000 x 1.25 = 7 500 m^3;
- parameters of auto truck (dumper) are shown in table 1.: dumper, body volume – 15 m^3, the average travel time from site to landfill and back is 30 min., financial costs of machine operation: fixed costs – 3000 CZK, variable costs – 1 000 CZK /hour, the maximum number of dumpers –

15; the failure rate is 2%/day; the average repair time – 2 hours; time of delivery to servicing machine is 2 minutes.

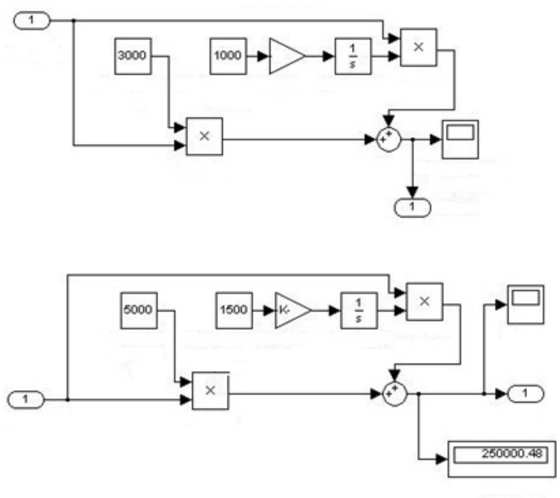

Fig. 5. Subsystems of the mathematical model in SW[1]

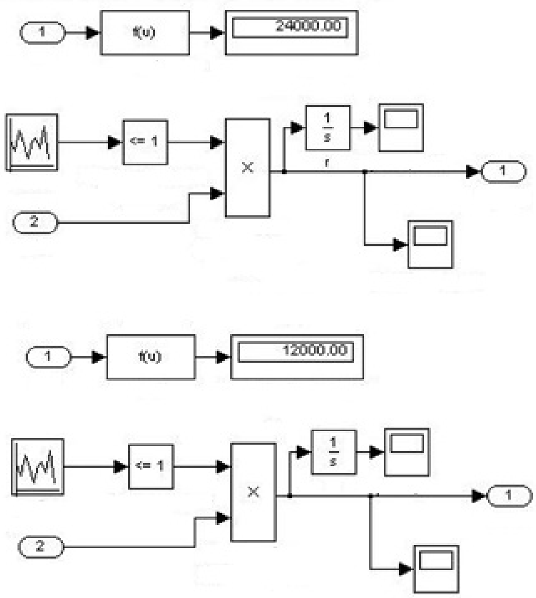

Fig. 6. Subsystems of the mathematical model in SW[1]

Parameter	Value
Body volume, [m^3]	15
Average travel time, [min]	40
Average time of maneuvering and unloading, [min]	2
Average loading time by excavator performance, [min]	15 x 60 / P
Fixed costs, [CZK]	3 000
Variable costs, [CZK/hour]	1 000
Average failure rate, [%/day]	2
Average repair time, [min]	60
Maximum number of dumpers, [-]	15

Table 1. Input parameters of the auto truck[7]

- Working shift T_S = 8 hours per day, i. e. 40 hours per week.
- Construction task has to be performed maximum in 2 weeks (T_M = 80 working hours, that is 4 800 minutes).
- The input parameters of proper excavators are described in Table 2[7]. Most of the input parameters of machines are given by the manufacturer or calculated and averaged on the basis of observation and monitoring. An appropriate output per hour of an excavator, without the influence of random disturbances, according to[9] is greater than

$$P_{app} = \frac{Q_{exc}}{TM \times N} = 7\,500 / 80 / 1 = 93.75 \text{ m}^3/\text{hour} \quad (7)$$

- Basic time unit is a minute.

Parameter	Variant 1	Variant 2	Variant 3
Bucket volume, [m^3]	2	3	4
Average duration of working cycle, [min]	0.83	0.67	0.625
Average hourly output, [m^3/hour]	100	120	150
Fixed costs, [CZK]	5 000	7 500	15 000
Variable costs, [CZK/hour]	2 000	3 000	4 000
Probability of failure, [%/day]	2	4	3
Average time of repair, [min]	60	80	90

Table 2. Input parameters of excavator[7]

The total average time of the truck working cycle for each variant without the influence of random factors, according to[9] is equal to:

TC_1 = 40 + 2 + 15 x 60 / 100 = 40 + 2 + 6 = 48 min.
TC_2 = 40 + 2 + 15 x 60 / 120 = 40 + 2 + 5 = 47 min.
TC_3 = 40 + 2 + 15 x 60 / 150 = 40 + 2 + 4 = 46 min.

