

# Suggestion of the ground stiffness estimative method with the running speed of a plate compactor

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## Abstract

A reversible plate compactor, that is one type of small compaction machine, travels by performing small jumps forward or backward according to its own vibration and resistance from the ground. So it is assumed that its running speed is affected by the stiffness of the ground. Therefore it is thought that it would be possible to estimate ground stiffness by measuring the running speed of a reversible plate compactor using an inexpensive method (stopwatch etc.).

## Keywords –

soil compaction, ground stiffness, reversible vibratory plate compactor, running speed

## 1 Introduction

The vibration behaviour of the vibratory compaction machine is impacted by the stiffness of the ground it contacts, so it is possible to estimate the stiffness of the ground by analysing the signal of the acceleration sensor which equipped on the machine [1][2][3][4]. This technology has attracted interest as far more rational ways of performing quality control of compaction. But this requires instruments called acceleration sensors, calculation modules, and display instruments in addition to the normal compaction machine, increasing the cost. Another problem obstructing their popularization is the difficulty in incorporating them in existing machines already in use

at construction sites.

On the other hand, a reversible plate compactor, that is one type of small compaction machine, travels by performing small jumps forward or backward according to its own vibration. So it is assumed that its running speed is affected by the stiffness of the ground too. Therefore it is thought that it would be possible to estimate ground stiffness by measuring the running speed of a reversible plate compactor using a simple method (stopwatch etc.).

So this research was done in an attempt to establish a method of estimating ground stiffness value based on the running speed of a reversible plate compactor, and evaluating whether or not the value obtained satisfies compaction quality control standards. (Japanese compaction quality control standard value for railway embankments.)

First, a numerical calculation was done to study the relationship of the running speed of the reversible plate compactor with the ground stiffness. Then an actual reversible plate compactor was used to perform an experiment in a laboratory test pit to verify the result of numerical calculation.

## 2 Numerical calculation model

The study of the running speed of the reversible plate compactor based on the numerical calculation was performed separately for vertical direction movement and horizontal direction movement.

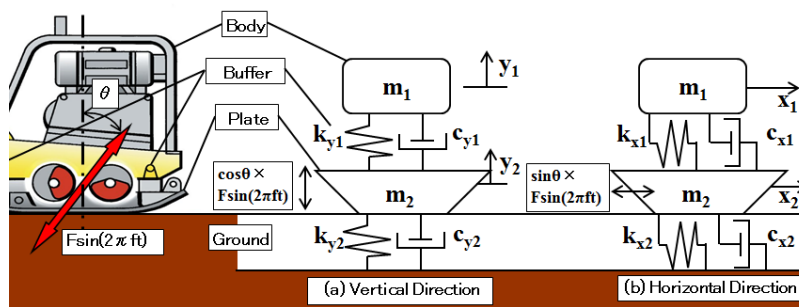


Fig. 1. Reversible plate compactor – ground system model

For the vertical direction, the reversible plate compactor and the ground were replaced with the 2-degrees of freedom vibration model like that shown in Figure 1(a). The equation of motion of this model is shown as formula (1), and solving this using the Runge-Kutta method calculated the vertical displacement accompanying change over time.

$$\begin{aligned} m_1 \ddot{y}_1 + c_{y1}(\dot{y}_1 - \dot{y}_2) + k_{y1}(y_1 - y_2) &= -m_1 g \\ m_2 \ddot{y}_2 + c_{y2} \dot{y}_2 + k_{y2} y_2 - c_{y1}(\dot{y}_1 - \dot{y}_2) - k_{y1}(y_1 - y_2) &= -m_2 g + \cos \theta \times F \sin(2\pi f t) \end{aligned} \quad (1)$$

