MOTION PLANNING TAKING INTO ACCOUNT THE DYNAMIC MODEL OF VEHICLES : APPLICATION TO THE COMPACTOR

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ABSTRACT: Motion planning is a necessary step before automation in road construction. This paper deals with motion planning for compactors. The compactor is an articulated frame steering vehicle used in road construction. Generally, motion planning uses the kinematic model of the robot. In the case of the compactor, this model is far away from reality because it does not take into account compactor masses and particular drum-soil contact forces. A motion planning which uses both the kinematic and the dynamic model of the compactor is presented in this paper. The advantages of this motion planning are:

- To generate feasible motion for the robot which implies a smoother control law for mechanics,
- Limited slips.

KEYWORDS: Compactor, Mobile robots, Motion planning, Dynamic model

1. INTRODUCTION

This article focuses on the compaction process during the road construction. This process is made up of several trajectories of several compactors. In the frame of automation in road construction, it can be seen that the compactor must follow an efficient trajectory defined in position, velocity and acceleration according to compactor degrees of freedom to guarantee the homogeneity and the expected density of considered compacted material. Those features should be improved taking into account a dynamic model compared with a simple path tracking using kinematic model. The compactor motion planning, presented here, is based on the dynamic modelling and identification of the compactor (Guillo *et al.*, 1999). Necessary elements of modelling are presented in this paper before the presentation of the method of motion planning with simulation results.

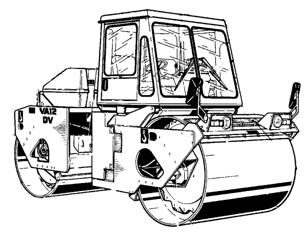


Fig. 1. A typical compactor: Albaret VA12 DV

2. COMPACTOR MODEL

A planar motion of the compactor is considered. Let R_g be a Galilean reference frame attached to the rolling plan Π. In these conditions, a vector of three generalized coordinates is needed to specify the compactor posture.

The front frame is chosen to be the reference body C_r of the compactor, *i.e.* its situation gives the compactor posture.

According to classical robot manipulator description (Canudas de Wit *et al.*, 1997), the compactor is considered as a mechanical system Σ composed of a tree structure of $n = 7$ rigid bodies C_j where C_0 is the base body and with the following body definitions (see Figure 2):

- C_1, C_2 are two virtual bodies (i.e. without mass and inertia) used to define the compactor position with respect to the frame R_0,
- C_3, C_5 are the front and rear drums,
- C_4, C_7 are the front and rear frames,
- C_6 is a virtual body (used to define a second frame R_6 attached to C_5).

The system Σ is provided with a frame R_j respectively attached to each of the $(n+1)$ bodies C_j. Let R_j be defined as $R_j = (O_j, x_j, y_j, z_j)$ (see Figure 2).

Classical tree structure description using the DHM notations (Khalil and Kleinfinger, 1986) applied to the system Σ defines the geometric parameters of the compactor (see Table 1 and Fig. 2) with respect to the position and orientation of the body C_0.

Table 1. Geometric parameters of the compactor

j	$i = a(j)$	σ_j	α_j	d_j	θ_j	r_j
1	0	1	$\pi/2$	0	$\pi/2$	r_1
2	1	1	$\pi/2$	0	$\pi/2$	r_2
3	2	0	$\pi/2$	0	θ_3	0
4	3	0	$-\pi/2$	0	θ_4	0
5	3	0	0	$-\mathrm{D}_5$	θ_5	0
6	5	2	0	$-\mathrm{D}_6$	0	0
7	6	0	$-\pi/2$	0	θ_7	0

According to the DHM description of the compactor, the vehicle motion is completely described by the vector q (see eq. 1) of six generalized coordinates where the vector ξ gives the compactor posture.

$$\begin{aligned} q &= \begin{bmatrix} r_1 & r_2 & \theta_3 & \theta_4 & \theta_5 & \theta_7 \end{bmatrix}^{\mathrm{T}} \\ \xi &= \begin{bmatrix} r_1 & r_2 & \theta_3 \end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{1}$$

2.1 *Geometric model*

The Direct Geometric Model (DGM) gives bodies situation with respect to the reference frame R_g as a function of q, the vector of joint variables.

