Mathematical Modelling of Phosphorus Accumulation in Photo-Biological Treatment Plants of a Biosphere-Compatible City

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ABSTRACT

We develop the idea of creating wastewater treatment facilities for biosphere compatible buildings with maximum-closed cycles with respect to water and energy. The idea started from experiments with closed-loop ecological Life Support Systems for space flights carried out in the 60s-80s last century. Water recycling is accomplished by functioning the microalgae-bacteria human ecosystem. One of the problems for systems of this type is high energy-consumption. However, besides water purification from nutrients, that organization of the purification process has multiple advantages, such as carbon dioxide recycling, wastewater disinfection, etc. One of the major benefits is the opportunity to get third-generation biofuels from algae. The closure of the system with respect to energy is performed by biofuel combustion. Thus, there is an objective to cultivate such an amount of biomass, which allows producing enough volume of biofuel. In projects, which have been presented on the exhibitions recently, for that goal phosphoruscontaining reagents are used. However, nowadays, depletion of phosphorus deposits is one of the most serious problems threatening food security, since its variously estimated deposits are enough only for a period of 30-80 years. One solution to this problem is to create a phosphorus closed-loop cycle system. We present the mathematical model describing the dynamics of phosphorus accumulation in the partially closed-loop biological system. The model can be used as a basis for the automatic control system of the photo-biological treatment plants to ensure energy independence of a residential building.

Keywords -

Biosphere Compatibility of Cities; Mathematical Modelling; Photobioreactor; Treatment Plants

1 Introduction

The urban population in developing countries is growing rapidly and wastewater recycling becomes one of the main problems of megacities. It is a common situation when the population increase exceeds the speed of construction of sewage networks and treatment facilities. According to the UN, the water factor is the reason of almost 80% of diseases and three million deaths every year in developing countries. This critical situation is a real threat to sustainability at the global level [1].

Existing technologies imply the creation of maximum centralized systems. As a result, they require extensive sanitary zones, which is impossible to provide in a dense urban environment, lengthy sewer networks associated with huge water losses, and large areas of water treatment plants deform the natural landscape. These listed negative factors are inherent in the technology in general but in developing countries, it is a very common situation when there is no opportunity to collect and treat sewage centrally due to economic reasons.

We present the alternative approach of local facilities. In [2, 3] our concept of the local photobiological wastewater treatment plant was described. The basic elements of the system are photoreactor units with microalgae, which might be placed on a special carcass covering separately located facilities or directly on the facade of a residential building. Figure 1 shows a possible option of a constructional solution for local treatment facilities.

The main requirement for such facilities is compliance with the principle of the biospherecompatibility. The term Biosphere Compatible Economic Activity or Biosphere-Compatibility means an activity that uses technology with closed circuit units of matter and energy, which is based on the laws of functioning of biological systems and principles of selfecological communities. Thus, our goal is to develop the system with maximum-closed cycles with respect to water and energy. The idea started from the experiments with closed-loop ecological life support systems for space flights carried out in the 60s-80s last century [4, 5]. Water recycling is accomplished by functioning the microalgae-bacteria-human ecosystem. One of the problems for systems of this type is high energyconsumption. That fact has been one of the major factors constraining the implementation of the system at space stations. Nevertheless, under earth conditions, it is possible to close the process with respect to energy by using excess algae and bacteria as a source of biofuel.



Figure 1. An architectural shell of the installation for photoreactors with algae at local treatment facilities: **1** is the carcass of an architectural shell, **2** is the photoreactor unit made from transparent ETFE membrane

The idea to use biological sediments (the sludge) formed during the purification process for getting biofuel is not new. The traditional way of sewage purification (which is also extremely energy-intensive) includes the stage of biological treatment by bacteria the activated sludge process and subsequent conversion of biomass residues into methane. Methane production is not the goal: the methane is simply a byproduct of utilization of sludge residues, which are epidemiologically dangerous. The current performance of methane production is very low and the overall process becomes profitable only for large treatment plants. At best, it may defray only 50% of power consumption of treatment facilities. Therefore, most often, there is a tendency to reduce residual amount of biomass as much as possible.

