Automation and Robotics in Construction XVI © 1999 by UC3M

END POSITION CONTROL OF LONG NON-RIGID ARMS

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Abstract: The position control of a load at the far end of a long, non-rigid arm by an actuator at the near end is considered. The actuator must integrate two tasks: position control of the load and active absorption of vibration to steady it in position. A control strategy based on the launching and absorbing of mechanical waves by the actuator is presented. It is shown to work remarkably well for longitudinal waves in lumped mass-spring systems, being robust, close to optimal and inherently adaptive. Only the first two masses and springs need to be characterised and observed to determine the required actuator movement The approach also applies to related continuous systems. The case of flexural, transverse waves is under study and looks equally promising.

Keywords: Position control, active vibration absorption, mechanical waves, flexible robot arms.

1 INTRODUCTION

Industrial robot arms generally achieve position and vibration control, by having relatively short and very stiff/massive arms. This makes them relatively heavy, power-hungry, expensive, and slow. Lighter arms are considerably more dynamic, are safer in operation, and less expensive to make and operate. But they have the obvious problem of being more flexible and therefore prone to large amplitude vibration. This makes rapid position control very difficult. A further complication is that, in general, the dynamic response of the arm cannot be predicted, because it depends strongly on the load mass which may be unknown or variable. Also the precise position of the arm tip may be difficult to ascertain.

Vibration could be inhibited by adding passive dampers, but this would also inhibit rapid dynamic response (and increase energy consumption). Active vibration control (using a controlled actuator) therefore seems attractive. Rather than add a new actuator, an obvious solution would be to use the existing actuator to do two jobs: position the load mass at the far end of the flexible arm and control the vibration. Combining the two tasks is not just expedient: it is essential. Because any movement of the actuator to achieve position control simultaneously and necessarily introduces vibrations into the flexible arm

This paper presents a control strategy which tells the actuator how to achieve the double task. It is based on the idea of mechanical waves. A movement of the actuator is considered as launching a wave into the flexible arm. The actuator must then absorb reflected waves actively, with precisely the correct amount of "give" (being neither too rigid nor too free) while simultaneously imposing the required position. When all movement has ceased, the load mass should end up in exactly the correct position, because further attempts to correct residual position errors will initiate further vibration.

The paper uses on a lumped mass-spring string model, moving longitudinally, to develop and illustrate the ideas, and then briefly considers their extension to continuous systems and flexural vibrations.

2. LUMPED, LINEAR MASS-SPRING MODEL.

As a first model, consider an actuator at one end of a string of masses and springs, giving a lumped-parameter model of a flexible arm, with a load mass at the far end. Fig. 1 shows a system of n mass-spring units. If the system is uniform, i.e. $k_1 = k_2 = \ldots = k_n = k$ and $m_1 = m_2 = \ldots = m_n = m$, then any movement of the actuator will propagate a transient wave down through the system. If the actuator undergoes a net displacement as a result of its movement, then the wave propagated by that movement will cause the same net displacement of any mass as the wave passes that mass. When the wave reaches the final mass, it will be reflected and start to travel back towards the actuator. Somewhat surprisingly, as the returning wave passes any given mass, it will cause the mass to displace by the same amount again (in the same direction as the original displacement).

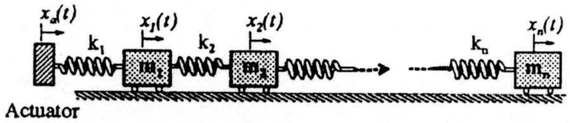

Fig. 1. Multiple degree of freedom, lumped parameter model of flexible robot arm.

Thus if the actuator propagates a wave with a net displacement of one unit then, after one reflection, the end-mass will have undergone a net displacement of two units. Now, if a zero reflection condition can be established at the actuator for the returning wave, then the actuator will effectively absorb the returning wave, and the entire system (including the actuator) will come to rest having undergone a net displacement of two units.