For example, equation 2 gives the situation of the rear frame (C_6) of the compactor with respect to the frame R_g. The symbolic modelling software SYMORO+ (see (Khalil and Creusot, 1997)) had been used to determine the expression of equation 2 from the table 1 of the geometric parameters of the compactor.

$$\begin{aligned} {}^{g}\mathrm{A}_6 &= \begin{bmatrix} \cos(\theta_3+\theta_5) & -\sin(\theta_3+\theta_5) & 0 \\ \sin(\theta_3+\theta_5) & \cos(\theta_3+\theta_5) & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ {}^{g}\mathrm{P}_6 &= \begin{bmatrix} r_1 - \mathrm{D}_5 \cdot \cos\theta_3 - \mathrm{D}_6 \cdot \cos(\theta_3+\theta_5) \\ r_2 - \mathrm{D}_5 \cdot \sin\theta_3 - \mathrm{D}_6 \cdot \sin(\theta_3+\theta_5) \\ 0 \end{bmatrix} \end{aligned} \tag{2}$$

2.2 *Kinematic model*

In the previous section, the geometric description of vehicles using robotics formulation is presented. From this description, the *Direct Kinematic Model* (DKM) of vehicles can be developed. It is composed of n_t relations (see eq. 3) corresponding to the velocity of each of the n_t vehicle terminal bodies (wheels, drums, tools,...).

The equation 3 gives the kinematic wrench of a vehicle terminal body C_j with respect to the reference frame R_g.

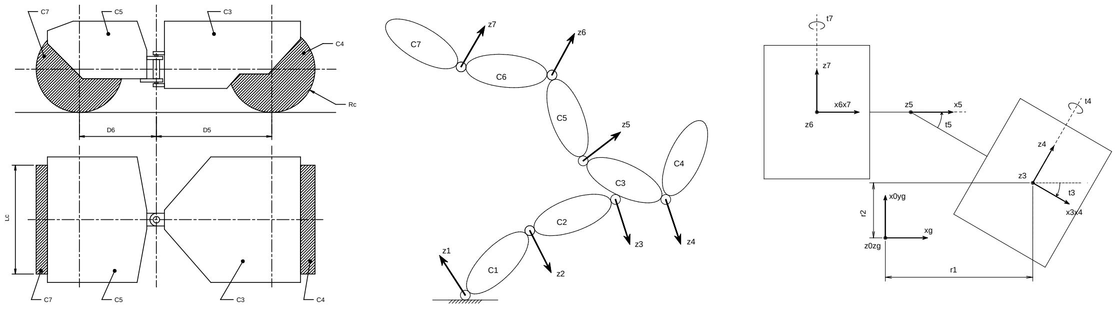
$$\begin{bmatrix} \mathrm{V}_{j,g}^{O_j} \\ \omega_{j,g} \end{bmatrix} = \Phi_j^{O_j} \, \dot{q} \tag{3}$$

where $\mathrm{V}_{j,g}^{O_j}$ and $\omega_{j,g}$ respectively are the velocity of the point O_j and the rotation velocity of the body C_j with respect to the reference frame R_g. $\Phi_j^{O_j}$ is called *base jacobian matrix* of the vehicle.

2.2.1. *Conditions of pure rolling and non-slipping*

Let be assumed that the contact between the rigid drum and the soil is reduced to a single point B_j of the rolling plan Π (see Figure 3). The contact between the drum and the soil is supposed to satisfy both conditions of *pure rolling* and *non-slipping* along the motion. This means that the velocity of the contact point B_j is equal to zero and implies that the two components of this velocity, respectively parallel to the plan of the drum and orthogonal to this plan (see eq. 4), are equal to zero.

$$\begin{cases} \mathrm{V}_{j,g}^{B_j} \cdot x_i = 0 \\ \mathrm{V}_{j,g}^{B_j} \cdot y_i = 0 \end{cases} \tag{4}$$



(a) Bodies definition (b) Equivalent tree structure (c) Projection frames definition

Fig. 2. Geometric description of the compactor using the MDH notations.

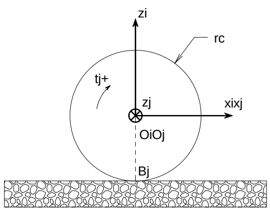


Fig. 3. Contact between a rigid drum and the soil reduced to a single point.

The constraints can be written in the general matrix form

$$J(q)\dot{q} = 0 \tag{5}$$

This means that whatever the type of vehicle, the velocity $\dot{q}$ is restricted to belong to a distribution Δ_c defined as

$$\dot{q} \in \Delta_c = \mathrm{span}\{\mathrm{col}(S(q))\} \tag{6}$$

where the columns of the matrix $S(q)$ form a basis of $\ker(K(q))$. This is equivalent to the following statement: for all time t, there exists a time-varying vector $\eta(t)$ such that

$$\dot{q} = S(q)\eta \tag{7}$$

The relation 7 is called the *Inverse Kinematic Model under constraints* of the vehicle.

