Fuzzy Set-based Contingency Estimating and Management

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Purpose Contingency estimating and management are critical management functions necessary for successful delivery of construction projects. Considering its importance, academics and industry professionals proposed a wide range of methods for risk quantification and accordingly for contingency estimating1. Considerably less work was directed to contingency management including risk mitigate during a project. Generally, there are two types of risks; (i) known risks which can be identified, evaluated, planned and budgeted for and (ii) unknown risks which may occur. These risks require a cost and time contingency, even if they were not planned for, in order to mitigate their impact in an orderly manner. In this respect, the importance of contingency management is critical in view of increasing project complexity and difficulty of estimating and/or allocating sufficient contingencies to mitigate risks encountered during project execution. This paper focuses on the contingency management from two perspectives; estimation and depletion of contingency over project durations. **Method** A new method is developed using fuzzy sets theory2 along with a set of measures and indices to model the uncertainty inherent in this process. This method includes a possibility measure, an agreement index, a fussiness measure, an ambiguity measure and a quality fuzzy number index. These measures and indices provide not only the possibility of having adequate contingency but also address issues of precision and vagueness associated with the uncertainty involved. The paper also presents a comparison between the commonly used Monte Carlo Simulation method and the proposed direct fuzzy-sets-based method. As to depletion, the paper presents a management procedure focusing on depletion of the contingency in a generic computational platform. The developed procedure makes use of policies and procedures3 followed by leading construction organizations and owners of major constructed facilities. The developed method and its computational platform were coded using VB.NET-programming. **Results & Discussion** A numerical example is analysed to demonstrate the use of the developed method and to illustrate its capabilities beyond those of the traditional Monte Carlo Simulation.

Keywords: *Contingency, Management, Estimating, Depletion, Fuzzy set theory.*

INTRODUCTION

Modeling uncertainties and quantification of risk are critical to successful management of engineering, procurement and construction projects. Generally, there are two types of risks; 1) known risks which can be identified, evaluated, planned and budgeted for and 2) unknown risks which may occurred. These risks required a cost and time contingencies, even if they weren’t planned for, in order to mitigate their impact in an orderly manner. Accordingly, it was recommended to divide project contingency fund into two components allocated and unallocated[19]. Contingency estimating and management become critical in view of increasing project complexity and difficulty of estimating and/or allocating sufficient contingencies to mitigate risks encountered during project execution. It should be noted that contingency estimating is commonly prepared prior to project execution; its management is an on-going process over project duration. There are several methods that deal with contingency estimation and allocation[2,3,10,20]. However, considerably fewer procedures and methods can be found in literature for contingency depletion and contingency management[27, 2,3]. Comparing to estimating and scheduling it also noted, the models used to manage contingency are not formal, standardized, document, or well organized by clearly defined procedures[6].

Contingency has different meanings for different estimators and management personnel[10, 8, 26]. For example: contractors consider it as a fund for more profit. Consultants count it as a fund to cover: design minor mistakes and / or owner change orders. The owner keeps this fund to address and mitigate unforeseen situations that may occur over project durations[26]. Risner (2010)[15] defined contingency as untapped funds which may generate a high risk for construction projects in case of misuse[15].

The literature contains different methods for contingency estimation divided into deterministic and probabilistic[10] and lately fuzzy system was used in contingency estimation[17] or a hybrid method between any two approaches (i.e. Monte Carlo and fuzzy system). Traditionally, the most widely used method is “Crystal Ball” method by setting a percentage of total project cost (i.e.: 5% 10% etc…). But this method is arbitrary and difficult to defend. Based on limitations of this method, deterministic methods have been developed for contingency estimation in order to add realistic approach such as: Expected value method, Risk Analysis Method, and Method of Moments. The commonly used probabilistic methods are those using Simulation (i.e., Monte Carlo Simulation). These methods are divided by two categories correlated methods and independent methods (i.e., Pareto Principle or 80/20 rule, PERT, Risk Ranking, etc…), but both categories are difficult to be adopted due to complexity and need of historical data which is not always available. In order to overcome this limitation a new fuzzy-based system methods have been developed[17]. Fuzzy system theory is considered an effective alternative to the random modeling of uncertainty and it doesn’t require any assumption about inputs[17] like PDF shapes, correlations (i.e. Poisson distribution[20]). As to contingency management, the literature[6,2] indicates that current practice depends largely on project managers rather on systematic and well-structured procedures. As well, management polices remain organization dependent. For example the policies of the department of transportation (DOT) are different from those of, department of energy (DOE) in the USA. Documented progress meetings of projects of these departments show different contingency management procedure followed by Turner (2011)[20], e-builder (2009)[11], Orange County Public Schools (2008)[13], Canadian department of transportation (2009)[5], U.S. Department of Energy (2009)[24,25,23], and U.S. Department of Defence (1996)[22]. Furthermore, the municipalities’ projects followed different procedures based on experience and municipality regulation such as: Metropolitan of St. Paul (2011)[18], and Palo- Alto (2011)[14].