$m_1$	Mass of upper body	(kg)
$m_2$	Mass of plate	(kg)
$k_{y1}$	Vertical spring constant of rubber buffer	(MN/m)
$k_{y2}$	Vertical spring constant of soil	(MN/m)
$c_{y1}$	Vertical viscous damping coefficient of rubber buffer	(Nsec/m)
$c_{y2}$	Vertical viscous damping coefficient of soil	(Nsec/m)
$F$	Max centrifugal force	(kN)
$f$	Frequency	(Hz)
	Angle of centrifugal force	
$\theta$	(Angle between resultant force of two eccentric shafts and vertical line)	(deg)

Because the ground and exciter plate are connected in the model in Figure 1(a), the subgrade reaction  $N$  (N) represented by formula (2) is also calculated in cases where it is negative. But, because these are not actually connected, the subgrade reaction  $N$  cannot become a negative and represents the exciter plate separating from the ground, or in other words, jumping. So in this calculation, the change of the subgrade reaction  $N$  from positive to negative is considered to represent the start of a jump, and the displacement in the air was calculated, by later applying  $k_{y2} = c_{y2} = 0$  to formula (1) by the time it landed.

$$N = -c_{y2} \dot{y}_2 - k_{y2} y_2 \quad (2)$$

To judge that it has landed, because if, at the same time,  $N = 0$  in formula (2), this formula represents displacement of the ground, solving this calculates the ground displacement while it is in the air and the time when the exciter plate displacement and the ground displacement are reversed is judged to be the landing time.

For the horizontal direction, the reversible plate compactor and ground are replaced by the 2-degrees of freedom vibrating model such as that shown in Figure 1(b). The equation of motion of this vibrating system is

represented by formula (3), and like formula (1), solving it using the Runge-Kutta method calculated the horizontal displacement accompanying change over time.

$$\begin{aligned} m_1 \ddot{x}_1 + c_{x1}(\dot{x}_1 - \dot{x}_2) + k_{x1}(x_1 - x_2) &= 0 \\ m_2 \ddot{x}_2 + c_{x2} \dot{x}_2 + k_{x2}(x_2 - x_0) - c_{x1}(\dot{x}_1 - \dot{x}_2) - k_{x1}(x_1 - x_2) &= \sin \theta \times F \sin(2\pi f t) \end{aligned} \quad (3)$$

$k_{x1}$	Horizontal spring constant of rubber buffer	(MN/m)
$k_{x2}$	Horizontal spring constant of soil	(MN/m)
$c_{x1}$	Horizontal viscous damping coefficient of rubber buffer	(Nsec/m)
$c_{x2}$	Horizontal viscous damping coefficient of soil	(Nsec/m)

To judge ground contact or jump, the results of the above vertical direction calculation were used to calculate displacement in air by applying  $k_{x2} = c_{x2} = 0$  to formula (3) while it was in the air. And it was presumed that while it was on the ground, no slippage between the exciter plate and ground occurred.

In the calculation, the vertical ground viscous damping coefficient  $c_{y2}$  was calculated from formula (4). Here, the damping ratio  $D_{y2}$  was set at 0.4. [5]

$$c_{y2} = 2D_{y2} \sqrt{m_2 k_{y2}} \quad (4)$$

The vertical rubber buffer viscous damping coefficient  $c_{y1}$  can be calculated in the same way as formula (4). The damping ratio  $D_{y1}$  was set as 0.1. [3]

As the horizontal soil spring constant  $k_{x2}$  was calculated using the relational formula (5) with the vertical soil spring constant  $k_{y2}$ . [5] Here the Poisson's ratio  $\nu$  of the ground was 0.4 as the normal value.

$$k_{x2} = \frac{8(1-\nu)^2}{7-8\nu} k_{y2} \quad (5)$$

The horizontal ground viscous damping coefficient  $c_{x2}$  can be calculated from formula (6). Here the damping ratio  $D_{x2}$  was set at 0.3. [5]

$$c_{x2} = 2D_{x2} \sqrt{m_2 k_{x2}} \quad (6)$$

The horizontal rubber buffer spring constant  $k_{x1}$  and viscous damping coefficient  $c_{x1}$ , were set equal to the vertical spring constant  $k_{y1}$  and viscous damping coefficient  $c_{y1}$  considering the shape of the rubber buffer and the way it is installed.