In other words, to move the end-mass a desired amount, the actuator is initially moved half this amount, and then is allowed to be "dragged" by the returning wave (in a precisely controlled way) which leads to a net actuator displacement of exactly the second half of the desired total displacement of the system. This has the effect of efficiently absorbing all vibration energy out of the system while leaving it at rest at the desired new position. Thus the "contending" requirements of position control and vibration absorption are seamlessly integrated, in a way which is independent of the load mass.

The concept may be extended to systems that are not uniform. Any non-uniformity in the system (differing masses or stiffnesses along the arm) will cause an incident wave to be partially reflected and partially transmitted. If all reflected waves are absorbed by the actuator, then it is easily shown that the end-mass will ultimately come to rest at the correct position. Furthermore, this control strategy continues to perform well even when the actuator response is not ideal, and when internal damping is present. It will also cope naturally with vibrations entering the system from an external source.

3. WAVE ABSORPTION

For the actuator to present a zero reflection (or total absorption) condition to any returning wave two requirements must be met. Firstly, the motion of the masses must be analysed so as to separate it into two counter-propagating waves, one outgoing from the actuator (which is to be "allowed through"), the other the returning transient from the system (which has to be absorbed, or passed out of the system). This separation problem is not trivial and is considered below. Secondly, to allow the returning wave to pass out of the system, there should be no dynamic mismatch between the first mass-spring unit and the actuator. To achieve this the strategy adopted is to make the actuator behave, to an incoming wave, as if it were the beginning of an infinitely long string of uniform masses and springs matching the mass-spring unit next to the actuator. This allows transient and steady-state waves from any source to be effectively absorbed by the actuator. In this context it is necessary to determine the response of an infinite string of mass-spring units to an impulse input.

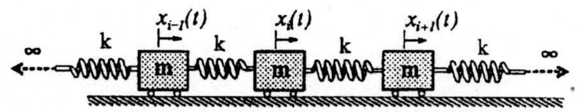

Fig. 2. Infinite-infinite uniform mass spring system.

Fig. 2 shows an infinite number of uniform mass-spring units extending in both directions. Equation of motion for an arbitrary mass (i) is:

$$mx_i = k(x_{i-1}(t) - 2x_i(t) + x_{i+1}(t)) \qquad (1)$$

Assuming all initial conditions are zero, and letting $\omega_n = \sqrt{k/m}$, this may be transformed to the Laplace domain, yielding:

$$s^2 X_i(s) = \omega_n^2 (X_{i-1}(s) - 2X_i(s) + X_{i+1}(s)) \qquad (2)$$

The dynamic response of the infinite mass-spring system can be characterised by a transfer function $G(s)$ which relates how any mass (i) moves in response to a movement of an adjacent mass ($i-1$). Thus:

$$X_i(s) = G(s) X_{i-1}(s) \qquad (3)$$

The same relationship will exist between mass ($i+1$) and mass (i), giving

$$X_{1+1}(s) = G^2(s) X_{i-1}(s) \qquad (4)$$

Substituting (3) & (4) into (2) and simplifying yields a quadratic in $G(s)$:

$$\omega_n^2 G^2(s) - (s^2 + 2\omega_n^2) G(s) + \omega_n^2 = 0 \qquad (5)$$

Hence:

$$G(s) = \frac{1}{2\omega_n^2}\left[(s^2 + 2\omega_n^2) \pm s(s^2 + 4\omega_n^2)^{\frac{1}{2}}\right] \qquad (6)$$

Only one solution for $G(s)$ tends to zero as s tends to infinity (the solution with the negative sign before the radical), making this the only real solution in the time domain. This solution corresponds to a wave moving from left to right through the system. The second solution corresponds to a wave moving from right to left.

The time-domain function $g(t)$, corresponding to $G(s)$ in the Laplace domain, is the unit impulse response between adjacent masses in the infinite mass-spring system. By convoluting an incoming wave with $g(t)$, it will be possible to produce a control signal that will cause the actuator to absorb the incoming wave.