The different practices in contingency management provide a motivation for researchers to propose a new procedure for contingency management. For example Ford (2002)[6] generalizes and formulates subjective procedures of contingency management based on project managers’ practice and experience. He identifies two types of strategies applied by most project managers: Passive Strategy and Aggressive Strategy (called also Active Strategy). The passive strategy encourages managers to not spend funds too early to manage unforeseen risks and assure project timely completion while aggressive strategy encourages managers to spend funds linearly from an early stage and to perform flexibility in contingency use.

This research focuses on contingency estimation based on fuzzy-set theory and contingency management based on its depletion over project’s durations. The outputs of this research are: a new methodology for contingency depletion project duration and development of a tool for project contingency estimation using fuzzy system coded using VB.net. The evaluation of this tool has been done by comparing its results with results of other methods using the same case studies. Finally a result discussion, recommendations and conclusion are drawn.

BACKGROUND

Elements of fuzzy set theory, developed originally by Zadeh (1965)[29] , for modeling uncertainties are summarized below:

Fuzzy number “A” can be represented by an ascending order quadruple [a_1, a_2, a_3, a_4] as shown in Fig. 1. Each fuzzy number defined by a membership function $\mu_{A,}$ which can be expressed as:

$$\mu_A(t) = \begin{cases} 1 \quad \text{when } a_2 < t < a_3 \\ 0 < value < 1 \quad when \begin{cases} a_1 < t < a_2 \\ or \\ a_3 < t < a_4 \end{cases} \\ 0 \text{ otherwise} \end{cases} \qquad (1)$$

Also, the classical set theory can be extended to fuzzy set theory[4, 29]. Suppose A [a_1, a_2, a_3, a_4] and B [b_1, b_2, b_3, b_4] are two fuzzy numbers then, the application of set operations such as intersection (*Fig. 1*) could be expressed as:

$$\mu_{A\cup B}(t) = \min\,[\mu_A(t), \mu_B(t)] \qquad (2)$$

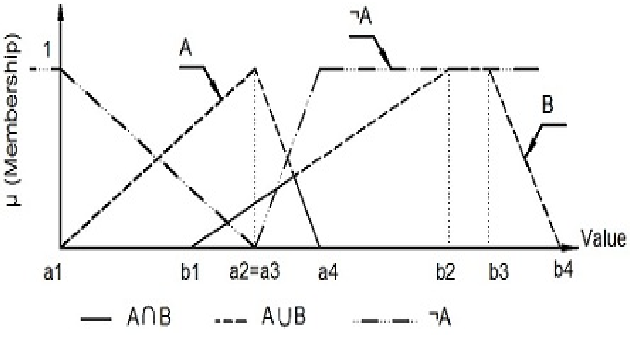

Fig. 1. The Fuzzy-Set operations

Clearly, the relation between A∩B and AUB should satisfy equation (3):

$$\mu_{A\cup B} = \mu_A + \mu_B - \mu_{A\cap B} \tag{3}$$

Furthermore, fuzzy arithmetic operations such as addition could be represented as[4]:

$$A + B = [a_1 + b_1,\ a_2 + b_2,\ a_3 + b_3\ , a_4 + b_4] \tag{4}$$

PROPOSED METHOD

The proposed method has two components; contingency estimating and contingency depletion and management. The estimating process encompasses the execution of the following operations:

Fuzzification

Cost items are represented by fuzzy numbers based on expert judgment. The output of fuzzification is fuzzy estimates A_{ij} for each cost item A_i given by each expert E_j.

$$Where, \begin{cases} \text{i } = 1\ldots\text{n, n is number of cost items} \\ \text{j} = 1 \ldots \text{m, m is number of experts participated} \\ \text{in estimating cost item Ai} \end{cases}$$

Unifying Inputs

All inputs will be re-expressed by a trapezoidal fuzzy number. Crisp fuzzy number will be rewritten [a, a, a, a] instead of [a], uniform [a, a, b, b] instead of [a, b] and triangular [a, b, b, and c] instead of [a, b, c].

The fuzzy contingency estimation is carried out at cost item, work package and project levels.

