Laser Scanning and the Continuous Wavelet Transform for Flatness Control

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ABSTRACT

Current methods for surface flatness control in construction are based on sparse measurements and therefore may lead to inaccurate and imprecise results. Previous research has shown that Terrestrial Laser Scanning (TLS), with the accuracy and density of 3D point clouds it can provide, could support more complete and reliable control of surface flatness in construction. However, these previous works have only applied to existing methods based on sparse measurements, or used defect detection methods that are not based on the analysis of surface waviness (i.e. the frequencies in the floor surface profile), although this generally constitutes the key information sought after in flatness assessment.

In this paper, we investigate the application of a frequency analysis technique, more particularly the Continuous Wavelet Transform (CWT), to TLS point clouds associated to surfaces. The aim is to make full use of the density of points provided by TLS and provide detailed results frequency-wise. We provide the reasoning behind employing the CWT for analyzing frequencies in this context, and report results obtained using data acquired from actual slabs. The CWT results are also compared with those obtained when applying the WAViness Index method. The encouraging preliminary results lead us to suggest a path forward for future development and testing with a view on possibly establishing a new standard test method for floor flatness.

Keywords - Floor; Flatness; WAViness; Continuous Wavelet Transform

1 Introduction

1.1 Flatness Control Methods

The construction of buildings, infrastructure and other facilities requires geometric accuracy, so that further works can be successfully conducted and the facility performs as planned. This need has led to the establishment of a wide range of dimensional tolerances, many of which standardized (at national, multi-national and even international level). And as a result, the control of dimensional tolerances is an important activity conducted on jobsites, requiring accurate, rapid and affordable measurement tools and procedures [1].

One important area of dimensional control is the control of the flatness of floors, most typically concrete slabs with or without screeds. For this, varying flatness specifications and control procedures have been developed over time, the most common being the Straightedge method, the F-Numbers method, and the WAViness Index method.

The Straightedge method [2] is the oldest one. It requires laying a 3-meter straightedge at varying (random) locations on the floor surface and measuring the largest deviations under it. The floor is within tolerance if none of the deviations exceeds a value specified based on the level of flatness required. This method is simple to understand and apply, and requires basic, inexpensive tools. However, its implementation is time consuming, prone to errors, and generally provides a very partial assessment of the floor flatness in space (measurements are done at few sparse locations on the floor) and in types of deviations (this method essentially considers only one surface wavelength).

With the development of modern measurement tools like profilometers, the F-Numbers method [3] was proposed. This method requires defining survey lines on the floor and measuring the floor elevations at one-foot intervals along them. A formula is then applied to the measured data that gives two numbers, \( F_F \) and \( F_L \), for each line and subsequently for the entire floor. The floor is within tolerance if neither \( F_F \) nor \( F_L \) is below its corresponding specified value. By its reliance on more modern measurement methods, the F-Numbers method is more time-efficient than the Straightedge method, and also more precise. However, it still is somewhat time-consuming and is hard to understand (the two F-numbers or unit-less and do not seem to relate to anything easily understandable by an operator). It also only provides sparse results both in space (measurements are done along few survey lines sparsely
The flatness of floor surfaces is particularly critical for places like warehouses where forklifts are operated, and Ytterberg [4] has theorized that the operation of forklifts tends to be particularly affected by floor undulations (waviness) with period ranging from 50% to 200% of their wheelbase length, which would typically translate to the range from 2ft to 10ft. The clear limitation of the F-Numbers to cover this range has led to the development of the Waviness Index (WI) method [5]. This method is actually very similar to the F-Numbers method as it is also based on one-foot elevation measurements along manually defined survey lines. However, it then calculates several values of waviness for undulations of period 2, 4, 6, 8 and 10ft – i.e. 60, 120, 180, 240 and 300cm. In addition to providing a coverage of the range of undulation periods discussed above, the outputted WI values are also expressed in centimeter (out-of-flatness deviation) which are easier to understand than the F numbers. However, the method still has three main limitations.

1. Its fairly tedious measurement process limits the amount of survey lines that can be measured, leading to spatially sparse results which may not be representative of the true level of flatness of the floor.
2. The flatness results enable the detection of discrepancies but not directly their localization, which is required for remediating them.
3. The method is still based on measurements along lines although floors are 2D surfaces.