To detect incoming waves, and therefore to distinguish between rightwards propagating and leftwards propagating transient waves in the system, consider again the system shown in Fig. 2. Suppose there are both leftwards and rightwards propagating waves in the system. The motion of mass *(i)* can be regarded as consisting of two components, motion due to rightward propagating waves = $R_i(s)$, and motion due to leftward propagating waves = $L_i(s)$. Hence:

$$X_i(s) = R_i(s) + L_i(s) \qquad (7)$$

Similarly:

$$X_{i+1}(s) = R_{i+1}(s) + L_{i+1}(s) \qquad (8)$$

Now:

$$R_{i+1}(s) = G(s)R_i(s) \qquad (9)$$

and, by symmetry, the characteristic transfer function G(s) is the same for motion in both directions, so

$$L_i(s) = G(s)L_{i+1}(s) \qquad (10)$$

Substituting (9) & (10) into (7) & (8) yields:

$$X_i(s) = R_i(s) + G(s)L_{i+1}(s) \qquad (11)$$

$$X_{i+1}(s) = G(s)R_i(s) + L_{i+1}(s) \qquad (12)$$

Substituting for $L_2(s)$ from (12) into (8) and solving for $R_i(s)$ gives the result

$$R_i(s) = \left(X_i(s) - G(s)X_{i+1}(s)\right) / \left(1 - G^2(s)\right) \qquad (13)$$

Substituting this result back into (12) and solving for $L_{i+1}(s)$ yields:

$$L_{i+1}(s) = \left(X_{i+1}(s) - G(s)X_i(s)\right) / \left(1 - G^2(s)\right) \qquad (14)$$

Thus, if it is possible to observe the position of two adjacent masses, and in particular $X_1(s)$ and $X_2(s)$, it will be possible to differentiate between incoming and outgoing waves, and in addition, to instruct the actuator to absorb any incoming (returning) waves. To do this, it is sufficient to have knowledge of the nature of the system close to the actuator (i.e. the nature of the first two spring-mass units which should be uniform), and to be able to observe the position of the first two masses.

Note that the transfer function *G(s)* is unusual in that, while it relates to two adjacent local masses, it does so while assuming that these masses are embedded in an infinite string of masses and springs. In this sense it is both local and global, and it remains valid no matter how many masses are in the string or how few. The effects of longer or shorter strings are modelled by the appropriate boundary conditions, not by modifying *G(s)*, which is like a "characteristic transfer function" of the string. The validity of this approach can be demonstrated down to the shortest possible "string" of just one spring and mass.

4 PRACTICAL IMPLEMENTATION

To test the concept, a computer simulation of the control algorithm has been developed. To convolute an input signal with *g(t)* (= $L^{-1}[G(s)]$), an impulse response corresponding in magnitude to the value of the input signal is set up at each simulation time-step, and added to the cumulative effect of all the impulse responses set-up for previous sampling points. The output is the sum of all these impulse responses.

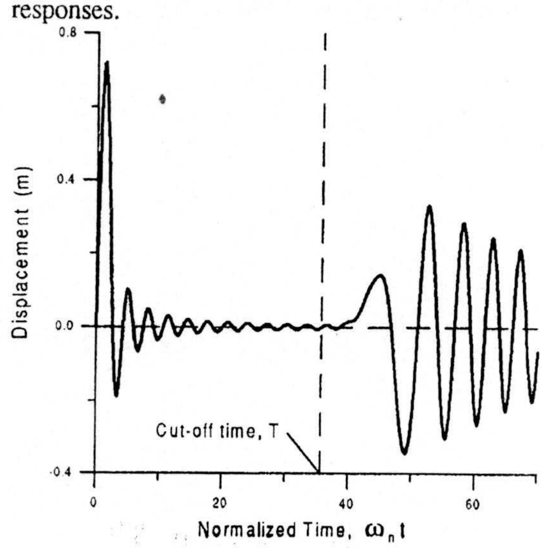

Fig. 3. Unit impulse response of mass (1) in a finite length mass-spring system.

