Reduced-order Models for Supporting Energy Audits of Buildings

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Abstract –

An energy audit is a standard process to support the decision-making in the area of energy management. Nevertheless, an energy auditor does not succeed as easily and quickly in identifying all the energy streams in a facility in order to decide whether retrofitting the audited object is beneficial. In fact, energy auditing of buildings is usually a time consuming and expensive process, due to efforts required for data collection and modelling of audited objects. Therefore, a decision-making tool should support the modelling phase and this purpose might be achieved by means of reduced-order modelling, after a quick data acquisition, through on-site measurements. As highlighted in literature, reduced-order models, also called grey-box models, showed their reliability to achieve a suitable description of the thermal response of buildings in a short time. Besides, they are more cost-effective and their thermal parameters can be usually extracted in short time. Consequently, deriving a lumped parameter model, by means of measurements collected in a rather short period, could allow the owners and managers of real estates to perform fast and cheap preliminary assessment on the opportunity to implement energy renovation actions. Starting from a test performed in the machine laboratory of our department at the Università Politecnica delle Marche, the investigated empirical procedure for deriving grey-box models will be provided and energy saving opportunities, within an energy retrofit, will be analysed as an example about the application of this procedure.

Keywords –

Energy audit; Reduced-order models; Energy retrofit

1 Overview of the energy management system

Buildings are responsible for almost half of the total primary energy use and the consequent greenhouse emissions worldwide. Even if current energy systems are improving, they do not satisfy yet the acceptable limits for the efficiency [1]. In this regard, the European Commission puts effort in promoting measures for the reduction of energy needs and greenhouse gas emissions, by recommending energy retrofitting projects. One of the targets of the European Directive 2012/27/UE is to retrofit 3% of the existing public stock, while upgrading it to current legislation [2]. Several actions in favour of the Energy Performance Contracting (EPC) market in Europe have been put into practice, e.g. within the Intelligent Energy - Europe (IEE) programme. The Energy service companies (ESCos) have been identified as key players in the implementation of EPC investments [3]. The EPC is a contractual arrangement between the beneficiary and the provider of an energy efficiency improvement, where investments in an energy efficiency project are paid in relation to the achievement of energy savings produced [2]. About this, contracts offered by ESCOs usually concern all the energy services, included energy audits. In such a way, facility owners and managers are supported in improving ageing and inefficient assets, transferring risks and reducing meanwhile energy costs [4]. The methodology of EPC is oriented to ensure the quality of performance. In fact, a strong commitment from the government to the use of EPC has been observed in several countries [5]. However, the success of EPC projects depends the most on the correct estimation of the expected energy savings. For the ESCOs, it is crucial to evaluate the feasibility of the proposed energy conservation measures (ECMs) to achieve an acceptable energy use baseline, according to a contractually agreed level of energy performance. A detailed energy audit is typically carried out at a first stage, in order to identify the energy saving opportunities.

Therefore, a decision support system for energy auditors might be beneficial in several scenarios, as for example:

- in case owners of multiple buildings must give a prority for refurbishment or every requalification;
- in case the energy enhancement of any existing buildings is conditioned upon the possibility of repaying back the initial investment with annual savings from energy reduction;
- in any case it is essential to select the most appropriate actions or strategies to reduce buildings' environmental impact.

Despite the fact that building simulation models are more effective to understand the implications in the energy use policies, they usually entail an expensive and time-consuming implementation process [6].

What makes the things worse, referring to an Ashrae level 3 of energy auditing [7], is that a detailed audit needs a substantial time to set the data and provide the detailed dynamic model of the audited object; besides, a considerable computational effort is necessary to run the large number of simulations, usually required. Furthermore, uncertainty in model inputs often does not guarantee an accurate prediction of the energy performance, particularly for existing buildings, in which a lot of information about intrinsic characteristics remain unknown. In order to develop quick and efficient diagnoses, reduced-order models represented by grey-box models are preliminary assessed in this paper. The reduced modelling can be applied in many fields in construction (i.e. safety, occupancy dynamics and structural damages) because of its reliability and usefulness for simulations, advanced controls, real-time predictions. Recently, it is widely used to improve energy management strategies. Relevant literature provides many examples and case studies about modelling the energy dynamics of buildings. In fact, parameters of the reduced models are extracted in real time; this makes the estimation of the thermal response of a building in its current state feasible, and this information reusable to make predictions about its expected behaviour in further analyses.