### 1.2 Laser Scanning for Flatness Control

Terrestrial Laser Scanning (TLS) is a technology that is revolutionizing geometric surveying in construction by its capacity to provide both accurate and dense point measurements very rapidly [1][6][7].

With regard to dimensional control, the current practice tends to apply existing methods to the TLS point clouds (e.g. making point-to-point measurements) so that professionals still do not really exploit the density of measurements. The first approaches to do so, simply visually plotted the deviations of points from a reference surface [8]. While this approach helps in the visual identification of potential defects, it still felt short of automatically detecting and quantifying deviations.

Tang et al. [9] then developed and tested an algorithm (with two variants) to detect flatness deviations in 2D TLS data. However, the detection methods they employ focuses on detecting deviation picks as opposed to characterizing surface waviness.

### 1.3 Contribution

In this paper, we present a novel approach to floor flatness control using TLS, as well as preliminary results. The approach is based on the application of the Continuous Wavelet Transform (CWT) to the floor elevation profile. We show that this method has the potential to address the limitations of current standard and state-of-the-art methods.

Section 2 presents the CWT method. Preliminary results are then reported in Section 3 and concluded with an overall discussion in Section 4.

### 2 Continuous Wavelet Transform (CWT)

The Wavelet Transform is a signal analysis method that emerged as a method overcoming the limitations of the Fourier Transform (FT) and Short-Time Fourier Transform (STFT) methods. The FT enables the accurate detection of frequency components in a signal, but is not able to report where along the signal the detected frequencies are located. The STFT partially addresses this limitation by windowing the input signal through its convolution with a fixed-width square signal. However, the method is ineffective at both accurately detecting and precisely locating frequencies spanning a wide range. In contrast, the Wavelet Transform (WT) aims to convolve the input signal with a wavelet function at different locations along it and at multiple scales. Wavelets take their name from the fact that their energy is contained within a short period, and they typically have one center frequency $c_f$. Therefore, the convolution of a wavelet at multiple scales and locations along an input signal leads to the detection of specific frequencies and specific locations.

Several variants of the WT exist and have been developed for very different applications. For example, the Discrete Wavelet Transform has seen application in signal compression. In contrast, the Continuous Wavelet
Transform (CWT) is more appropriate to pattern detection in a signal (the pattern being that of the wavelet). The CWT thus appears theoretically well suited to the problem of surface waviness characterization.

Applying the CWT, like any other WT, requires the selection of the mother wavelet. One common CWT wavelet is the Mexican Hat wavelet. As show in Figure 1, the wavelet is composed of one main, centered frequency undulation, and thus seems appropriate to the detection of flatness defects. The center frequency of the Mexican Hat wavelet is 0.252. By convolving an input signal with the Mexican Hat at a given scale $a$, undulations of characteristic frequency $f$ can be detected; $f$ can be simply calculated as [11]:

$$f = \frac{f_c}{\delta_p a}$$

where $\delta_p$ is the point sampling period in the input signal.

More details about the Wavelet Transform, its variants and fields of applications can be found in [11].

![Figure 1. The Mexican Hat wavelet.](image)

3 Preliminary Results

3.1 Dataset

We have conducted preliminary experiments using two existing concrete slabs of university laboratories. The slabs are the same as those used in [10]. The first Acoustic Lab (AC) slab is 6.4m x 6.7m. The second Drainage Lab (DL) slab is 4.8m x 8.1m. The two slabs have been laser scanned using a Faro Focus 3D using standard resolution and medium-high data accuracy settings. Note that the Acoustic Lab slab required two scans that were subsequently merged. Figure 2(a) shows the DL slab.

![Figure 2. The DL slab (a) and the point cloud associated to its top surface (b)](image)

3.2 Preprocessing

Using the approach described in [10], the subset of cloud points corresponding to the top surface of the slabs is segmented out of the entire point cloud (Figure 2(b)). Because this point set can still contain millions of points, it is subsequently organized in a 2D square array structure that enables fast nearest-neighbor searches. Finally, to reduce the impact of laser scanning measurement noise on the calculation of point elevations, a mean filter is applied to the points’ elevations (i.e. $z$ coordinates) using a neighborhood radius $\rho = 25$ mm.