The inclusion of a purification stage performed by algae changes the established approach to wastewater treatment radically. What once was considered as a byproduct of life activity is a valuable resource now. Now we have the aim to produce as much algal biomass as possible. The point is that microalgae is currently considered as the best plant to get biofuel. Comparison of oilseeds shows that the specific energetic value of algae with a 50% lipid content is 15.5 times greater than this indicator of the most energy saturated oilseed -Chinese Tallow-Tree. Moreover, getting biofuel from microalgae (so-called third generation biofuel) occurs without withdrawing agricultural land from circulation and using drinking-quality water. Microalgae do not pose a threat to food security in contrast to the biofuel production from agricultural products.

Using photoreactors with algae in construction has also many other benefits. For example, algae disinfect water by ousting bacteria, including pathogenic to humans, from micro biocenosis by a competitive relationship. One more property that is important is that a photoreactor perfectly works as a natural filter absorbing flue gases and greenhouse gas. All these unique features of algae attract the attention of biologists, construction engineers, and architects. Examples of architecture projects with photoreactors, which have been presented at the exhibitions recently include: BIQ House, Hamburg International Building Exhibition [6]; Urban Algae Canopy Module by ecologic Studio [7]; the project "SymBio2 photoreactors at the incineration plant [8]; the project "ENERGY.2010.3.4-1: Bio-fuels from algae" [9].

Phosphorus and nitrogen compounds contained in wastewater are essential elements for the growth of algae. The problem is that if we want to receive such biomass volume that it would be possible to get enough biofuel and close the purification process with respect to energy then there is the need to use phosphorus- and nitrogen- containing reagents additionally. Unlike nitrogen compounds synthesizable from air, phosphates can be only got by mining. However, nowadays, depletion of phosphorus deposits is one of the most serious problems threatening food security, since its variously estimated deposits are enough only for period of 30-80 years [10]. One solution to this problem is to create phosphorus closed-loop cycle systems. Creation of the phosphorus accumulating system (which accumulates nitrogen as well) makes a treatment plant stable with respect to wastewater flow rate oscillations. That is a very important property for small facilities where those oscillations might be extremely high. Even in case of total reduction of waste volume, bacteria and algae can stay alive by accumulated reserves for a long time and provide robust operation of treatment plant.

For detailed study of the prospects for the biofuel production from microalgae growing by wastewater nutrients, the mathematical model of phosphorus dynamics in the partially closed-loop artificial ecosystem was built. The model predicts biogen concentrations in the system and can be used for optimization of the biomass accumulation process and biofuel yield. The calculations have been made for the original technological scheme with several recirculation loops.

The model describes concentration evolution of following matters:

- Substances dissolved and suspended in water: bacteria, algae, phosphates, nitrates, oxygen, and organic carbon in the form of BOD.
- Biogenic elements within cells of microorganisms.

2 The Mathematical Model of Phosphorus Accumulation

2.1 Computational Domain and Technological scheme

Figure 2 shows the structure of the original technological scheme of a sewage treatment plant with

the recirculation of sludge mixture between an anaerobic tank, an aerobic tank and a photoreactor. The scheme also provides the return of biogens from the methane tank back into the system.

Coupled an anaerobic tank, an aerobic tank and a photoreactor is a computational domain. For every unit we write a number of ordinary differential equations describing evolution of substance concentrations in this unit during the time. We assume that the units are ideal mixing reactors, wherein diffusion coefficients are high and consequently, substance concentrations are distributed uniformly in the unit's volume.

The features of the technological scheme are conditioned by the peculiarity of the biochemical process of phosphorus accumulation. The accumulation process occurs by two steps with participation of phosphate-accumulating organisms (FAO). Under anaerobic conditions, FAO consume low molecular weight fatty acids and accumulate them intracellularly in the form of poly-hydroxy-alkanoates (PHA). This process is accomplished due to the decomposition of polyphosphates, which have been preliminary accumulated in the cells of microorganisms.