Our study started with a fact-finding survey on the reduced-order models and the use of identification techniques from literature. Then, we investigated the robustness of such techniques in a real case, with the purpose of defining experimentally an efficient decision support tool for the energy retrofitting. Therefore, we studied the possibility of identifying the unknown properties of a mathematical model based on partial observations of the heat dynamics of the building, that is, in our case, a small data collection of temperature measurements. To sum up, the modelling phase is discussed in Section 2. The preliminary test is described in Section 3, including the experimental set up and data generation. The results of the study are provided in Section 4. Conclusions are given in Section 5.

2 The modelling phase

Buildings modelling for advanced control, in particular to derive a total model for the heat dynamics of buildings, has been the purpose of several research groups in the last years, also related to the increasing attention for the energy savings. Therefore, several different approaches have been described and many different methods for the dynamic analysis of energy use in buildings have been developed. In literature, the problem of deriving a suitable reduced model is solved referring to two different approaches. The first one is a purely physical approach, in which it is possible to investigate the physical behaviour of a building. However, the inaccuracy of control strategies relying on these detailed models represents a clear weakness, since the real building parameters are often unknown in existing buildings [8]. Contrarily, the second approach concerns the statistical black-box models, in which the parameters result from a statistical relation between the input and the output of the system, because they are generally disconnected from physical systems. The later introduction of the grey-box models represents a "half-way" modelling to overcome the lacks of both modelling methods. In this work, the word "grey" is meant as defined by Reynders and Madsen [8-9], namely the combination of a prior knowledge of system dynamics, to define the model structure, and statistic methods, to estimate the unknown parameters.

In order to turn energy audits into fast and cheap processes, no effort must be employed for extensive simulations, thus our proposal is to train the model from a limited set of experimental data. Consequently, grey-box modelling represents an interesting solution because of the direct link between the physical laws of building dynamics and a strong framework of continuous stochastic differential equations formulated in a state space form [10]. The state-space representation provides a mathematical description to analyse dynamical systems with multiple inputs and outputs. A dynamical system is characterized by its state variables. The state variables are stacked in a time varying state vector, referred to as the system state.

Regarding grey-box models, the state-space form derives from the physical laws, which are formulated by first-order stochastic differential equations. Therefore, the model structure is a linear time-invariant model, represented by the following equations:

$$dX(t) = A(\theta)X(t) + B(\theta)U(t) + \sigma(\theta)d\omega \quad (1)$$
$$Y(t) = C(\theta)X(t) + D(\theta)U(t) + \varepsilon \quad (2)$$

The states of the system are represented by the vector *X(t)* which correspond to the temperatures of the relevant building components, in the case of modelling the thermal response of buildings. *U(t)* is a vector containing the measured inputs of the system, which can be either inputs or controlled outputs (heating and ventilation system, solar gains, outdoor temperature, etc.). The measured output *Y(t)* is function of the states and the inputs. A, B, C, D are nonlinear functions, depending on the parameters' vector θ. The matrix A characterizes the dynamic behaviour of the system and the matrix B concerns the influence of input signals entering the system. ω is a random function of time, assumed as a standard Wiener process with independent increments; the mutual independent ε is a white noise process representing the measurement error.

To identify the unknown parameters of the vector θ, the ML estimation method is adopted, because it was validated by Madsen [9-11] and successfully applied in several applications. According to statistics, the MLE is the hypothetical population value that maximizes the likelihood of the observed sample, that is, the value most likely to have generated the sample actually observed. The maximum likelihood method thus provides a means of estimating a set of parameters characterizing a distribution of an observed phenomenon, if we know the form of this distribution.