At the cost item level, the fuzzy estimate is calculated as average of all fuzzy estimates given by participating experts:

$$Ai = \frac{1}{m}\sum_{j=0}^{j=m} A_{ij} \tag{5.a}$$

$$= \frac{1}{m}\left[\sum_{j=0}^{j=m} a_{ij}, \sum_{j=0}^{j=m} b_{ij}, \sum_{j=0}^{j=m} c_{ij}, \sum_{j=0}^{j=m} d_{ij}\right] \tag{5.b}$$

$$= \left[\frac{1}{m}\sum_{j=0}^{j=m} a_{ij}, \frac{1}{m}\sum_{j=0}^{j=m} b_{ij}, \frac{1}{mi}\sum_{j=0}^{j=m} c_{ij}, \frac{1}{m}\sum_{j=0}^{j=m} d_{ij}\right] \tag{5.c}$$

For a package that contains n_i items A_i (i= 1... ni), calculate the sum of all fuzzy estimates of its items:

$$P_k = \sum_{i=1}^{i=ni} A_i^k \tag{6}$$

Similarly, for a project that contains n_p packages P_k (k= 1…n) calculate the project range cost estimate as the sum of fuzzy estimates of its packages:

$$C = \sum_{k=0}^{k=np} P_k \tag{7}$$

Defuzzification

The commonly used method for defuzzification is the center of area (COA)[1] which could be expressed as:

$$y^* = \frac{\int x\mu_i(x_i)}{\int \mu_i(x_i)} \tag{8}$$

$$Where, \begin{cases} \text{y} * = \text{ defuzzified value} \\ \mu(\text{x}) \ = \text{ aggregated membership function} \\ \text{x } = \text{ output variable} \end{cases}$$

The defuzzification of a fuzzy number can be represented by its expected value (Table 1) and variance (

Table 2). Several researchers presented methods for calculating expected value and variance of fuzzy number [26, 17, 4].

Expected value (EV) and variance (V) of a fuzzy number could be calculated as:

$$EV = b - \frac{1}{2}\int_{a}^{b} \mu(x).dx + \frac{1}{2}\int_{b}^{c} 1.dx + \frac{1}{2}\int_{c}^{d} \mu(x).dx \qquad (9)$$

$$V(X) = EV(X^2) - [EV(X)]^2 \qquad (10)$$

Table 1. Fuzzy Number Expected Value

Fuzzy Number Type	Expected Value Formula
Crisp	a
Uniform	$\frac{a+b}{2}$
Triangular	$\frac{a+2b+c}{4}$
Trapezoidal	$\frac{a+b+c+d}{4}$

Table 2. Fuzzy Number Variance

Fuzzy Number Type	Variance Formula
Crisp	0
Uniform	$\frac{(b-a)^2}{12}$
Triangular	$\frac{a^2+b^2+c^2-ab-ac-bc}{18}$
Trapezoidal	$\frac{(b-a)}{(d+c-b-a)}\left(\frac{1}{6}(a+b)^2+\frac{1}{3}b^2\right) + \frac{1}{(d+c-b-a)}\left(\frac{2}{3}(c^3-b^3)\right) + \frac{1}{(d+c-b-a)}\left(\frac{2}{3}(c^3-b^3)\right) + \frac{(d-c)}{(d+c-b-a)}\left(\frac{1}{3}c^2+\frac{1}{6}(c+d)^2\right) - EV^2$

Uncertainty Modeling

Uncertainty could be represented by introducing several measures and indices. In this paper fuzziness measure, ambiguity measure, possibility measure, fuzzy number quality index (FNQI), and agreement index are incorporated to model the uncertainty associated with fuzzy number.

Fuzziness Measure

The fuzziness measure could be expressed as[17]:

$$F(A)=\int_{-\infty}^{+\infty}(1-|2\mu_A(x)-1|)\,dx \tag{11.a}$$

By solving equation 11.a, fuzziness measure of a trapezoidal fuzzy number formula could be re-written as[17]:

$$F(A)=\frac{(b-a)+(d-c)}{2} \tag{11.b}$$

Ambiguity Measure

The ambiguity measure could be expressed as[17]:

$$AG=\frac{d+2c-2b-a}{6} \tag{12}$$

Possibility Measure

The possibility measure (P) of a fuzzy number (A) describes the chance of a fuzzy event ($A \in [a,b]$) to occur[27]. The possibility measure could be expressed as:

$$P(A\in[a,b])=\operatorname*{Sup}_{x\in[a,b]}\mu_A(x) \tag{13}$$

Fuzzy Number Quality Index

The fuzzy number quality index (FNQI) could be expressed as[17]:

$$FNQI=\frac{W_F\times F(A)+W_{AG}\times AG(A)}{(W_F+W_{AG})} \tag{14}$$

Where, W_F and W_{AG} are the fuzziness and ambiguity weights respectively.