Figure 2. The technological scheme, where Q is the wastewater flow rate, m^3/c ; Q_{met} is the rate of flow returning from the methane tank back into the system, m^3/c ; R_1 is the recirculation coefficient of the sludge-water mixture in the wastewater mineralisation unit (the denitrifier plus the aeration tank), *fraction of Q*; R_2 is the recirculation coefficient of the compacted activated sludge, *fraction of Q*; R_3 is the recirculation ratio of the clarified water from the photobioreactor, *fraction of Q*; R_4 is the recirculation ratio of compacted microalgae, *fraction of Q*

Thus, under the anoxic conditions and in the presence of easily oxidizable organic substances, phosphate accumulating organisms secrete phosphorus into the medium. However, under aerobic conditions, by oxidation of PHA, polyphosphates are synthesized in amount that is much larger than that, which has been released under anaerobic conditions. FAO are heterotrophic microorganisms that oxidize aerobically various organic substrates. There are also denitrifying FAO that reserve polyphosphates using chemically bound oxygen of nitrates.

To accumulate phosphorus, FAO need the alternation of purification modes between an oxygen treatment and an anaerobic regime. That will accumulate biologically available phosphorus in the best form for re-using and supplying it to a photobioreactor.

The maximum content of polyphosphate in bacteria is 50%, which corresponds to the phosphorus content of

15-20%. At pH above 8.5 part phosphates precipitate as calcium salts, therefor it is very important to control the acidity during the phosphorus accumulation process.

It is also necessary to provide supply of volatile fatty acids into the anaerobic reactor, so it is advisable to make a preliminary acidification of raw sludge before wastewater supplying into the denitrifier (the anaerobic tank).

2.2 The System of Equations

The concentration of substances in the system with recirculation between the units is described by the following system of equations.