The structures for modelling the heat dynamics are derived from the analogy with electric circuits. According to the model order, the thermal mass of the building is lumped to a discrete number of capacitances. In the thermal network, the capacitor *C* represents the active thermal capacity of the building zone for storing heat, while the thermal resistance *R* is linked to material properties and affects the heat flow across building layers at different temperatures. However, a given parameter is not identified with the same physical correspondence in each model. The models used in this work (Figure 1) are referred to the 1st, 2nd and 3rd-order models. To simplify the problem, in this early stage, the case study does not entail the internal and solar gains.

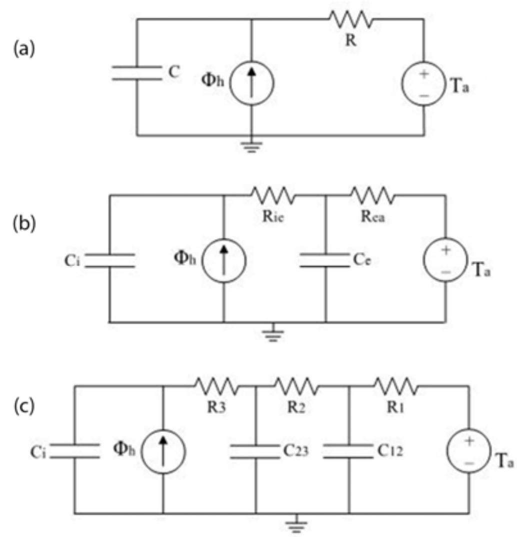

Figure 1. RC-networks of the grey-box models: 1st, 2nd, 3rd order models, from top to bottom

The stochastic differential equations, related to the state space form, are deduced from principles of energy balance at nodes and of heat transfer. For the first order model (Figure 1a), the only state is described by:

$$dT = \frac{1}{RC}(T_a - T)dt + \frac{1}{C}\Phi_h dt + \sigma d\omega \quad (3)$$

The two states grey-box model TiTe (Figure 1b), is defined by the following equations:

$$dT_i = \frac{1}{R_{ie}C_i}(T_e - T_i)dt + \frac{1}{C_i}\Phi_h dt + \sigma_i d\omega_i \quad (4)$$

$$dT_e = \frac{1}{R_{ie}C_e}(T_i - T_e)dt + \frac{1}{R_{ea}C_e}(T_a - T_e)dt + +\sigma_e d\omega_e \quad (5)$$

Consequently, the third order model (Figure 1c) is represented by these ordinary differential equations:

$$dT_i = \frac{1}{R_3C_i}(T_{23} - T_i)dt + \frac{1}{C_i}\Phi_h dt + \sigma_1 d\omega_1 \quad (6)$$

$$dT_{23} = \frac{1}{R_3C_{23}}(T_i - T_{23})dt + + \frac{1}{R_2C_{23}}(T_{12} - T_{23})dt + \sigma_2 d\omega_2 \quad (7)$$

$$dT_{12} = \frac{1}{R_2C_{12}}(T_{23} - T_{12})dt + + \frac{1}{R_1C_{12}}(T_a - T_{12})dt + \sigma_3 d\omega_3 \quad (8)$$

Whereas Φ_h [kW] and T_a [°C] constitute the input measured values of the power provided by the heater and the ambient temperature, the resistance/capacitance parameters, R [°C/kW] and C [kWh/°C], are estimated as well as the stochastic terms [°C] and the initial values of the other temperatures [°C]. The time is expressed in hours. For every system of differential equations, the output equation is represented by the discrete time measurement equation, which takes into consideration the data-driven:

$$Y_k = T_{ik} + e_k \tag{9}$$

where k is the point in time t_k of a measurement,
where Y_k [°C] is the measured interior temperature,
where T_{ik} [°C] is the state at the time t_k,
where e_k [°C] represents the measurement error, which is assumed to be a Gaussian white noise process with variance σ^2 [9]. In fact, a real context provides a number of disturbances to be taken into account in the modelling of the system, to not obtain largely distorted results.