In the above numerical calculation, the ground stiffness is represented as the vertical soil spring constant  $k_{y2}$  based on rectangular loading (plate of compactor). On the other hand, the compaction quality control standard values are provided as the modulus of subgrade reaction  $K_{30}$  caused by circular loading (Plate loading test). Therefore, it is necessary to replace the value  $K_{30}$  with  $k_{y2}$  in order to consider whether or not the ground stiffness value satisfies the compaction quality control standard based on the running speed obtained by the numerical calculation. Formula (7) is used to transform the modulus of subgrade reaction  $K_{30}$  value to the vertical ground spring constant  $k_{y2}$ . [6]

$$k_{y2} = \frac{4\pi br}{L} \cdot K_{30} \quad (7)$$

Here,

$$L = \ln\left(\frac{\sqrt{1+m^2}+m}{\sqrt{1+m^2}-m}\right) + m \cdot \ln\left(\frac{\sqrt{1+m^2}+1}{\sqrt{1+m^2}-1}\right)$$

$$m = \frac{b}{a}$$

$2a$                       Width of plate                      (m)  
 $2b$                       Length of plate                      (m)

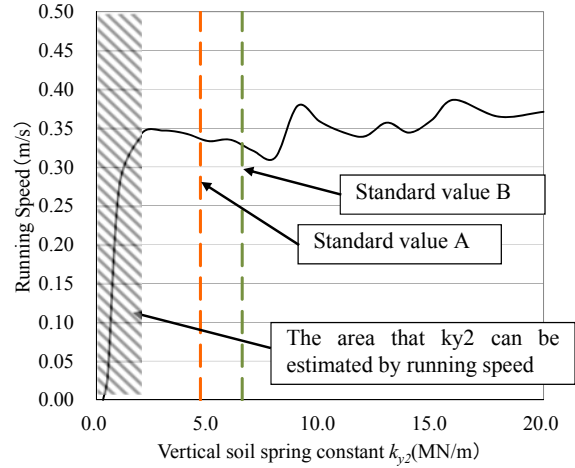
### 3 Study of running speed by numerical calculation

Table 1 shows the machine specifications of the reversible plate compactor and Table 2 shows the ground conditions used for the calculation. Here, the machine specifications were the specifications for the reversible plate compactor that are now used most widely in Japan. The vertical direction soil spring constant  $k_{y2}$ , confirmed behavior of running speed for a broad range of spring constants from extremely soft ground to hard ground that has been compacted, so it was set at  $k_{y2} = 0.3 \sim 20.0$  MN/m for this study.

Figure 2 shows the results of calculating under the conditions shown in Table 1 and Table 2. The horizontal axis represents the vertical soil spring constant  $k_{y2}$ , and the vertical axis represents the running speed calculated based on the horizontal displacement of the plate of the reversible plate compactor. The value obtained by converting the compaction quality control standard value in a railway embankment shown on Table 3 to the soil spring constant by formula (7) is represented by the broken line.

This shows that outside the diagonal line area in Figure 2, the running speed is almost constant even when the soil spring constant changes, and it is difficult