3.3 1D CWT Implementation

We have implemented a 1D CWT algorithm that applies a Mexican Hat-based CWT to elevation profile survey lines defined using the same approach as [10], that is:

Survey lines are defined at regular intervals $\delta_i$. In the results reported here, we use $\delta_i = 30$cm, which leads to 34 survey lines for the DL floor, and 38 survey lines for the AC floor. A survey line cannot extend closer to the floor boundary than $d_{\text{boundary}}$ (we use $d_{\text{boundary}} = 30$cm). Then, survey points are sampled along each line at regular interval $\delta_p$, leading to the establishment of the survey line elevation profile. In order to achieve a good resolution in the localization of undulation frequencies
along survey lines, we use $\delta_p = 1cm$. The CWT is then applied to each survey line elevation profile. For this, we have used the free library cwtlib [12]. Note that the maximum scale $a_{max}$ at which the CWT may be applied can be identified using Equation (1) in a reverse manner. Considering a maximum undulation period of $10 ft \approx 300 cm$ (see Section 1.1), that is a maximum characteristic frequency $f = 0.0033 cm^{-1}$, we get $a_{max} = 75$.

3.4 1D CWT Results

Figure 4 illustrates results obtained for six survey line elevation profiles randomly selected from the DL and AC cases. The CWT transform plots shown in lines 2 and 4 of the figure are commonly called scalograms. With the colormap employed here, a red color indicates positive correlation between the signal and the wavelet, i.e. convex undulation is detected at the given location and scale. A blue color indicates a negative correlation, i.e. a concave undulation is detected. The shaded parts of the scalograms are discarded because they correspond to locations at which the wavelet falls partially outside the profile data, and therefore the results are not meaningful.

The results show several things. First of all, as expected, the Mexican Hat-based CWT reacts well to waviness in the elevation profiles present at varying scales. Furthermore, it shows the distinct advantage of dissociating concave elevations profiles from convex ones. Finally, the scale at which the CWT reacts appears to correspond to the period of the corresponding undulations. For example, the elevation profile of Figure 4(e) – enlarged in Figure 3 – shows a convex undulation of period $\approx 150cm$ centered at $\approx 180cm$ along the survey line. The CWT response then shows a convex pick at the scale $\approx 35$ that does correspond to a characteristic period of $\approx 150cm$. Similarly, the elevation profile in Figure 4(c) shows a singular concave undulation of period $\approx 100cm$ centered at $\approx 320cm$ along the line. At that location, the CWT response does detect a clear concave pick at the scale $\approx 25$ that corresponds to a characteristic period of $\approx 100cm$.

While these results are clearly very promising, it is unclear at this stage what level of response from the CWT constitutes a defect. While this question will not be fully answered in this manuscript, we have conducted some further analysis aiming to provide some preliminary answer to this question, by comparing the CWT results with those obtained with the Waviness Index method.

3.5 Comparison with Waviness Index

We have also developed an algorithm that automatically applies the Waviness Index method to the point set of a floor surface. The method follows the procedure described in Section 3.3 and in [10] to define survey lines. It then automatically applies the procedure defined in the standard ASTM R 1486 [5]. First of all, note that we use the exact same survey lines as for the CWT method, so that results can be compared per line. Second, the advantage of the Waviness Index method over other existing standard methods is that it provides separate results (called LAD) for undulations of period $60, 120, 180, 240$ and $300cm$ (these periods are obtained by having an underlying variable $k$ taking the values 1, 2, 3, 4 and 5), so that the comparison with the CWT method can not only be done overall, but also for each undulation period.
Figure 4. Results obtained when applying the Mexican Hat -based CWT to six elevation profile lines of the DL and AC labs. Rows 1 and 3 show the elevation profiles along the survey lines. Rows 2 and 4 show the CWT transform plots (scalograms) for the elevation profile just above.
To perform the comparison between the two approaches we calculate CWT responses corresponding to the LAD values using a similar root mean square RMS formula [5], that is:

$$\text{CWT}_{L,a} = \sqrt{\frac{\max_{i=1}^{\text{im}ax} CWT_{L,a,i}^2}{\text{im}ax_{L,a}}}$$  \hspace{1cm} (2)

where \( CWT_{L,a,i} \) is the CWT response at the scale \( a \), at the \( i^{th} \) sampled location along the line \( L \); \( \text{im}ax_{L,a} \) is the number of incremental locations where \( CWT_{L,a,i} \) can be calculated along the line.