3 The preliminary test

The main purpose of our experimental approach is the investigation of grey-box modelling in simulating real and complex cases, in order to implement it in a decision support tool for energy audits of existing buildings. To this aim, a first controlled situation was simulated in a machine laboratory at the Università Politecnica delle Marche and it will be described in this section.

3.1 Experimental set up

The entire laboratory of our department was used to create a first controlled situation. The changing room of our laboratory was chosen as the test-room, while the entire laboratory was one of the boundary conditions of temperature. The floor of the changing room, whose area is about 13 m^2, was covered with polystyrene sheets and bricks to insulate it from the ground. Besides, in this testing area an electric heater of nominal power 400 W and real measured power 377,6 W, generated the heat flux. This heat input was controlled by means of a timer, programmed with the sequence of PBRS signal, according to Madsen’s procedure [11].

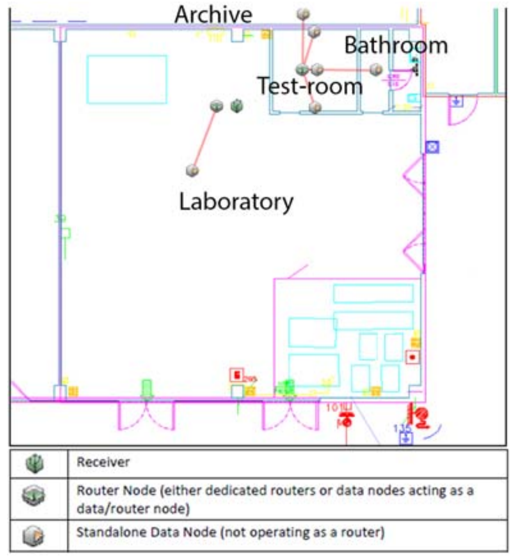

Figure 2. Map extracted from HoboNode Manager, representing the laboratory with sensors deployment

3.2 Data generation

A Hobo Zw Series Wireless Network was installed in the test room to measure temperatures of the various environments. The map in Figure 2, extracted from the hobo software interface (HoboNode Manager), shows the deployments of the data node, the routers and the receiver. The receiver was connected to a computer, which was used to set up the network and to offload the data from the receiver at the site. The six available sensors were installed in the area: three in the test-room, two on the internal walls and one hanging from the ceiling; one close to in the bathroom; another one in the laboratory and the last sensor was hung in the archive. This deployment of the sensors was dictated by the requirements posed by the adoption of numerical techniques (see section 2). In addition, a thermo-anemometer, VT200 Kimo, was placed in the changing room with two probes to measure the temperature of the internal air and of the floor. Finally, a Kestrel climate station was positioned on the roof of the changing room. These measurements were useful to test the network acquisition. Only the Hobo monitoring network provided the data for the identification process. The applied data covered a period of about 18 days, from 2nd to 20th July 2015. The number of total observations was equal to 27131, corresponding to an acquisition per minute. Prior to estimating the model parameters, the data were resampled at a sampling time of 15 minutes. This sampling time was chosen, according to Shannon’s theorem, based on the smallest period of the electric heater that is 30 minutes. Temperature data from the sensors on the walls of the changing room were not considered in computing because too much affected by the convective motion of the air inside. At the same time, also temperature measurements of the bathroom were later excluded because they were too similar to the ones of the laboratory. Data of the first day of the test, July 2nd, were cancelled because corrupted. Some photos of the test set up are shown in Figures 3 to 6.

Figure 3. The testing area of the changing room

Figure 4. The sensor positioning in the laboratory

Figure 5. Overview of the laboratory

Figure 6. The archive bordering the laboratory

4 Results

In order to evaluate the robustness and reliability of the reduced-order modelling, in the data elaboration both the suitable structure of the lumped model and the influence of data period were investigated.