2.2.1 Model of Mineralization

$$\begin{split} \frac{dS_{i}^{aen}}{dt} &= \frac{(S_{i}^{0} + S_{i}^{air} \cdot (R_{1} + R_{2}) + S_{i}^{por} \cdot R_{3}) \cdot Q(t)}{W_{den}} + \sum_{j=1}^{6} N_{i,j}^{mnr} \cdot \mu_{j}(S^{den}, X^{den}) - \frac{S_{i}^{aen} \cdot (1 + R_{1} + R_{2} + R_{3}) \cdot Q(t)}{W_{den}}, \\ \frac{dP_{1}^{den}}{dt} &= \frac{(P_{1}^{air} \cdot R_{1} + P_{1}^{sed} \cdot R_{2}) \cdot Q(t)}{W_{den}} + \sum_{j=1}^{6} N_{i,j}^{mnr} \cdot \mu_{j}(S^{air}, X^{air}) - \frac{P_{1}^{den} \cdot (1 + R_{1} + R_{2} + R_{3}) \cdot Q(t)}{W_{den}}, \\ \frac{dP_{2}^{den}}{dt} &= \frac{(P_{2}^{air} \cdot R_{1} + P_{2}^{sed} \cdot R_{2}) \cdot Q(t)}{W_{den}} + \sum_{j=1}^{6} N_{i,j}^{mnr} \cdot \mu_{j}(S^{air}, X^{air}) - \frac{P_{2}^{den} \cdot (1 + R_{1} + R_{2} + R_{3}) \cdot Q(t)}{W_{den}}, \\ \frac{dX^{den}}{dt} &= \frac{(X^{air} \cdot R_{1} + Y^{sed} \cdot R_{2}) \cdot Q(t)}{W_{den}} + \sum_{j=1}^{6} N_{i,j}^{mnr} \cdot \mu_{j}(S^{air}, X^{air}) - \frac{X^{den} \cdot (1 + R_{1} + R_{2} + R_{3}) \cdot Q(t)}{W_{den}}, \\ \frac{dX^{den}}{dt} &= \frac{(X^{air} \cdot R_{1} + X^{sed} \cdot R_{2}) \cdot Q(t)}{W_{den}} + \sum_{j=1}^{6} N_{i,j}^{mnr} \cdot \mu_{j}(S^{air}, X^{air}) - \frac{X^{den} \cdot (1 + R_{1} + R_{2} + R_{3}) \cdot Q(t)}{W_{den}}, \\ \frac{dS_{i}^{air}}{dt} &= \frac{S_{i}^{den} \cdot (R_{1} + R_{2}) \cdot Q(t)}{W_{air}} + \sum_{j=1}^{6} N_{i,j}^{mnr} \cdot \mu_{j}(S^{air}, X^{air}) - \frac{S_{i}^{air} \cdot (R_{1} + R_{2}) \cdot Q(t)}{W_{air}}, \\ \frac{dP_{1}^{air}}{dt} &= \frac{P_{1}^{den} \cdot (R_{1} + R_{2}) \cdot Q(t)}{W_{air}} + \sum_{j=1}^{6} N_{i,j}^{mnr} \cdot \mu_{j}(S^{air}, X^{air}) - \frac{P_{1}^{air} \cdot (R_{1} + R_{2}) \cdot Q(t)}{W_{air}}, \\ \frac{dP_{2}^{air}}{dt} &= \frac{P_{2}^{den} \cdot (R_{1} + R_{2}) \cdot Q(t)}{W_{air}} + \sum_{j=1}^{6} N_{i,j}^{mnr} \cdot \mu_{j}(S^{air}, X^{air}) - \frac{P_{1}^{air} \cdot (R_{1} + R_{2}) \cdot Q(t)}{W_{air}}, \\ \frac{dX^{air}}{dt} &= \frac{X^{den} \cdot (R_{1} + R_{2}) \cdot Q(t)}{W_{air}} + \sum_{j=1}^{6} N_{i,j}^{mnr} \cdot \mu_{j}(S^{air}, X^{air}) - \frac{P_{2}^{air} \cdot (R_{1} + R_{2}) \cdot Q(t)}{W_{air}}, \\ \frac{dX^{air}}{dt} &= \frac{X^{den} \cdot (R_{1} + R_{2}) \cdot Q(t)}{W_{air}} + \sum_{j=1}^{6} N_{i,j}^{mnr} \cdot \mu_{j}(S^{air}, X^{air}) - \frac{Y^{air} \cdot (R_{1} + R_{2}) \cdot Q(t)}{W_{air}}, \\ \frac{dX^{air}}{dt} &= \frac{Y^{den} \cdot (R_{1} + R_{2}) \cdot Q(t)}{W_{air}} + \sum_{j=1}^{6} N_{i,j}^{mnr} \cdot \mu_{j}(S^{air}, X^{air}) - \frac{Y^{air} \cdot (R_{1} + R$$

where S_i^{den} is the concentration of the *i*-th solute in the denitrifier (the anaerobic tank), g/m^3 ; S_i^{air} is the concentration of the *i*-th solute in the aeration tank, g/m^3 ; S_{i}^{pbr} is the concentration of the *i*-th solute in the photobioreactor, g/m^3 ; S_i^0 is the concentration of the *i*-th solute in the wastewater, g/m^3 ; X_i^{den} , X_i^{air} , and X_i^0 are the concentrations of the *i*-th strain of activated sludge in the denitrifier, the aeration tank, and the wastewater, accordingly, kg/m^3 ; X_i^{sed} is the concentration of the *i*-th strain of activated sludge in the compacted activated sludge, g/m^3 ; P_1^{den} , P_1^{air} and P_1^{sed} are is concentrations of poly-hydroxy-alkanoates, PHA, in the denitrifier, the aeration tank and the desilter, accordingly, g/m^3 ; P_2^{den} P_2^{air} and P_2^{sed} are is concentrations of poly-phosphates, PP, in the denitrifier, the aeration tank and the desilter, accordingly, g/m^3 ; $N^{mne}_{i,j}$ is the stoichiometric coefficient for the *i*-th substrate in the *j*-th chemical reaction for the mineralization block; μ_j is the speed of the *j*-th biochemical reaction, 1/h; W_{den} is the volume of

the denitrifier, m^3 ; W_{air} is the volume of the aeration tank, m^3 .