Hourly output of the truck for each variant according to[9] is the following:

$P_{truck\ 1} = V_{bucket}/TC_1 x60$ = 15 / 48 x 60 = 12.50 m^3/hour
$P_{truck\ 2} = V_{bucket}/TC_2 x60$ = 15 / 47 x 60 = 12.77 m^3/ hour
$P_{truck\ 3} = V_{bucket}/TC_3 x60$ = 15 / 46 x 60 = 13.04 m^3/ hour

The minimum number of trucks for maximum occupation of the excavator for each variant without the influence of random factors, according to[9] is:

$PV_1 = N \times P_{exc\ 1} / P_{truck\ 1}$ = 100 / 12.5 = 8,0 => 8 dumpers
$PV_2 = N \times P_{exc\ 2} / P_{truck\ 2}$ = 120 / 12.77 = 9,4 => 10 dumpers
$PV_3 = N \times P_{exc\ 3} / P_{truck\ 3}$ = 150 / 13.04 = 11,5 => 12 dumpers

Other calculations for determining the random effects will be performed in the simulation software[1]. After the introduction of random values (machine failures) in the mathematical model, according to the simulation, the time of machine service extends:

$TN_1 = 6 + 0.058 = 6.058$ min.
$TN_2 = 5 + 0.084 = 5.084$ min.
$TN_3 = 4 + 0.059 = 4.059$ min.

In the same way the intervals between inputs of elements into service will increase:

$TV = TV_1 = TV_2 = TV_3 = 42 + 0.052 = 42.052$ min.

The total average time of the truck working cycle for each variant with the influence of random factors (failures) is equal to:

$TC_1 = 42.052 + 6.058 = 48.11$ min
$TC_2 = 42.052 + 5.084 = 47.14$ min
$TC_3 = 42.052 + 4.059 = 46.11$ min

The described calculation is arranged in Tables 3, 4, 5 for each variant. Working cycles of the excavator and trucks are interdependent at the service place. Due to the random intervals between arrivals of vehicles, a queue to service will arise here. It is therefore possible to examine the dependences between the transport system and an excavator with the use of the theory of waiting lines[9].

To make the optimal choice of machinery it is necessary first of all to determine other parameters of the queuing theory. The intensity of service is equal to:

$\mu_1 = 1 / TN_1 = 1 / 6.058 = 0.1650709 \text{ min}^{-1}$
$\mu_2 = 1 / TN_2 = 1 / 5.084 = 0.1966955 \text{ min}^{-1}$
$\mu_3 = 1 / TN_3 = 1 / 4.059 = 0.2463661 \text{ min}^{-1}$

The intensity of the input of elements into the queuing system:

$\lambda = \lambda_1 = \lambda_2 = \lambda_3 = 1 / TV = 1/42.052 = 0.023780 \text{ min}^{-1}$

The intensity of the system operation:

$\rho_1 = \lambda / \mu_1 = 0.023780 / 0.1650709 = 0.14405974$
$\rho_2 = \lambda / \mu_2 = 0.023780 / 0.1966955 = 0.12089794$
$\rho_3 = \lambda / \mu_3 = 0.023780 / 0.2463661 = 0.09652335$

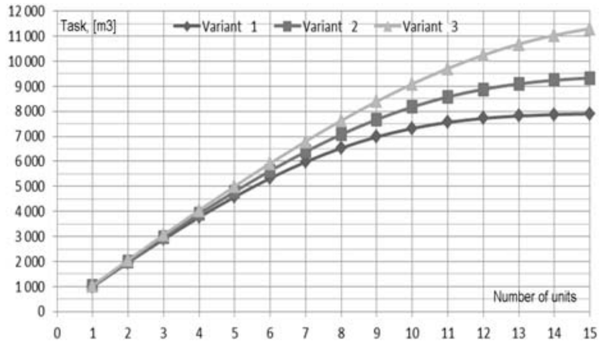

Fig. 7. Calculation the number of units for completing task

After that we will calculate the basic characteristics of the system. According to the calculation, that to fulfil the excavation Q = 7 500 m^3 within up to 80 working hours (4 800 minutes) for the first variant 11 dumpers will be needed, the second requires 9, and the third requires only 8 dumpers. The initial period of the working cycle will be greater due to waiting in line for the service and will continually extend; the efficiency of the entire transport system with an increasing number of dumpers does not grow linearly, see Figure 5. In the last step of selecting the optimal system we will focus on the assessment of the cost parameters for each variant. The optimal solution is a variant with the lowest total costs.
Calculation of the cost characteristics of the system is shown in Tables 3 and 4 for each variant. In Figure 7 the optimal set of machinery is evaluated. The calculation and the selection of an optimal system include the operational costs for the excavator and trucks. The total working time does not exceed 80 working hours.
In Table 3 it is shown that the total costs of trucks for the third variant are the lowest of all options and are equal to 664,000 CZK, respectively 89 CZK per 1 m^3 of excavation. Table 4 shows, that the total costs of the excavator by the first variant are the lowest of all and are equal to 165,000 CZK, respectively 22 CZK per 1 m^3 of excavation.
On the basis of 8 we choose the second variant. For the task fulfilment, the second variant is chosen: excavator with output of 120 m^3/hour and 9 trucks for the transport of soil.