As found in the referenced literature, the simplest model, that is 1R1C network (Figure 1a), was analysed in the data period 3-10 July. As already known, the first reduced-order model was quite unrealistic, in fact it revealed insufficient to describe the heat dynamics of our test room. In fact, evaluating results of parameters estimation in CTSM-R, errors were meaningful and model validation [12] was not satisfied. After that, the 2R2C network (Figure 1b) was considered in the same data period of the first model, because measured data were more stable. The estimated results of this second-order model better fitted with measured plots. Three second-order models were tested, differentiated as for the boundary condition T_a in Equation 5. In the first case, the ambient temperature T_a was represented only by the laboratory temperature T_l. In the second case, the weighted average of the laboratory temperature (T_l) and the archive one (T_a), weighted on the respective surface, calculated it. In the last case, a coefficient (K_a) was introduced to weight the archive temperature as compared to the one of laboratory. This coefficient was estimated by CTSM-R improving estimation results and the Equation 5 was rewritten in this way:

$$dT_e = \frac{1}{\mathrm{R}_{ie}\mathrm{C}_e}(\mathrm{T}_i - \mathrm{T}_e)\mathrm{dt} + \\ + \frac{1}{\mathrm{R}_{ea}\mathrm{C}_e}(K_a\mathrm{T}_a + (1 - K_a)T_l - \mathrm{T}_e)\mathrm{dt} + \sigma_e \mathrm{d}\omega_e \tag{10}$$

Then, to fix the best data period for the parameters estimation, a 3C3R model (Figure 1c) was considered; it was a natural extension of the second-order model (including the K_a parameter), for taking into account the more variable dynamics of the last test.

Four intervals were implemented for the ML estimation: 3-10 July, 8-15 July, 3-15 July, 3-20 July. The root of the mean squared error for the 1-step prediction and simulation of the identified model were investigated for each data set. The best data set was chosen depending both on the long period of acquisition and on the smallest RMSE value in simulation, that was equal to 0.1574. It corresponded to the period 3-15 July.

4.1 The reduced model validation

Finally, the last portion of data was used to validate the model of Equations 6-7-11. The validation was carried out by performing a simulation of the test-room indoor temperature from 16th until 20th July, using the 3C3R model with the estimated values in CTSM-R.

$$dT_{12} = \frac{1}{\mathrm{R_2 C_{12}}}(\mathrm{T_{23}} - \mathrm{T_{12}})\mathrm{dt} + \\ + \frac{1}{\mathrm{R_1 C_{12}}}(K_a \mathrm{T_a} + (1-K_a)T_l - \mathrm{T_{12}})\mathrm{dt} + \sigma_3 \mathrm{d}\omega_3 \tag{11}$$

The validation phase was alike performed for the second and the first reduced-order models seen before, using the best data set for both the parameter estimates (always including the K_a parameter). The first order model resulted as non-representative of the heat dynamics of the test room, while the third order model, which fitted better, presented some parameters with a Pr-value over 0.05 that indicated an over-parametrization of the model. Thus, the second order model was considered the best choice in terms of parameters validation in R and in terms of RMSE value in simulation, which was equal to 0.0745, as shown in Figures 7-8.