The rate of changing of dissolved phosphorus concentration in the medium is determined by a difference between its rate of absorption during the oxidation (μ_5) and its release during PNA accumulating (μ_6).

To describe the mineralization process we consider the dynamics of concentrations of following substances: S_1 is the biological oxygen demand, g/m^3 ; S_2 is the ammonia nitrogen, g/m^3 ; S_3 is the nitrate nitrogen, g/m^3 ; S_4 is the dissolved oxygen, g/m^3 ; S_5 is the dissolved phosphorus compounds, g/m^3 ; X is the activated sludge, g/m^3 .

The formulas describing the speed of the j-th biochemical reaction in the photobioreactor and the aeration tank are given below. To make the record briefer we omit the indexes *pbr* and *air*.

$$\begin{split} \mu_1(S_1,S_2,S_4,X) &= \frac{\mu_1^{\max}}{3600} \cdot \frac{S_1 \cdot S_2 \cdot S_4 \cdot X}{(S_1 + K_{1,1}) \cdot (S_2 + K_{1,2}) \cdot (S_4 + K_{1,4})}, \\ \mu_2(S_2,S_4,X) &= \frac{\mu_2^{\max}}{3600} \cdot \frac{S_2 \cdot S_4 \cdot X}{(S_2 + K_{2,2} + \frac{S_2^{-2}}{K_{2,2} \cdot I_{2,2}}) \cdot (S_4 + K_{2,4})}, \\ \mu_3(S_1,S_2,S_3,S_4,X) &= \frac{\mu_3^{\max}}{3600} \cdot \frac{S_1 \cdot S_2 \cdot S_3 \cdot K_{3,4}^{ing} \cdot X}{(S_1 + K_{3,1}) \cdot (S_2 + K_{3,2}) \cdot (S_3 + K_{3,3}) \cdot (S_4 + K_{3,4}^{ing})}, \\ \mu_4(S_4) &= \frac{Q \cdot Q_{air} \cdot H_{air} \cdot K_{air}}{W} \cdot \frac{(1 + \frac{H_{air}}{20,6}) \cdot S_4^{\max} - S_4}{(1 + \frac{H_{air}}{20,6}) \cdot S_4^{\max}}, \\ \mu_5(S_1,S_3,S_4,P_1,X) &= \frac{\mu_5^{\max}}{3600} \cdot \frac{S_1 \cdot K_{5,3} \cdot K_{5,3}^{ing} \cdot (S_4 + K_{5,4}^{ing}) \cdot (K_1^{PP} + \frac{P_1}{X})}{(S_1 + K_{5,1}) \cdot (S_3 + K_{5,3}^{ing}) \cdot (S_4 + K_{5,4}^{ing}) \cdot (K_1^{PP} + \frac{P_1}{X})}, \\ \mu_6(S_4,S_5,P_1,P_2,X) &= \frac{\mu_6^{\max}}{3600} \cdot \frac{S_4 \cdot S_5 \cdot (\frac{P_1}{X}) \cdot (K_{6,6} + \frac{P_1}{X}) \cdot (K_{6,7} + K^{\max} - \frac{P_2}{X})}{(S_4 + K_{6,4}) \cdot (S_5 + K_{6,5}) \cdot (K_{6,6} + \frac{P_1}{X}) \cdot (K_{6,7} + K^{\max} - \frac{P_2}{X})} \end{split}$$

where $K_{j,i}$ is the half-saturation constant for the *i*-th substrate and the *j*-th biochemical reaction, g/m^3 ; $K^{ing}_{j,i}$ is the inhibition constant for the *i*-th substrate and the *j*-th biochemical reaction, g/m^3 ; K^{max} is the maximum content of polyphosphates in FAO, g/m^3 .