To find *g(t)*, it is possible to expand (6) to an infinite series, and convert back to the time domain, resulting in a type of infinite power series. This is computationally inefficient however and a better approach exists. The impulse response of a finite string of *m* mass-spring units consists of *m* harmonics. If *m* is sufficiently large (that is, if the string is sufficiently long), then the vibration of the initial mass will have decayed to almost zero before the wave set up by the impulse returns after reflection at the far end (see Fig. 3).

A pseudo-infinite mass-spring system response can be obtained by taking the first section of this waveform (up to the cut-off time, *T*), and thereafter setting $g(t)$ identically equal to zero. In practice this means setting up a series of about twenty harmonics for each sampling interval for a period of time equal to T , and then 'switching off' those particular harmonics. This approach is not only computationally efficient: by a happy coincidence it also ensures the stability of the feedback system, as can be shown. Formally therefore, *g(t)* is described as

$$g(t) = \sum_{i=1}^{m} a_i \sin f_i \omega_n t, \quad \text{for } 0 < t < T/\omega_n,$$

$$\text{and } g(t) = 0, \ \text{for } t > T/\omega_n \qquad (15)$$

where a_i are the amplitudes of the contributing harmonics and f_i are the ratios of these frequencies to ω_n, with $0< f_i <2.0$

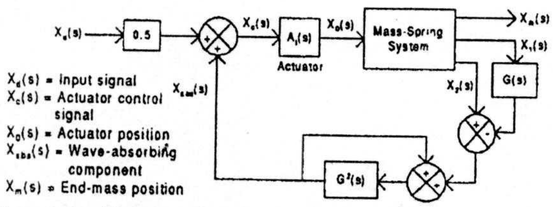

Fig. 4. Block diagram representation of the control system.

In practice, incoming waves may be distinguished from outgoing waves using a positive feedback loop to implement (14). Fig. 4 shows a block diagram representation of the complete control system.

For the record, as it were, the transfer function giving the response of the actuator $X_0(s)$ to any desired position input $X_d(s)$ can be shown to be

$$X_0(s)=X_d(s)\left(\frac{\frac{1}{2}Q(s)\left[1-G^2(s)\right]}{Q(s)U(s)-G^2(s)\left[Q(s)U(s)+P_2(s)-Q(s)P_1(s)\right]}\right)$$

so that the term in the large brackets is effectively the overall transfer function of the control system. This follows from Fig. 4 and some manipulation, with $1/U(s) = A_t(s)$, the transfer function of the actuator, and the response $X_i(s)$of any mass in the system to an input $X_0(s)$ from the actuator being given by

$$X_i(s) = X_0(s)\left[\frac{P_i(s)}{Q(s)}\right] \qquad (17)$$

so that Q(s) is the characteristic polynomial of the multiple-degree-of-freedom system under control and $P_i(s)$ is a polynomial which depends on i.

5. PERFORMANCE EVALUATION AND COMPARISON

Fig. 5 shows the response of the system to a step input when controlling a uniform three mass system with $m_1 = m_2 = m_3 = 1$ kg and $k_1 = k_2 = k_3 = 300$ Nm^{-1}, and with an ideal (zero order) actuator, i.e. an actuator with a transfer function equal to one. For a simple system such as this, the two distinct phases of the actuator movement are clearly distinguishable. The initial phase has the actuator moving 0.5 m and coming momentarily to rest; the second phase, when the feedback signal arrives, has the actuator moving the remaining 0.5 m and oscillating around the final position as it absorbs the vibrating energy out of the system while bringing it to rest at the desired new position. During the initial phase, a step wave is propagated and during the second phase the reflected wave is absorbed.