We also calculate, for each line \( L \), the overall CWT response (i.e. integrating the results at multiple scales) using the same weighted root mean square formula as the WI method [5], that is:

$$\text{CWT}_L = \sqrt{\frac{\sum_{a=1}^{\text{am}ax} \max_{i=1}^{\text{im}ax} \text{CWT}_{L,a,i}^2}{\sum_{a=1}^{\text{am}ax} \max_{i=1}^{\text{im}ax} \text{CWT}_{L,a,i}}}$$  \hspace{1cm} (3)

where \( \text{am}ax \) is the number of scales considered. In the context of the comparison conducted here, this is the set of five scales that correspond to the same undulation periods as those considered by the WI method (i.e. 60, 120, 180, 240 and 300cm; or \( k = 1 \) to 5). The scales are calculated by using Equation (1) in a reverse way.

Figure 5 shows a scatter plot of the 295 pairs of values \((\text{LAD}_{L,k}, \text{CWT}_{L,a})\) obtained for each of the 76 survey lines \( (L) \) and the five \( k \) (and corresponding \( a \)) values. Figure 6 shows a scatter plot of the pairs of values \((\text{WI}_L, \text{CWT}_L)\) obtained for all 76 lines. The results in Figure 5 show a certain correlation between the WI and CWT results. In fact, the correlation \( R^2 \) value is only 0.67. Yet, a level of disparity between the \( \text{WI}_L \) and \( \text{CWT}_L \) values remains present. The results in Figure 6 indicate an even stronger correlation when combining all five undulation periods. The correlation \( R^2 \) value is in fact 0.84.

While these results altogether show a strong positive correlation between the WI and CWT values, thereby confirming the value of the proposed approach, this correlation is not as strong at the period level (Figure 5). A more in-depth analysis of the results offers a likely explanation for this. Indeed, it is observed that the correlation is poorer for shorter undulation periods, especially 60cm \((k = 1)\), for which the \( R^2 \) value is 0.60. Looking at the measurement profiles considered by both approaches, it is observed that the measurement sampling of the WI method, i.e. every 30cm can easily lead to failed detections of undulations of period 60cm. Figure 7 shows an example of elevation profile for one line as measured using our method (i.e. with measurements every 1cm) and as measured using the WI method. The undulation of period 60cm centered at the location 130cm along the line is essentially missed by the WI measurement method, while it is not by the CWT-based method. A similar observation can be made for the following concave defect with shorter undulation period. This leads to the conclusion that the lower correlation observed in Figure 5 seems due to the sparse measurement required by the WI method leading to imprecise estimations of the \( \text{LAD}_{L,k} \), particularly at low periods.

![Figure 5. CWT_{L,a} vs. LAD_{L,k} for each survey line and for undulation periods 60, 120, 180, 240 and 300cm](image)

![Figure 6. CWT_{L} vs. WI_{L} for each survey line](image)

### 4 Conclusion

This paper presented a novel approach to floor flatness control that harnesses the measurement density that TLS can deliver and the power of the CWT to accurately detect and locate undulations of any frequency in a floor elevation profile.
Results obtained using the Mexican hat mother wavelet and data acquired from two existing concrete slabs have shown very promising results. The proposed method has unique advantages over existing methods, including the most recent Waviness Index method:

1. Floor surfaces can be analyzed densely and efficiently;
2. The analysis provides a very high resolution in undulation periods – the results in Figure 4 includes results for 100 frequencies, while the WI method only considers five;
3. The outputted scalograms enable an effective visualization of the results, and detection and localization of potential defects.
4. It is theoretically possible to assess floor flatness directly in 2D – although this has not been demonstrated in this paper.

A comparison of the results obtained with the proposed method against those obtained with the WI method shows that the proposed method provides at least as good results as the WI method, and is in fact likely superior to it.

Nonetheless, further experiments clearly need to be conducted to confirm this potential. These should particularly consider many more, also larger, concrete floors. Further, work should establish the correlation between CWT response levels with actual defects (i.e. thresholds) based on varying specified levels of flatness. Finally, the possibility to directly analyze the 2D elevation profile of floors (as opposed to using survey lines) remains to be demonstrated – although this possibility is theoretically supported by the literature.

References