2.2.2 Model of Growth of Microalgae

The processes of nitrification in the mineralization

block of the considered technological scheme are virtually absent due to high load on activated sludge. However, during the microalgae cultivation, there are acceptable conditions for the oxidation of ammonia nitrogen, which is assimilated by microorganisms at the same time. Therefore, we assume that the output water of the photobioreactor will only contain nitrate nitrogen: it is quite justified when the process takes more than 12 hours.

$$\begin{split} \frac{dS_{3}^{pbr}}{dt} &= \frac{(S_{2}^{den} + S_{3}^{den}) \cdot \left[(1+R_{3}) \cdot Q(t) - Q_{bac}(t) \right]}{W_{pbr}} + \sum_{j=7}^{9} N_{9,j}^{pbr} \cdot \mu_{j} (S^{pbr}, A^{pbr}) - \frac{S_{3}^{pbr} \cdot \left[(1+R_{3}+R_{4}) \cdot Q(t) - Q_{bac}(t) \right]}{W_{pbr}}, \\ \frac{dS_{5}^{pbr}}{dt} &= \frac{S_{5}^{den} \cdot \left[(1+R_{3}) \cdot Q(t) - Q_{bac}(t) \right]}{W_{pbr}} + \sum_{j=7}^{9} N_{10,j} \cdot \mu_{j} (S^{pbr}, A^{pbr}) - \frac{S_{5}^{pbr} \cdot \left[(1+R_{3}+R_{4}) \cdot Q(t) - Q_{bac}(t) \right]}{W_{pbr}}, \\ \frac{dP_{4}^{pbr}}{dt} &= \frac{P_{3}^{sed} \cdot R_{4} \cdot Q(t)}{W_{pbr}} + \sum_{j=7}^{9} N_{11,j} \cdot \mu_{j} (S^{pbr}, A^{pbr}) - \frac{P_{3}^{pbr} \cdot \left[(1+R_{3}+R_{4}) \cdot Q(t) - Q_{bac}(t) \right]}{W_{pbr}}, \\ \frac{dP_{4}^{pbr}}{dt} &= \frac{P_{4}^{sed} \cdot R_{4} \cdot Q(t)}{W_{pbr}} + \sum_{j=7}^{9} N_{12,j} \cdot \mu_{j} (S^{pbr}, A^{pbr}) - \frac{P_{4}^{pbr} \cdot \left[(1+R_{3}+R_{4}) \cdot Q(t) - Q_{bac}(t) \right]}{W_{pbr}}, \\ \frac{dA^{pbr}}{dt} &= \frac{A^{sed} \cdot R_{4} \cdot Q(t)}{W_{pbr}} + \sum_{j=7}^{9} N_{12,j} \cdot \mu_{j} (S^{pbr}, A^{pbr}) - \frac{P_{4}^{pbr} \cdot \left[(1+R_{3}+R_{4}) \cdot Q(t) - Q_{bac}(t) \right]}{W_{pbr}}, \\ \frac{dA^{pbr}}{dt} &= \frac{A^{sed} \cdot R_{4} \cdot Q(t)}{W_{pbr}} + \sum_{j=7}^{9} N_{13,j} \cdot \mu_{j} (S^{pbr}, A^{pbr}) - \frac{A^{pbr} \cdot \left[(1+R_{3}+R_{4}) \cdot Q(t) - Q_{bac}(t) \right]}{W_{pbr}}, \\ \frac{dA^{pbr}}{dt} &= \frac{A^{sed} \cdot R_{4} \cdot Q(t)}{W_{pbr}} + \sum_{j=7}^{9} N_{13,j} \cdot \mu_{j} (S^{pbr}, A^{pbr}) - \frac{A^{pbr} \cdot \left[(1+R_{3}+R_{4}) \cdot Q(t) - Q_{bac}(t) \right]}{W_{pbr}}, \\ \frac{dA^{pbr}}{dt} &= \frac{A^{sed} \cdot R_{4} \cdot Q(t)}{W_{pbr}} + \sum_{j=7}^{9} N_{13,j} \cdot \mu_{j} (S^{pbr}, A^{pbr}) - \frac{A^{pbr} \cdot \left[(1+R_{3}+R_{4}) \cdot Q(t) - Q_{bac}(t) \right]}{W_{pbr}}, \\ \frac{dA^{pbr}}{dt} &= \frac{A^{sed} \cdot R_{4} \cdot Q(t)}{W_{pbr}} + \sum_{j=7}^{9} N_{13,j} \cdot \mu_{j} (S^{pbr}, A^{pbr}) - \frac{A^{pbr} \cdot \left[(1+R_{3}+R_{4}) \cdot Q(t) - Q_{bac}(t) \right]}{W_{pbr}}, \\ \frac{dA^{pbr}}{dt} &= \frac{A^{sed} \cdot R_{4} \cdot Q(t)}{W_{pbr}} + \frac{A^{sed} \cdot R_$$