Agreement Index

The agreement index (AI) of two fuzzy numbers A and B could be expressed as[16]:

$$AI=\frac{Area\ (A\cap B)}{Area\ (A)} \tag{15}$$

Application of the developed method and the use of its indices and measures will be demonstrated in the numerical example considered in the case study.

As to contingency management, different types of depletion curves (Fig. 2) were reported in the literature. As described below these curves depend largely on organization practice, manager's management strategy and experience, and project characteristics.

Linear Depletion

This is an ideal depletion curve and the easiest to be followed as planned contingency depletion curve. For Example Turner (2011)[21] used it in Mitchell Park Library Community Center project for Palo-Alto City in United States[21].

Basic Depletion (S-Curve)

This curve is called "typical depletion", since it follows the project cost baseline. The contingency drawdown should follow the same curve which is represented by S-curve[12].

Front-End Loading Depletion

This depletion curve is more complicated than linear and basic curves since it needs a bit more effort to be applied. It represents aggressive strategy[6] implemented by managers who believe that project start-up is more risky and fuzzy which rationalize earlier contingency depletion[11].

Back-End Loading Depletion

This curve is opposite of front-end loading depletion and it represents passive strategy implemented by managers who prefer to keep funds for timely project completion[6].

Custom Depletion

This is a newly proposed contingency depletion curve. It could be named also "tailored depletion curve" since it is generated based on periodically estimated / allocated contingency of project. This curve is a project dependent and it assumes within any period that the depleted contingency should be less or equal to estimated contingency for that period. For example, let consider d_j is one of project milestones (i.e. completion), CE_{ij} is estimated contingency for cost item "i" over period d_j and CD_{ij} is its depleted contingency for the same period. The total estimated and depleted contingency of project could be expressed as:

$$\sum_{dj=1}^{n} CE_{ij} \geq \sum_{dj=1}^{n} CD_{ij} \tag{16}$$

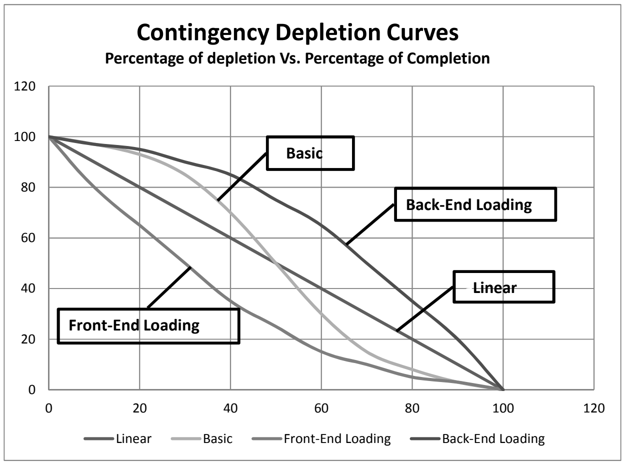

Fig. 2. Depletion Curves types

Based on each company or project manager experience one of the depletion curves could be selected as planned contingency depletion based on several factors affecting depletion. However, affecting factors and selection procedure are beyond scope of this paper.

CASE STUDY

The case study considered here is drawn from the literature for a tunneling project for the city of Edmonton "North Edmonton Sanitary Trunk" (NEST) where the city had an initial estimate of \$6 million and a maximum allocated budget of \$8.8 million by (Shaheen, 2007)[17] . The results generated by the proposed method is compared to those of Shaheen (2007)[17], Monte Carlo Simulation (MCS)[18], and PERT as shown in (Table 3).

Table 3. Comparison of Expected Value Results

Method	EV formula	Project cost estimate
Proposed Method	$\frac{a+b+c+d}{4}$	\$5, 456, 526
PERT[9]	$\frac{a+4.\left(\frac{b+c}{2}\right)+d}{6}$	\$ 5, 352, 680
Method Proposed by Shaheen (2007)[17]	$a+\frac{1}{3(c-b+d-a)}\times$ $[2(c-b)^2+(b-a)\times$ $(d-a)+(d-a)^2]$	\$ 5, 548, 706
MCS (500 iterations)[17]	***based on generated distribution function***	\$ 6, 059, 263