2.3 *Dynamic model*

The *Inverse Dynamic Model* (IDM) of a vehicle is written as following

$$M(q)\ddot{q} + H(q, \dot{q}) = \mathbb{L} + \mathbb{Q}^c \tag{8}$$

where:

- $M(q)$ is the mass matrix of the system Σ.
- $H(q, \dot{q})$ is the vector of centrifugal, Coriolis and gravity terms.
- $\mathbb{L}$ is a vector depending on the internal forces between the vehicle bodies: motor torques, friction, lumped elasticity.
- $\mathbb{Q}^c$ is a vector depending on the external contact forces between the soil and the drums.

2.3.1. *Internal forces* The vector $\mathbb{L}$ of internal forces is composed of three components :

- the actuation vector $\mathbb{L}^a$ which is composed of motor torques,
- the friction vector $\mathbb{L}^f$ which is composed of friction component on different joints,
- the elastic forces vector $\mathbb{L}^e$ which represent the stiffness of joints.

2.4 *Contact forces between drums and the soil*

Using the principle of virtual powers, the vector $\mathbb{Q}^c$ of external forces is developed to obtain

$$\mathbb{Q}^c = \sum_j {}^i\Phi_j^{O_j T}\ {}^i\mathbb{T}^{O_j}_{\Pi_j \to C_j} \tag{9}$$

where ${}^i\mathbb{T}^{O_j}_{\Pi_j \to C_j}$ points out the wrench of the resultant contact forces between the soil Π_j on the drum C_j at point O_j projected in frame R_i.

2.5 *Linearity property of the inverse model*

The expression of kinematic and potential energies are linear in relation to a set of ($n_p = 11$) parameters, X_s. Consequently, the expression of the Inverse Dynamic Model is also linear in relation to the same set of parameters and then it is possible to write it as following

$$Y_s = D_s(q, \dot{q})X_s \tag{10}$$

with $Y_s = \mathbb{L}^a$.

Using this property of the Inverse Dynamic Model, a Weighted Least Squares method of identification is proposed by (Gautier, 1997) to obtain the values of the dynamic parameters X_s of robot manipulators and applied to a mobile machine, the compactor, in (Guillo *et al.*, 1999).

2.6 *General expressions of the dynamic model*

The general expression of the Inverse Dynamic Model is given by the relation 8. According to the expression of the internal and external forces which take part in the model, there are two possible expression of this model:

- The conditions of pure rolling and non-slipping are satisfied. Then, the Inverse Dynamic Model has the following expression

$$\begin{cases} M(q)\ddot{q} + H(q,\dot{q}) = \mathbb{L}^a + \mathbb{L}^f + \mathbb{L}^e + J(q)^T\lambda \\ J(q)\dot{q} = 0 \end{cases} \tag{11}$$

- The conditions of pure rolling and non-slipping are not satisfied. Then, the Inverse Dynamic Model has the following expression

$$\begin{cases} M(q)\ddot{q} + H(q,\dot{q}) = \mathbb{L}^a + \mathbb{L}^f + \mathbb{Q}^c \\ \mathbb{Q}^c = \sum_j {}^i\Phi_j^{O_j T}\ {}^i\mathbb{T}^{O_j}_{\Pi_j \rightarrow C_j} \end{cases} \tag{12}$$

3. MOTION PLANNING

Motion planning is to determine joint position, velocity and acceleration. The method developed is composed of four steps

(1) Path determination in order to respect the geometric constraints of the compactor task.
(2) Timing law determination in order to respect the kinematic constraints of the compactor task.
(3) Generalized coordinates determination using the inverse kinematic model under constraints.
(4) Actuator torques and drum-soil contact forces determination in order to compare them with physical limitations.

3.1 *Path determination*

According to the geometric model, a planar motion of the compactor is considered. The path is determined by the compactor posture

$$\xi = \begin{bmatrix} r_1 & r_2 & \theta_3 \end{bmatrix}^T = \begin{bmatrix} x(s) & y(s) & \theta(s) \end{bmatrix}^T \tag{13}$$

where :

- θ is the path orientation,
- x and y are the path cartesian coordinates.

The path is characterized by its curvature K along the arc length s

$$\mathrm{d}\theta = K\mathrm{d}s \tag{14}$$

$$\theta(s) = \theta_0 + \int_0^s K(u)\mathrm{d}u \tag{15}$$

$$x(s) = x_0 + \int_0^s \cos(\theta(u))\mathrm{d}u \tag{16}$$

$$y(s) = y_0 + \int_0^s \sin(\theta(u))\mathrm{d}u \tag{17}$$

Concerning asphalt compaction, the compactor has to compact the bituminous mix spread by the paver. But a paver is wider than a compactor, so the compactor path is composed of juxtaposed tracks (see figure 4).