where A^{pbr} is the concentration of algae in the photobioreactor, kg/m^3 ; A^{sed} is the concentration of compacted algae from the desilter, kg/m^3 ; P_3^{pbr} and P_3^{sed} are concentrations of nitrogen as a cell-quota (contained into the cells), accordingly, in the photobioreactor and the desilter, g/m^3 ; P_4^{pbr} and P_4^{sed} are concentrations of phosphorus a cell-quota, accordingly, in the photobioreactor and the desilter, g/m^3 ; W_{pbr} is the volume of the photobioreactor, m^3 .

In this mathematical model, the rate of the algae growth is depends on the internal concentrations of

biogenic elements hyperbolically (Droop model). The rate of accumulation of cell quotas depend on both the intracellular and external concentrations of the relevant nutrients. The rate of absorption of a certain nutrient from the medium is equal to the rate of accumulation cell quotas by phytoplankton. In this paper, we consider processes for averaged algocoenosis and not take into account the presence of several strains of microalgae.

As well as for mineralization processes, for brevity, we will omit index *pbr* in formulas for μ_j .

$$\begin{split} \mu_{7}(S_{3},P_{3},A) &= \mu_{7} \cdot A \cdot \left(\frac{P_{3}^{\max} - \frac{P_{3}}{A}}{P_{3}^{\max} - P_{3}^{\min}}\right) \cdot \left(\frac{S_{3}}{K_{8,3} + S_{3}}\right), \quad \mu_{8}(S_{5},P_{4},A) = \mu_{8}^{\max} \cdot A \cdot \left(\frac{P_{4}^{\max} - \frac{P_{4}}{A}}{P_{4}^{\max} - P_{4}^{\min}}\right) \cdot \left(\frac{S_{5}}{K_{8,5} + S_{5}}\right) \\ \mu_{9}(P_{1},P_{2},A) &= \mu_{7}^{\max} \cdot F(I) \cdot E(T) \cdot A \cdot \left(\frac{\frac{P_{3}}{A} - P_{3}^{\min}}{P_{3}}\right) \cdot \left(\frac{\frac{P_{4}}{A} - P_{4}^{\min}}{P_{4}A}\right), \\ F(I) &= \frac{1}{H_{pbr} \cdot (k_{W} + k_{A} \cdot A)} \cdot \ln \frac{I + K^{rad}}{I \cdot \exp\left(-H_{pbr} \cdot (k_{W} + k_{A} \cdot A)\right) + K^{rad}}, \quad E(T) = \exp\left(-\frac{\left(T - c_{i}\right)^{2}}{2 \cdot \sigma^{2}}\right), \end{split}$$

where *I* is the illumination, W/m^2 ; K^{rad} is the radiation constant, $kcal/(m^2 c)$; k_W is the light absorption coefficient of water; k_A is the light absorption coefficient of algae; H_{pbr} is the depth of the photobioreactor, *m*; *T* is the temperature, ${}^{\circ}C$; c_i is the optimal temperature, ${}^{\circ}C$; σ is the tolerance interval for temperature, ${}^{\circ}C$.

The model is a simplified scheme of ASM2 (Activeted sludge model 2) [11, 12], which supplemented by the microalgae description that is similar to [13]. Succinct description of the processes allows using model as a basis for an automatic control system.