**Table 1.** Machine specification



**Fig. 2.** Soil spring constant – running speed relationship according to numerical calculation

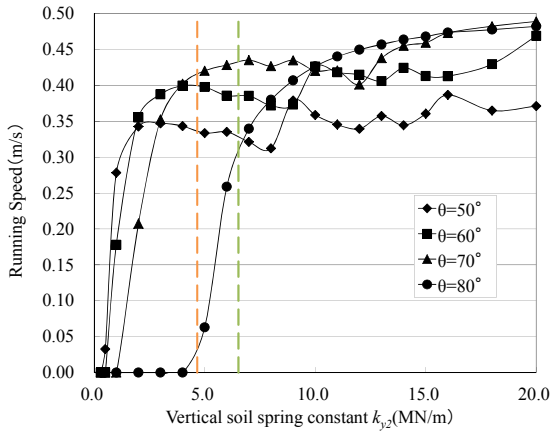
$\theta$ (deg)	50
$m_1$ (kg)	200
$m_2$ (kg)	130
$k_{y1}$ (MN/m)	1.0
$F$ (kN)	45
$f$ (Hz)	73
$2a$ (m)	0.4
$2b$ (m)	0.3
$c_{y1}$ (Nsec/m)	$c_{y1} = 2D_{y1}\sqrt{m_1k_{y1}}$ $D_{y1} = 0.1$
$k_{x1}$ (MN/m)	$k_{x1} = k_{y1}$
$c_{x1}$ (Nsec/m)	$c_{x1} = c_{y1}$

**Table 2.** Ground condition

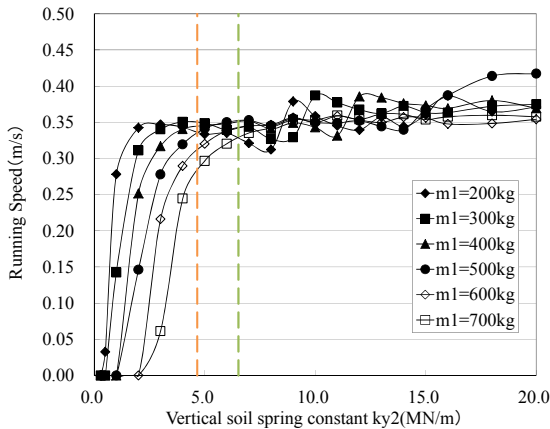
$k_{y2}$ (MN/m)	0.3 ~ 20.0
$c_{y2}$ (Nsec/m)	$c_{y2} = 2D_{y2}\sqrt{m_2k_{y2}}$ $D_{y2} = 0.4$
$k_{x2}$ (MN/m)	$k_{x2} = \frac{8(1-\nu)^2}{7-8\nu}k_{y2}$ $\nu = 0.4$
$c_{x2}$ (Nsec/m)	$c_{x2} = 2D_{x2}\sqrt{m_2k_{x2}}$ $D_{y2} = 0.3$
$\nu$	0.4

**Table 3.** Japanese compaction quality control standard value for railway embankments

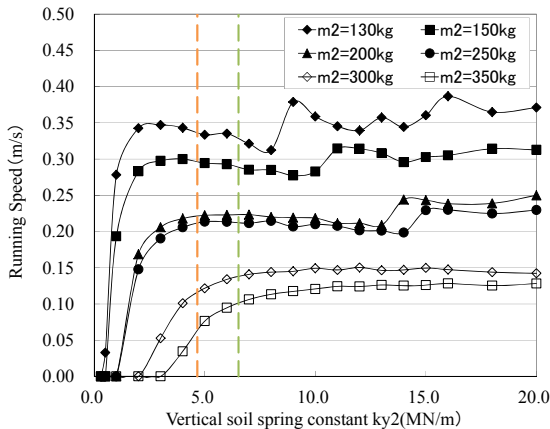
Standard value A	Minimum $K_{30}$ value : 50 MN/m
Standard value B	Minimum $K_{30}$ value : 70 MN/m



**Fig. 3.** Soil spring constant – running speed relationship according to numerical calculation (Difference in angle of centrifugal force)



**Fig. 4.** Soil spring constant – running speed relationship according to numerical calculation (Difference in upper body weight)



**Fig. 5.** Soil spring constant – running speed relationship according to numerical calculation (Difference in plate weight)

speed. In Figure 2, it is possible to estimate the soil spring constant based on running speed in only the diagonal line area, which is a range much lower than the quality control standard value. Therefore, it is impossible to judge whether or not the soil spring constant satisfies the quality control standard value by running speed.