The results shown in (Table 3) illustrate the accuracy of the proposed method in comparison to other methods. The proposed method can determine the possibility of the cost estimate being at a set crisp value (using the possibility measure). This is not possible using the probability theory, which will yield close to zero[7]. The method can test the vagueness and imprecision of the estimated cost using the AG and F measures[18]. Also, the method requires one fuzzy iteration only rather than a number of simulations and does not require data as in Monte Carlo simulation to construct the probability density functions associated with the cost items involved in contingency estimating. The measures and indices incorporated in this paper are: 1) fuzziness measure (F), 2) ambiguity measure (AG), 3) possibility measure (P), 4) fuzzy number quality index (FNQI), and 5) agreement index (AI) between the proposed method fuzzy estimation of NEST project "N_1" and "N_2" as shown in Table 4:

Table 4. Calculation of Measures and Indices

Measure / Index	Formula	Value
Fuzziness Measure F(A)	$\frac{(b-a)+(d-c)}{2}$	\$1,076,555
Ambiguity Measure AG(A)	$\frac{d+2c-2b-a}{6}$	\$ 376,352
FNQI (α=β=0.5)	$\frac{\alpha\times F(A)+\beta\times AG(A)}{(\alpha+\beta)}$	\$ 726,453
AI(N1, N2)	$\frac{Area(\mathrm{N}_1\cap\mathrm{N}_2)}{Area(\mathrm{N}_1)}$	0.93
P(A∈[6.0M, 8.8M])	$\mathbf{\sup_{x\in[a,b]}}\ \boldsymbol{\mu_A(x)}$	0.507

$$Where,\begin{cases} \mathrm{N_1} = \text{fuzzy estimation (in \$M) of NEST project} \\ \quad \text{by proposed method } \mathrm{N_1}[4.64, 5.16, 5.2, 6.83] \\ \\ \mathrm{N_2} = \text{maximum fuzzy value (in \$M)of NEST} \\ \quad \text{project in this case a symetric triangular fuzzy} \\ \quad \text{number was considered } \mathrm{N_2}[3.2, 6.0, 6.0, 8.8] \\ \\ \alpha, \beta = \text{the fuzziness and ambiguity measures} \\ \qquad\qquad \text{weights respectively} \end{cases}$$

Now we can examine a number of scenarios. As can be seen from (Fig. 3), the possibility of exceeding the allocated budget is 0.0 while the possibility of exceeding the expected value of the project estimated cost is P (A ∊ [5.46M,+∞]) =0.84 with an agreement index of 0.52. Also, the possibility of having project cost at 6.0 million is P (A ∊ [6M,+∞]) = 0.49. It should be noted here that it would not have been possible to get a meaningful probability value at this cost estimate using the theory of probability.

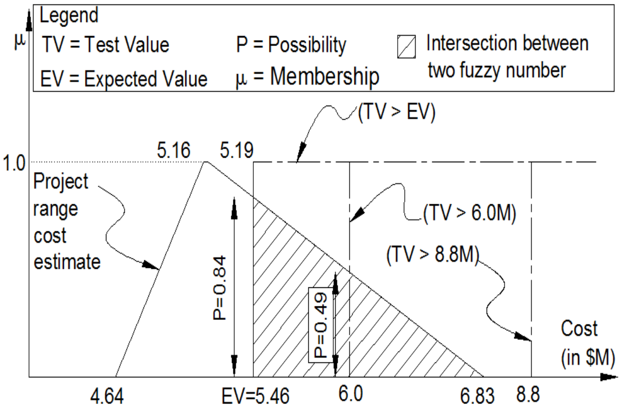

Fig. 3 Possibility and Agreement Index

CONCLUSIONS AND RECOMMENDATIONS

In this paper, contingency estimation using fuzzy set theory was presented as an effective and accurate in comparison to commonly used methods. Using fuzzy set theory overcomes limitations of probabilistic methods and it gives expert ability to express their knowledge based on their experience. The proposed method circumvent the limitations of simulation; not requiring historical data records to construct probability density functions for the cost items involved and not requiring large number of simulations. It also offers a set of indices and measures that address vagueness and imprecision associated with estimated cost at the cost item, work package and project levels, as well as possibility of having project cost at a specific crisp value or within a specified cost range.

The proposed contingency depletion methodology standardizes contingency management practice and offers a flexible unified procedure making use of policies and procedures adopted by companies and project managers based on their gut-feel, management skills, learned lessons, and regulations generated by experience. It proposed tailored contingency depletion curve so as to improve decision making based on contingency depletion performance over project duration. The limitation of this method is the assumption that all risks associated with project are well identified, reliably evaluated, and effectively responded to by risk management plan.

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