The automatic control system gradually regulates the volume of water circulating between the units: R_1 , R_2 , and R_3 . For that goal, the frequency control of electric centrifugal pumps is used. The functions of speeds of biological processes can change over time, but relatively slowly, during 10-30 days. For this reason, starting-up and adjustment processes last up to 3 months. To keep a constant oxygen concentration in the aeration tank and the carbon dioxide in the photobioreactor, the system contains the blowers with adjustable air output.

3 Computational experiments

Calculations are made for a period of 15 days. The examples of graphs of calculations for the recirculation modes with ratios $R_2=0.5$, $R_3=0$, $R_4=0.3$ and variable R_1 is given bellow. The model parameters was taken from





Figure 3. Circadian rhythms of the sewage flow rate



Figure 4. The graph of the illumination variation



Figure 5. The diurnal temperature variation

Figure 6 shows the results of calculations for dissolved phosphorus.



Figure 6. The concentrations of dissolved phosphorus in the photobioreactor for different recirculation modes: a) $R_1 = 0$, b) $R_1 = 1$, b) $R_1 = 2$, b) $R_1 = 3$

Figure 6 shows that increasing the recycle ratio R1 makes the efficiency of wastewater treatment from phosphorus higher.

The intracellular content of polyphosphates in FAO is presented in the figure 7.



Figure 7. The intracellular content of polyphosphates in FAO for different recirculation modes

Figure 8 shows the intracellular content of nitrogen and phosphorus in the microalgae with mode $R_1 = 3$. The dashed line indicates the maximum possible cell-quota.



Figure 8. The intracellular content of nutrients in the microalgae with mode $R_1 = 3$: a) nitrogen, b) phosphorus

The concentrations of nitrate nitrogen and microalgae in the photobioreactor are shown in Figures 9 and 10.



Figure 9. The concentrations of nitrate nitrogen in the photobioreactor with the mode $R_1 = 3$



Figure 10. The concentrations of microalgae in the photobioreactor with mode $R_1 = 3$

Phosphorus is removed out of the wastewater effectively if at the stage of pre-mineralization, the activated sludge is circulating between the aerobic and anaerobic reactors. That is due to the fact, that by increasing the ratio R_1 , polyphosphates accumulate within FAO better. That property is graphically represented in Figure 7.

Figure 8 shows that it is possible to enlarge the rate of growth of microalgae by increasing the cell-quota for phosphorus. Then it is necessary to ensure maximum return of phosphorus from the methane-tank back to the beginning of the technological chain of treatment facilities.

The photobioreactor creates optimal conditions for nitrification of ammonium nitrogen, so the model takes into account that at the output from the treatment plant, nitrogen is present in the form of nitrates predominantly, as shown in Figure 9.

Optimization objective is the growth rate of algae, which eventually determines the biogas output from methane-tanks. Figure 10 shows the dynamics of increasing amount of microalgae in the photobioreactor, which largely depends on the amount of phosphorus that repeatedly returned into the beginning of the system.

Computational experiments have shown that the flow with parameters $R_1 = 3$, $R_2 = 0.5$, $R_3 = 0$, $R_4 = 0.3$ is fully functional. After the occurrence of steady state mode the intracellular concentration of phosphorus in algae is 22% of the maximum possible cell quota.

In this case the average specific yield of methane is 5.57 m3/day and amount of electricity is 24.3 kW/h. This amount of energy 3-5 times more than the energy consumption of treatment facilities.

4 Conclusion

The presented mathematical model describes nutrient uptake and storage processes in a photobiological treatment plant. The calculations have been made for the original scheme of local wastewater treatment facilities with a load up to 100 people. The facilities can be placed directly into a residential building or in close proximity to it. The model can be a basis of creation a system with maximum closed circles with respect to water, phosphorus and energy. The intracellular stocks of phosphorus can provide smooth operation of facilities (i.e., ensuring the survival of microorganisms even when the wastewater flow rate is reduced extremely).

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