So a study was done to learn if it is possible to judge whether or not the soil spring constant satisfies the quality control standard value by performing minor modifications to the machine.

Figures 3 to 5 shows the results of performing numerical calculations while changing the angle of centrifugal force  $\theta$  from 50° to 80°, upper body weight  $m_1$  from 200kg to 700kg, and the plate weight  $m_2$  from 130kg to 350kg.

According to Figure 3 to Figure 5, in order to judge whether or not the soil spring constant satisfies the quality control standard value based on running speed, the angle of centrifugal force should be increased to 80° or the upper body weight should be increased to 700kg or the plate weight should be increased to 350kg. But, it is not realistic, in terms of both weight and space, to increase the upper body weight by 500kg or the plate weight by 220kg. Therefore, the most realistic and effective modification is increasing the angle of centrifugal force to 80°.

#### 4 Verifying applicability by a test pit experiment

In order to investigate the applicability of the results of performing numerical calculations, an experiment was done in a laboratory test pit.

The experiment was done in a test field simulating an actual execution site prepared in a test pit with dimensions width 5m×length 44.8m×height 4m shown in Figure 6.

**Table 4.** Material properties of the test soil

	Soil(1)	Soil(2)	Soil(3)
Density of soil particle $\rho_s$ (g/cm <sup>3</sup> )	2.675	2.665	2.647
Maximum grain size $D_{max}$ (mm)	9.5	4.75	9.5
Fine fraction content $F_c$ (%)	15.3	57.1	4.3
Maximum dry density $\rho_{dmax}$ (g/cm <sup>3</sup> )	1.674	1.531	1.571
Optimum water content $w_{opt}$ (%)	16.0	24.9	18.2

to estimate the soil spring constant from the running

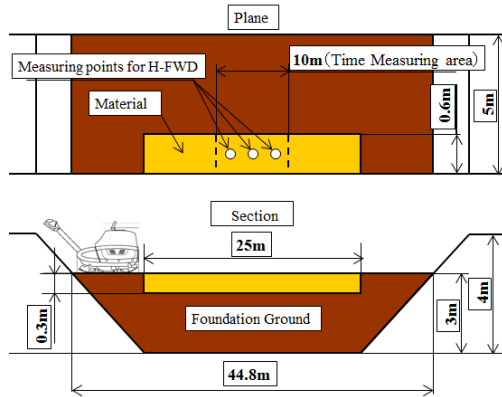


Fig. 6. Test field

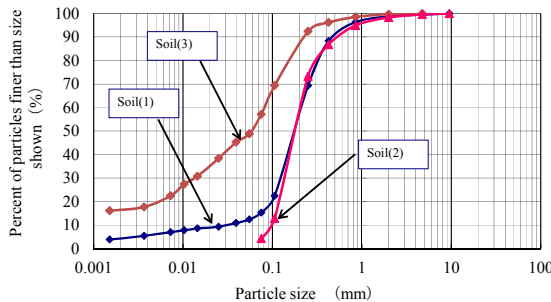


Fig. 7. Particle size accumulation curve

The test field was made according to following procedure. First, soil (1) shown in Table 4 was used to prepare enough compacted foundation ground with height of 3m in the test pit. Next, beside the wall of the foundation ground, a ditch with dimensions of width 0.6m×length 25m×depth 0.3m was excavated. Finally, test soil was embanked so that its finished thickness was 0.3m. Three kinds of test soil were used. The properties of the test soil are shown in Table 4 and Figures 7.

This test field was compacted with a reversible plate compactor with the specifications shown in Table 1 (Compactor1) and with a reversible plate compactor with the machine’s angle of centrifugal force changed to 80° (Compactor2). Therefore other specifications of both compactors are exactly same. And the time required for it to travel 10m in the field was measured with a stopwatch to calculate the running speed. At the same time, the modulus of subgrade reaction after passage by the reversible plate compactor was measured with a compact FWD. And the measurement by the compact FWD was done at three points in the 10m where the above speed was measured, and the average value of the measurements was used.