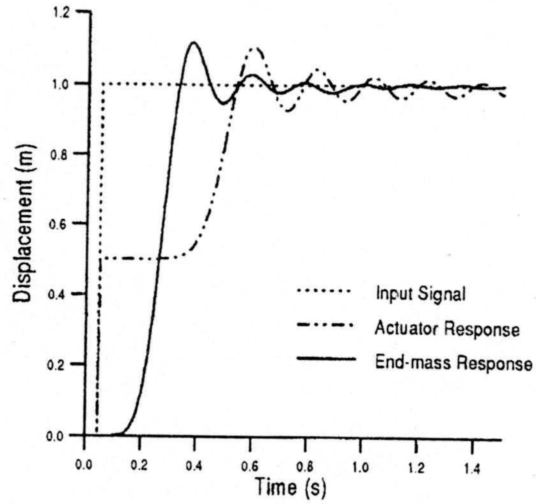

Fig. 5. Step response of control system with idealised actuator.

Fig. 6 shows the step response of the same system, but with a DC servomotor as the actuator. Because the actuator is no longer ideal, the reflected wave is not fully absorbed by the actuator, resulting in partial absorption and partial propagation back out into the system. This process will continue with each reflected wave being partially absorbed, and partially re-propagated, resulting in successive removal of energy from the system in a limiting sequence. It may be shown by considering the overall transfer function for the control system (16), that the final position of the end-mass will be correct (i.e. zero steady-state error) provided that the steady-state gain of the actuator transfer function is unity.

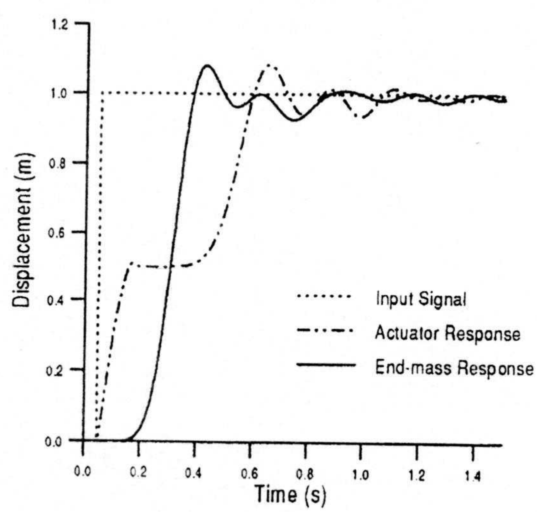

Fig. 6. Response to step input with actuator modelled as DC servomotor.

After stability and steady state error, typically the two most important performance issues of a position control system are response time and residual vibration amplitude. Fig. 7 shows a comparison between the response of the wave-absorption control system (with idealised actuator) and that of a multi-switch bang-bang forcing function, which has been shown to give time-optimal performance if the actuator is not force-limited (Bellman et al. (1956) and LaSalle (1960)).

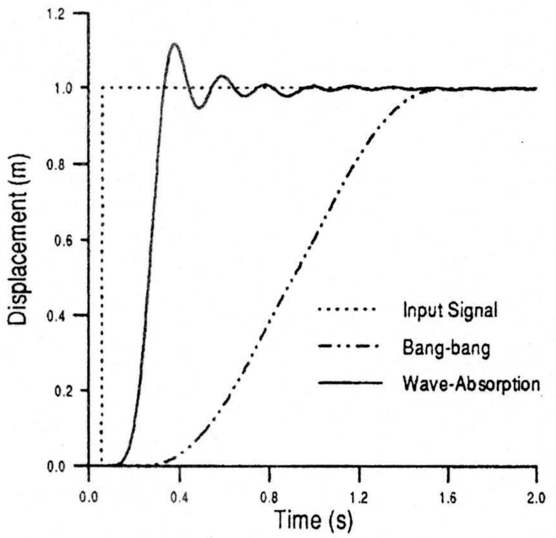

Fig. 7. Comparison of wave-absorption control with multi-switch bang-bang control.

Wave-absorption control compares very favourably both in terms of response time and degree of residual vibration. Furthermore, wave-absorption control is inherently more adaptive than bang-bang control and other open-loop control strategies such as those proposed by Meckl and Seering (1985), which are not as robust to changes in the natural frequencies of the system being controlled. Fig. 8 shows the step response of the same system (again with idealised actuator) with m_3 increased to 2 kg. Although this change affects the natural frequencies of the system, and thus its dynamic response, it does not impair the performance of the control system.

