# Integrating Activity Scheduling and Site Layout Planning

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**Abstract –**

**A temporal-spatial scheduling model is used to consider the location and movement of equipment, material and temporary structures on a construction site over time, ensuring efficient yet safe processes. The model ties together activity scheduling and dynamic site layout planning, fully integrating two domains that have so far been treated separately. Their integration is facilitated by modeling temporary site objects like cranes as singularity functions. These objects are explicitly linked in the model with work spaces through Boolean operators.**

**Keywords –**

**Scheduling, Singularity Function, Site Layout, Cranes**

## 1 Introduction

Construction activities are tightly interlinked with a construction site in terms of the timing of said activities, the resources they require, and the physical space that they occupy on the site. As type and nature of activities change throughout the project, construction sites are correspondingly dynamic, and their geometric layout will change in accordance with the changing activities. Yet while advances have been made in developing planning methods that take into account the location of activities on site, activity planning and site planning have so far not yet been fully integrated. This prevents them from being planned simultaneously and efficiently. This research presents a foundation for fully integrated planning of both construction activities and sites.

### 1.1 Phased and Dynamic Approaches to Site Layout Planning

Andayesh and Sadeghpour [1] distinguished three approaches of static, phased, and dynamic site layout planning. Traditionally, the *static* approach has assumed that the layout of a construction site remains the same throughout the project duration. This prevents using the same space on the site for different objects, even if they are not required at the same moment in time (e.g. [2]).

An improvement upon was the *phased* approach, in which the project was divided into discrete phases, and a site layout was planned for each phase [3]. It allowed that one location could be used for different objects in different phases. But as Andayesh and Sadeghpour [1] criticized, objects can enter or leave the site at moments other than the beginning or end of a phase. By reserving their space for an overly long duration the phased approach reduced the efficiency of the final site layout. Consequently, they introduced a *dynamic* approach, which considered the actual duration for which objects must occupy space, and implemented this approach in a model based on energy principles [4]. Nevertheless, this model presupposed the existence of a schedule for the activities, upon which the site layout plan can be based.

### 1.2 Space and Time Approaches

Representation of physical space within schedules has been an ongoing effort. Location-based scheduling methods were developed that considered both the timing and location of work [5]. They used diagrams to track how work progresses from one location to another. But such locations have been limited to a physical division of the site based on either a single type of repetitive unit (such as floors in buildings), or a single path across the site (for linear infrastructure projects). This has failed to reflect the actual use of space on real construction sites, which is typically much more complex than that.

Other studies focused on linking specific building components with scheduled activities and defining the required work spaces in computer-aided design (CAD) and building information modeling (BIM) systems [6]. The main goal of these four-dimensional (4D) systems has been to identify spatial conflicts between activities that have been scheduled to be simultaneously executed.

### 1.3 Need and Justification

Efficient planning of construction project execution requires a model that can integrate activity scheduling, work-space planning and site layout planning fully. In comparison, existing methods for dynamic site layout planning can be used only after all activities have been scheduled. Yet layout of the construction site is often *less* flexible than the timing of activities. Simultaneous planning can therefore give significantly more efficient solutions than those that existing methods provided.

# 2 Methodology

## 2.1 Goal and Objectives

The goal of this research is to define a temporal-spatial scheduling model that integrates the planning of construction site layout and activities by representing and explicitly linking these constructs mathematically.

Three research objectives are established as follows:

a. Modeling temporary site objects, including their changing location and work space envelopes, as singularity functions;
b. Linking temporary site objects with work spaces with logical relationships that reflect efficiency and safety requirements;
c. Verifying that site objects are eventually optimally located in accordance with their defined relations with planned activities.

## 2.2 Research Approach

This research defines an integrated mathematical model which can be used to implement planning and optimization methods for scheduling and site layout planning. Singularity functions will be used to define the required constructs and relations, as they provide a flexible mathematical framework that allows modeling the different aspects of construction projects.