Figure 8 shows the results of the test. In Figure 8, the modulus of subgrade reaction obtained by the compact FWD ( $K_{30}$  value) was converted to the spring constant of the soil by formula (7).

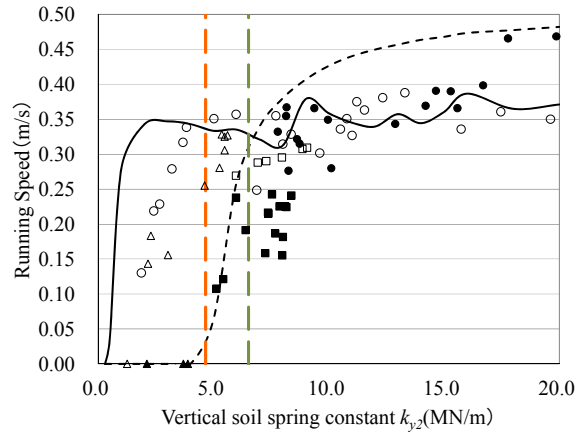


Fig. 8. Soil spring constant – running speed relationship according to numerical calculation

According to Figure 8, the test results and the calculation results show same tendency, when increasing the angle of centrifugal force from 50° to 80°. At angle of centrifugal force of 50°, it is difficult to judge whether or not the soil spring constant satisfies quality control standard values A and B respectively based on the running speed, but changing it to angle of centrifugal force of 80° permits judgments of the correctness of quality control standard values A and B.

However, it is expected that the compaction effect will be decreased because the vertical direction of the centrifugal force decreases when increasing the angle of centrifugal force from 50° to 80°. Therefore, a verification study was done.

Table 5 shows the compaction result (density and compaction degree) after 16 passes with compactor1 (angle= 50°) and compactor2 (angle= 80°) on soil(3).

Table 5. Compaction result after 16 passes

	Compactor1 (angle= 50°)	Compactor2 (angle= 80°)
Density (g/cm <sup>3</sup> )	1.559	1.511
Compaction degree (%)	99.2	96.2

According to Table 5, the compaction degree of compactor2 is lower approximately 3% than the compaction degree of compactor1. Therefore, it is better to use compactor1 for compaction work, and to use compactor2 for measurement work only.

## 5 Conclusions

The relationship of the running speed with soil spring constant according to numerical calculation was considered. The results clarified the following points.

(1) In the case where the reversible plate compactor that was not modified (angle= 50° ) was used, the range in which it is possible to estimate the soil spring constant using the running speed is in a range far lower than quality control standard values and it is difficult to use the running speed to judge whether or not the quality control standard value is satisfied.

(2) By changing the angle of centrifugal force to 80°, it is possible to judge whether or not the soil spring constant value satisfies the compaction control standard value in a railway embankment based on the running speed.

And the results of the numerical calculation were confirmed by an experiment using two reversible plate compactors. Compactor1 has the angle of centrifugal force as 50°. And compactor2 has the angle of centrifugal force as 80°. The results clarified the following points.

(3) Increasing the angle of centrifugal force to 80°, permits the estimation of whether or not the soil spring constant satisfies the compaction quality control standard value in a railway embankment based on the running speed.

(4) Because the compaction effect of compactor2 (angle= 80° ) is lower than the compaction effect of compactor1 (angle= 50° ), it is better to use compactor1 for compaction work, and to use compactor2 for measurement work only.

The method applied for this research is extremely simple if a stopwatch etc. is used, and it will help to easily rationalize quality control methods.

One major problem is the gap between the numerical calculation results and the test results. We wish to continue studies to bridge this gap in the future.

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