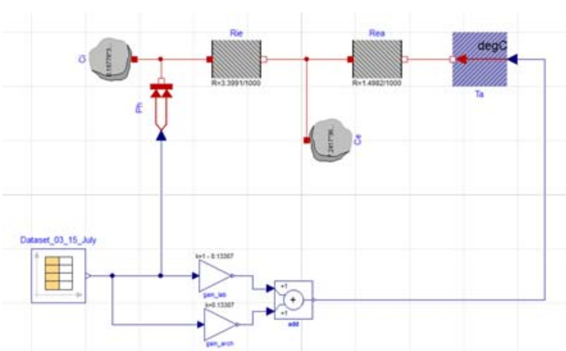

Figure 7. The 2nd-order model in simulation environment

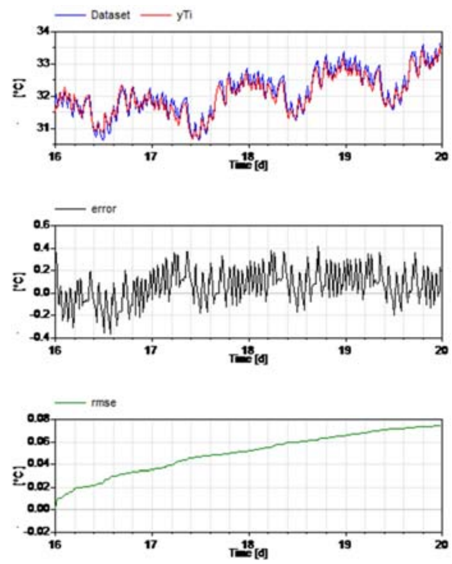

Figure 8. Results of the 2nd-order model for the validation phase

It was also possible to verify that replacing the K_a in the Equation 10 with its estimated value, that was 0.13307, the results of the estimation (Table 1) were the best for model validation, without any Pr-value over 0.05 (Table 2), that is, the model is not over-parametrized. The Pr-value represents the probability that the particular initial state or parameter is insignificant, i.e. equal to zero [12]. The variance of the system noise is represented in CTSM-R by the exponential function of the estimated parameters p11 and p22; while the variance of the measurement error in the output equation (see Equation 9) is the exponential function of the estimated parameter e11 (Tables 1-2).

Table 1. Parameter estimation in CTSM-R for the 2nd reduced-order model with fixed K_a

	Estimate	Std. error	t value
Ti0	2.9692e+01	4.9569e-02	5.9901e+02
Te0	2.9224e+01	5.7916e-02	5.0458e+02
Ce	7.4006e+00	9.8259e-01	7.5317e+00
Ci	1.7640e-01	1.6073e-03	1.0974e+02
e11	5.6405e+00	4.8150e-02	1.1714e+02
p11	1.0883e+01	1.6143e-01	6.7417e+01
p22	2.9891e+00	8.6550e-02	3.4536e+01
Rea	1.4677e+00	1.8143e-01	8.0896e+00
Rie	3.4447e+00	4.8751e-02	7.0658e+01

Table 2. Validation of the estimation results in CTSM-R for the second reduced-order model with fixed K_a

	Pr(>|t|)	dF/dPar	dPen/dPar
Ti0	0.0000e+00	-5.2349e-05	0

Te0	0.0000e+00	-4.8034e-06	0
Ce	9.7255e-14	-1.2173e-05	0
Ci	0.0000e+00	5.2610e-05	0
e11	0.0000e+00	-4.9432e-06	0
p11	0.0000e+00	8.7844e-05	0
p22	0.0000e+00	2.1533e-05	0
Rea	1.3323e-15	-1.3020e-05	0
Rie	0.0000e+00	-4.9613e-05	0

4.2 A validation of the reduced model for the energy audit

Grey-box modelling was applied to set reduced models and estimate thermodynamic properties of a building, starting from a data acquisition campaign. In the end, in order to provide the auditor a further support in decision-making, the influence of data acquisition period was investigated. Thus, identified the reduced model structure, a parametric study was carried out to evaluate the optimal season for a data acquisition campaign. In this validation for the energy audit, the annual energy consumptions of the heating system were analysed. Results were provided in terms of NMBE and CV-RMSE values, calculated on an hourly basis relatively to the reduced model and to the baseline. Therefore, they were compared and limits imposed by the ASHRAE guidelines were verified [13]. The followed procedure is summarized below.

Using our case study (see Section 3), the input data implemented in the identified 2nd-order model (see Paragraph 4.1) were derived from an annual simulation of the detailed building model (Figure 9), which was carried out by means of the Dymola software.