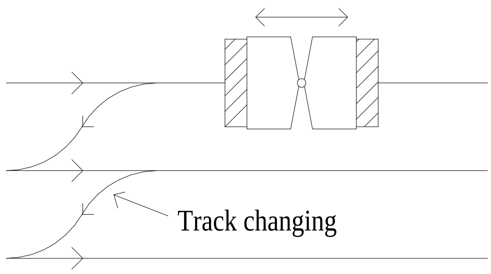


Fig. 4. Compactor typical path

The compactor path is essentially composed of straight lines and track changing. Determination of the path along a straight line is obvious.

$$K = 0 \tag{18}$$

$$\theta(s) = \theta_0 \tag{19}$$

$$x(s) = x_0 + s\cos(\theta) \tag{20}$$

$$y(s) = y_0 + s\sin(\theta) \tag{21}$$

A track changing path is characterized by its width D and its maximum curvature K_{max}. Parametric equations are determined in this way : there are two symmetrical parts in the path, if the path length is $4l$ then the curvature $K(u)$ is an odd function for $u = s - 2l$. These specifications are enough to compute $K(s)$ for $s \in [0, 2l]$.

The path curvature must be smooth to be compatible with the compactor dynamics. Constraints of symmetry lead to a second order function for the curvature such as :

$$K(s) = k_0 + k_1 s + k_2 s^2 \tag{22}$$

$$K(0) = 0,\ K(2l) = 0,\ K(l) = K_{max} \tag{23}$$

Solution for $K(s)$ with $s \in [0, 2l]$ is :

$$K(s) = 2K_{max}\frac{s}{l} - K_{max}\left(\frac{s}{l}\right)^2 \tag{24}$$

Cartesian coordinates could not be computed explicitly, so they are computed numerically. But all these calculations need the path length which is not known. To solve this problem, this equation is used :

$$y(2l) = \frac{D}{2} \tag{25}$$

that makes it possible to compute l according to D and K_{max}.

3.2 *Timing law*

This section deals with the compactor kinematics along the path which determines the timing law of the motion. According to the compactor task, different timing law are used :

- Acceleration,
- Braking,
- Constant speed.

For each timing law, acceleration must be continuous in order to respect physical characteristics of actuators. A polynomial of the second degree is used to respect the constraints of continuity. For an acceleration from 0 to V_{max}, the velocity law is given by :

$$v(t) = -\frac{1}{\tau^3} V_{max}(2t - 3\tau)t^2 \tag{26}$$

$$\tau = \frac{3V_{max}}{2\gamma_{max}} \tag{27}$$

with :

- V_{max} is the top speed of the motion,
- γ_{max} is the top acceleration of the motion,
- τ is the duration of the acceleration.

For the braking timing law, the acceleration is inverted, but the idea is the same. Generalized coordinates of the motion can be computed by combining the path and the timing law to generate inputs of the inverse kinematic model under constraints.

3.3 *Generalized coordinates computation*

The motion is specified by the path curvature function of its arc length and by the velocity function of the time. The path curvature can be computed as a function of the time :

$$s(t) = \int_0^t v(u)\mathrm{d}u \tag{28}$$

$$K(t) = K(s(t)) \tag{29}$$

Inputs of the inverse kinematic model under constraints can be computed :

$$\omega(t) = K(t)v(t) \tag{30}$$

$$\eta(t) = \begin{bmatrix} v(t) \\ \omega(t) \end{bmatrix} \tag{31}$$

So the generalized coordinates are computed :

$$\begin{cases} \dot{q}(t) = S(q)\eta \\ q(t) = q_0 + \displaystyle\int_0^t \dot{q}(u)\mathrm{d}u \\ \ddot{q}(t) = \dfrac{\mathrm{d}\dot{q}}{\mathrm{d}t} \end{cases} \tag{32}$$

The compactor motion is entirely defined by joint position, velocity and acceleration. The generalized coordinates have been computed with four parameters, the width (D) and the maximum curvature (K_{max}) for the path and the top speed (V_{max}) and the top acceleration (γ_{max}) for the timing law. The Dynamic model is used to tune this parameter in order to respect the capacity of actuation and the limit due to the drum-soil interaction.