This research will focus exemplarily on the task of planning the location of cranes on construction sites [7]. Cranes often constitute a bottleneck resource that leads to delays as crews wait for material or components to be lifted. At other times, cranes may be underutilized and remain idle, because activities have not been optimally scheduled. Cranes could also create safety hazards as other activities need to be conducted underneath areas into which objects must be lifted and placed. Given these problems, it is important to simultaneously plan the timing and location of activities and cranes on sites.

# 3 Singularity Functions

## 3.1 Definition

In its general form, the pointed-bracket operator per Equation 1, which is the basic term within all singularity functions, performs a case distinction. A function of $x$ is either zero before the activation cutoff $a$ on the $x$-axis or is evaluated as a polynomial with strength $s$ and growth $n$ from $a$ onward. Exponents $n$ can model shapes, e.g. a constant ($n$ = 0), linear growth ($n$ = 1), parabolic growth ($n$ = 2), etc. Such singularity functions enable unlimited additive composition for behaviors of any complexity from simple terms. Multiple terms can be simplified if their $n$ and $a$ are identical to shorten longer equations. To deactivate a function, its term is simply subtracted at the later cutoff (including any already attained value). An index R will mark its default right-continuity (active at $x \geq a$), index L the analogous left-continuity ($x > a$).

$$y(x) = s \cdot \langle x-a \rangle^n = \begin{cases} 0 & if\ x < a \\ s \cdot (x-a)^n & if\ x \geq a \end{cases} \quad (1)$$

## 3.2 Extension to 3D

Since their introduction to project management [8], systematic applications have been broadened beyond work and time in linear schedules to also encompass resource counts that remain linked with the underlying schedule to gain a flexible resource leveling model [9]; cost and pay over time to generate cash flow equations [10], time value of money that grows from interest [11], and calendar date conversions including holidays [12].

Recently the possibility to unify spatial and temporal aspects of project plans has begun to be explored. Their three-dimensional (3D) formulation multiplies pairs of on-and-off terms along both coordinate axes [13], which model two projections of the same 3D volume. On both axes they are activated and deactivated at the start and finish of the range. This allows block and ramp shapes within a 3D coordinate system of two horizontal spatial axes $x_1$ and $x_2$ and a vertical time axis $z$. But this model was limited to just shapes that are parallel to an axis, which limits its usefulness to only sites that are shaped like a city block. Other shapes were not explored. Float, the flexibility of activities regarding their timing, was defined as postponing or a productivity change (i.e. shift or rotation) within an overlap zone with a successor. Its availability was represented in form of a 'heat map' [14].

Besides work areas, these authors have defined paths on a site within a 3D space-time coordinate system. This approach has overcome the previous limitation that only axis-parallel shapes could be defined. A vector-inspired formulation has been created [15], which first defines a line segment and then gives it a width. It could act as a safety buffer for traveling equipment, and more broadly allows defining irregularly shaped work areas. Also, the need to introduce Boolean operators has been addressed, so that work areas can be related by AND (intersection), OR (inclusion), and XOR (exclusion) [16]. These are applied by multiplying, adding, or subtracting so-called signal functions of two geometric shapes. An improved version of a vector-based model for any rectangular geometric area that is rotated in the $x_1$-$x_2$-plane against the two spatial axes is given by Equation 2. From its start to its finish coordinates ($S$ to $F$), the value of the variable $h_1$ grows from 0 to 1 along the vector direction ($\Delta x_1$, $\Delta x_2$, $\Delta z$). Defining a perpendicular half width $w$ / 2 to the left and right with variable $h_2$ grows from 0 to 1 creates the desired area $h_1 \times h_2$. These widths per geometry are secondary vectors, which are scaled to a unit length of 1.0 by dividing by the vector length $l = \sqrt{(\Delta x_1^2 + \Delta x_2^2 + \Delta z^2)}$. Each is then multiplied with the desired width $w$ / 2. Their two directions are ($\Delta x_2$, -$\Delta x_1$, $\Delta z$) and (-$\Delta x_2$, -$\Delta x_1$, $\Delta z$). Their vertical $z$-value can be omitted for purely spatial elements without time aspect.