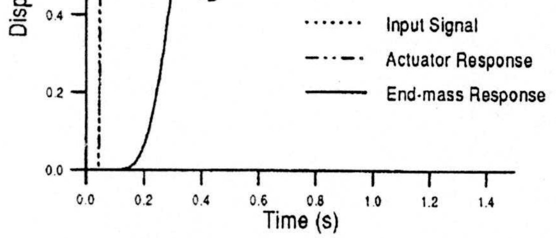

Fig. 8. Step response for system with m_3 = 2 kg.

Natural or inherent damping in such systems is frequently very light, so a damper-less model is appropriate. If required however, the effects of damping can be built into the model without difficulty.

6 CONTROLLED AND OBSERVED VARIABLES

From the perspective of state-space control, or modal analysis, this control strategy is remarkable as it performs very well without attempting to identify, or to control, the states of the system or the modes of vibration directly. An n degree of freedom system (n arbitrarily large) is being controlled by controlling *one* variable (actuator position), whilst observing just *two* variables (the position of the first two masses after the actuator).

Better physical insight is obtained by thinking in terms of mechanical waves. When the actuator moves for any reason, it launches a transient wave (or disturbance) into the system which must travel out to the end-mass before returning. Any such additional wave sent out by the actuator will be either chasing existing outgoing waves already in the system or passing through existing returning waves. The only waves the actuator can absorb or "cancel" are those coming towards it, and it must await their arrival to do so. Hence it does not need (and, curiously, cannot use) information about "waves" or state variables some distance away.

A practical implication is that the most useful sensors are those that give information about the section of the arm closest to the actuator. These are easier to mount and monitor than sensors monitoring the far end.

Active vibration damping is well known, for example in controlling structural vibrations and in vehicles. What is novel here is its combination with position control. Also novel is the "merging moieties" control concept: that of moving a load by launching a wave of *half* the required amount and then letting the system "drag" the actuator the remaining half in a precise way to reach rest at the required position.

7. WIDER IMPLICATIONS

The proposed strategy also works well for the corresponding continuous (distributed) mass-spring systems, where the mechanical waves now have a continuous medium. In fact, in this case the problem of separating the returning from the outgoing waves is easier. Also extension of the ideas to torsional vibrations (whether in lumped or continuous systems) is almost trivial. So, for example, the twisting vibrations of a vertical crane structure due to a rotating actuator at the base could be treated in an exactly analogous manner.

Flexural vibrations are another matter however. Here the limiting equation is not the wave equation (with 2^{nd} order derivatives in space and time), but the Euler-Bernouli vibrating beam equation, in which the acceleration-causing effect (beam bending) has a fourth order derivative term with respect to space. This whole matter is an on-going research topic. Nevertheless it has already been established that the basic concepts of launching and absorbing waves does work for flexural waves. The actuator now needs to be able to "match" both shear force and bending moment "waves", and the actuator frequency response typically needs to be an order of magnitude higher. Paradoxically, the more flexible the system (and therefore the worse the problem) the less demanding are the requirements on speed and force.

Finally, these ideas have application to the problem of controlling the position of a load at the end of a crane cable or chain as the crane moves, particularly when the cable/chain is heavy. Space does not permit development of the idea, but the wave launching and absorbing strategy will work, with some modification to allow for varying wave speed with varying tension.

8. CONCLUSION

Position control of flexible robot arms aims to achieve fast response times with a minimum of residual vibration. The wave-absorption technique provides an inherently adaptive and robust control strategy that compares very favourably with existing strategies in terms of response times and degree of residual vibration. It's performance with non-ideal actuators is only marginally impaired. Although a relatively large degree of computational effort may be required for real-time implementation, only a limited amount of sensory information is necessary, and comparatively little information on the nature of the controlled system is required. Work is at present under way to extend the technique to the control of flexible beams, already with promising results.

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