Parameter	Variant 1	Variant 2	Variant 3
Number of trucks, [-]	11	9	8
Fixed costs, [CZK]	3 000	3 000	3 000
Total fixed costs, [CZK]	33 000	27 000	24 000
Variable costs, [CZK /hour]	1 000	1 000	1 000
Total variable costs, [CZK]	880 000	720 000	640 000
Total costs, [CZK]	913 000	747 000	**664 000**
Costs per 1 m^3 of excavation, [CZK /m^3]	122	100	**89**

Table 3. Calculation of costs characteristics of auto trucks system

Parameter	Variant 1	Variant 2	Variant 3
Number of excavators, [-]	1	1	1
Total fixed costs, [CZK]	5 000	7 500	15 000
Variable costs, [CZK /hour]	2 000	3 000	4 000
Total variable costs, [CZK]	160 000	240 000	320 000
Total costs, [CZK]	**165 000**	247 500	335 000
Costs per 1 m^3 of excavation, [CZK /m^3]	**22**	33	45

Table 4. Calculation of costs characteristics of excavator system

Parameter	Variant 1	Variant 2	Variant 3
Total costs of the auto trucks, [CZK]	913 000	747 000	664 000
Total costs of the excavator, [CZK]	165 000	247 500	335 000
Total costs, [CZK]	1 078 000	**994 500**	999 000
Costs per 1 m^3 of excavation, [CZK /m^3]	144	**133**	134

Table 5. Calculation of cost characteristics of the system „excavator + auto trucks“

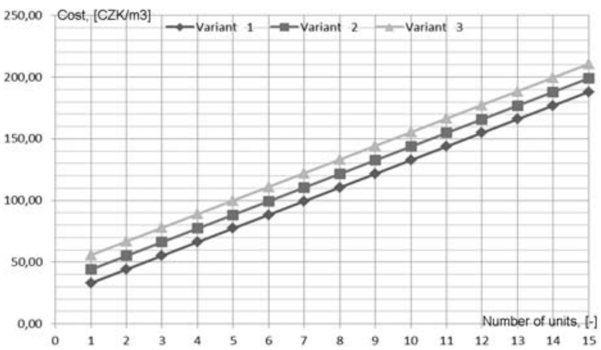

Fig.8. Calculation of cost parameters of the system

SUMMARY AND CONCLUSIONS

To concentrate on reducing the costs of construction means to avoid unnecessary delays in construction, as well as unnecessary additional costs due to a poor technology or wrong mechanization. The aim is to minimize these problems or even entirely eliminate them. The one of the main aim of our research is to eliminate unnecessary delays and additional costs. The mathematical modelling in the special simulation software proved the applicability of the queuing theory for construction processes. In the case of the introduction of additional parameters into the system, the mathematical model will be closer to an actual construction process in reality. The model can include a variety of random factors, including climatic and geographic conditions. On the basis of the simulation results, a construction manager can justify a decision on choice of machinery according to various criteria.

The mathematical simulation should show that some variants of machines combination fail to fulfil the task under the given conditions, and some will not be optimal in terms of costs or other parameters, while other variants will be optimal from the view of costs required to fulfil the construction task. The mathematical simulation of the basic example showed that the first option fails to perform the task under the given conditions; the second variant proved to be optimal in terms of the costs of the construction task fulfilment. The simulation software[1] allows us to look at the results in a graphical form or to export the data to other programs. The application of the queuing theory allows us to introduce into the system a waiting time for the servicing elements and to approximate the mathematical model to a real working task on site.

The application of queuing theory in the construction Industry is well known in the world and very wide and well described in the technical literature. But the first time, we use a mathematical modelling in the simulation software[1]. The present study demonstrates that it is possible to model mathematically and technically the whole complicated construction process containing hundreds of constituent construction processes, with a number of simplifications, and then to perform various calculations and changes for an effective, efficient and long-term planning of construction. Of course, we cannot state that our model is flawless and absolutely accurate. It is therefore important to verify the results of the study under real conditions.

References

1. Simulation SW Matlab Simulink v. R2010b.
2. Liška, V., Macroeconomics, 2.vyd. Praha: Professional Publishing, 2004.
3. Přikryl, P., Numerical methods of mathematical analysis. Praha: SNTL, 1985.
4. Rektorys, K., Overview of Applied Mathematics. Praha: Prometheus, 2000.

5. Kendall, D.G., “Stochastic processes occurring in the theory of queues and their analysis by the method of the imbedded Markov chain“, *The Annals of Mathematical Statistics*, Vol. 24, pp. 338-354, 1953.
6. The theory of queues, www.mti.tul.cz/files/oa/obsluha/index.html
7. Building machinery KOMATSU, www.kuhn-mt.cz
8. Klvaňa, J., Operation Research 2. Praha: FSv ČVUT, 2000.
9. Jarský, Č., On to the mathematical modelling of construction processes. Disertační práce. Praha: FSv ČVUT, 1981.
10. Jarský, Č., Automated Preparation and Management of Realisation of Project. Kralupy n.Vlt: CONTEC, 2000.