Then, the parameters estimation was run in CTSM-R, varying the data input period, that is a month of the year. In this case, the heat input was a PRBS signal. At this level, the reduced model was defined in its physical properties, as seen before. Subsequently, the dynamics of the model was simulated in Dymola environment, according to the different simulation periods for the comparison. In the simulations, the heat input was represented by a thermostatic control, in which the set-point temperature was fixed at 18°C (Figure 10).

Before comparing the energy consumptions, a comparison of temperatures between the reduced model and the baseline was carried out. ASHRAE limits of NMBE and CV-RMSE [13] were verified; moreover, it was observed that temperatures simulated in the model had the same trend of the baseline ones, even with a thermostatic control operating. Ultimately, the comparison of the energy consumptions in terms of NMBE and CV-RMSE showed that the optimal season for data acquisition campaigns must match the one of the heating system operating (Figure 11).

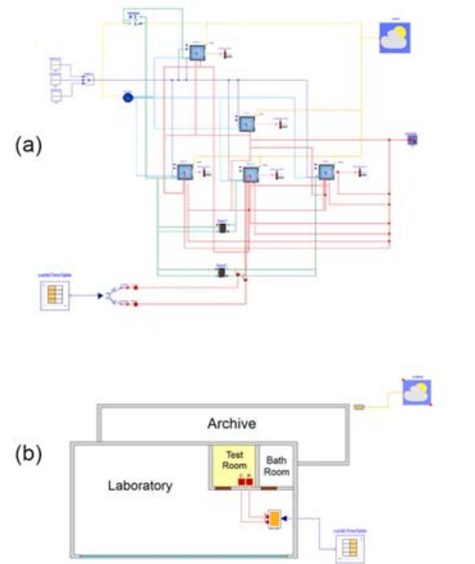

Figure 9. Detailed model of the building in Dymola environment (a) and the top layer of Dymola model (b)

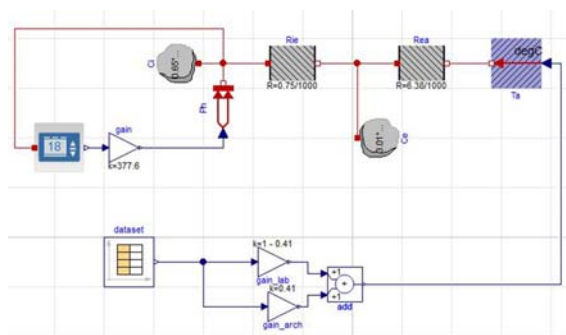

Figure 10. Identified reduced model of the building with thermostatic control

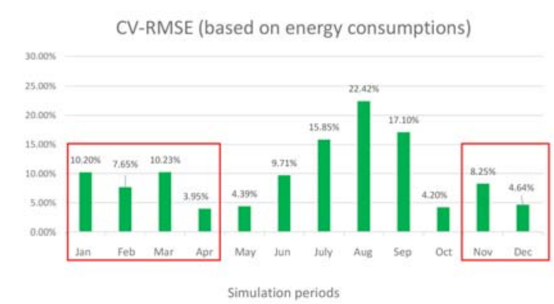

Figure 11. Comparison between CV-RMSE values in terms of energy consumptions for the different simulation periods

5 Conclusions

An empirical approach based on reduced-model strategy is provided to support energy audits of buildings. For this purpose, a preliminary test of a controlled real situation is performed, starting from a very little set of data. The influence of the data period is investigated and the results show the importance of measurements containing adequate dynamics, for the robustness of the identified reduced models.

The best predictions are obtained for the 2nd-order model; nevertheless, the reliability of the procedure is balanced on the comparison with real conditions. Subsequently, results of the parametric study prove that annual predictions on energy consumptions require a model structure of limited complexity; besides, the procedure shows its reliability in real operating conditions.

Further research should be able to generalize, as much as possible, this model identification approach. Future work should aim at testing other real and more complex cases, finding out saving opportunities of energy retrofits, in order to provide an efficient decision support tool for the energy audits of buildings.

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