3.4 *Dynamic tuning*

It is possible to compute motor torques and drum-soil interaction forces with the inverse dynamic model. In this equation :

$$M(q)\ddot{q} + H(q, \dot{q}) = \mathrm{L}^a + \mathrm{L}^f + J^T(q)\lambda \tag{33}$$

the *Lagrange multipliers* $\lambda = [\lambda_1 \dots \lambda_4]^T$ correspond to tangential contact force between drums and soil such as the pure rolling and non-slipping constraints are respected; the generalized actuation vector L^a correspond to motor torques. As drum-soil interaction forces and motor torques are computed, it is possible to analyze them and to tune motion parameters to optimize them.

4. SIMULATION RESULTS

Two sample motions to test this method have been used. The path is a 1.5 meter width track changing which is a typical compactor path (see section 3.1). Parameters of these motion are set in tables 2 and 3. The motion described in table 2 is a base motion, the motion described in table 3 is the base motion which has been optimized with our method.

Track width	$D = 1.5$ m
Maximum curvature	$K_{max} = 0.15$ m^{-1}
Top speed	$V_{max} = 1.4$ m.s^{-1}
Maximum acceleration	$\gamma_{max} = 3$ m.s^{-2}

Table 2. Motion planning parameters

Track width	$D = 1.5$ m
Maximum curvature	$K_{max} = 0.17\text{m}^{-1}$
Top speed	$V_{max} = 1.4$ m.s^{-1}
Maximum acceleration	$\gamma_{max} = 1.2$ m.s^{-2}

Table 3. Optimized motion planning parameters

These motion have been used as a reference in our compactor simulator. Simulation results are presented in Fig. 5 Fig. 6 and Fig. 7.

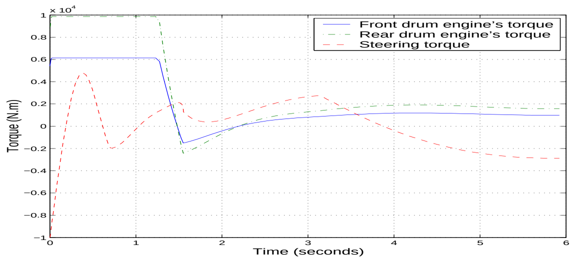


Fig. 5. Actuator torques for the base motion

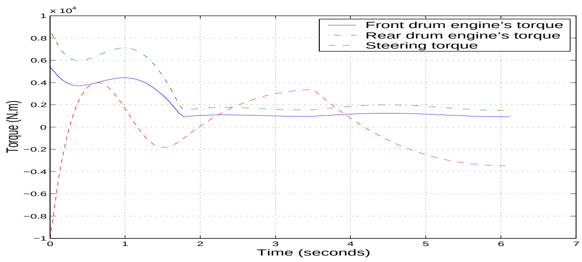


Fig. 6. Actuator torques for the optimized motion

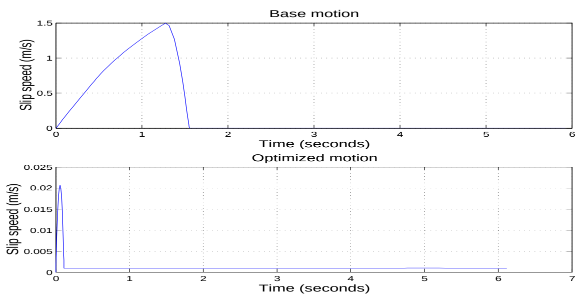


Fig. 7. Actuator torques for the optimized motion

These results show actuators torques which saturate and reach motors characteristics limits (Fig. 5). With our motion planning, it is possible to avoid saturation of actuators (Fig. 6). This optimization of the motion leads also to limited slips of the compactor drums on the soil (Fig. 7) which implies a better quality of asphalt compaction. In the frame of compaction automation, this optimized motion planning allows to design more efficient control laws. Indeed, with an optimized motion, the control of the compactor will be focus on rejecting modelling errors and external perturbations (the wind, ...). While with ordinary motion, the control law will also try to follow an unfeasible motion which implies important slips on the asphalt so a lower quality for the road.

5. CONCLUSION

A motion planning which takes into account the dynamic model of the compactor is presented in this paper. Path computing for specific tasks of compaction and the computation of timing laws have been developed to this purpose. Then the dynamic model of the compactor is used to check if the computed motion respects physical limitations of the compactor.

Motion planning is a necessary step in the implementation of a dynamic control for the compactor. It uses all works done on the compactor automation in the past years, as well for modelling as for the identification of the compactor. Now, we can consider the automation process of the compactor in order to improve compaction quality.

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