$$Area(h) = \begin{pmatrix} x_{1S} \\ x_{2S} \\ z_S \end{pmatrix}_{Start} + \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta z \end{pmatrix} \cdot \left( \langle h_1 - 0 \rangle_R^1 - \langle h_1 - 1 \rangle_R^1 - \langle h_1 - 1 \rangle_L^0 \right) \pm \frac{w}{2 \cdot l} \cdot \begin{pmatrix} -\Delta x_2 \\ +\Delta x_1 \\ 0 \end{pmatrix} \cdot \left( \langle h_2 - 0 \rangle_R^1 - \langle h_2 - 1 \rangle_R^1 - \langle h_2 - 1 \rangle_L^0 \right) \quad (2)$$

The model for a geometric solid is derived from said area model by adding a third variable $h_3$ that grows from 0 to 1 along the vector direction (0, 0, $\Delta z$) per Equation 3. Since the vertical axis $z$ represents time, the $\Delta z$ in the vector is the duration of work. Following a vector from ($x_1$, $x_2$, $z$) = (28, 61, 0) to (52, 93, 0), $h_1$ moves into the direction that is the difference (24, 32, 0) of these two vectors. It travels $\sqrt{(24^2 + 32^2)} = 40$ length units as a line segment. To create an area, $h_2$ moves perpendicularly into two opposite directions. Its vectorized singularity function for $w$ = 30 is (28, 61, 0) + (24, 32, 0) · ($\langle h_1 - 0\rangle^1_R - \langle h_1 - 1\rangle^1_R - \langle h_1 - 1\rangle^0_L$) ± 15/5 · (-4, +3, ±0) · ($\langle h_2 - 0\rangle^1_R - \langle h_2 - 1\rangle^1_R - \langle h_2 - 1\rangle^0_L$). Moving it along the time axis $z$ with $h_3$ into direction (0, 0, 8) creates the full temporal-spatial activity per Figure 1. Its singularity function extends the insufficient traditional vector into a solid with the custom location and orientation as desired.

$$Solid(h) = Area(h_1, h_2) + \begin{pmatrix} 0 \\ 0 \\ \Delta z \end{pmatrix} \cdot \left( \langle h_3 - 0 \rangle_R^1 - \langle h_3 - 1 \rangle_R^1 - \langle h_3 - 1 \rangle_L^0 \right) \quad (3)$$

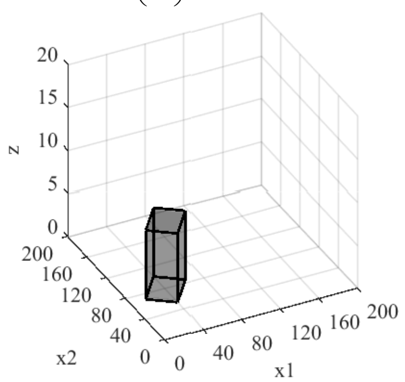


Figure 1. Rectangular solid of work

## 3.3 Crane Movements

As a vital productive resource on site, arguably the most important one [7] due to supplying many trades with materials and assemblies, cranes – here specifically tower cranes – have a dominant role. The operating area of such crane is cylindrical, where the mast is located at {$x_{1C}$, $x_{2C}$}. Its jib of radius $R$ rotates around it. Along it the trolley can move to any position $r$, where $0 \le r \le R$ (or realistically, starting about 3 m from the mast itself). Its hook location is determined by the mast and trolley, i.e. at {$x_{1C}$, $x_{2C}$} plus an offset {$\Delta x_{1C}$, $\Delta x_{2C}$}, where $\Delta x_{iC}$ can be positive or negative and $-R \le \Delta x_{iC} \le R$. Any singularity function is active from its cutoff onward. But $r > R$ should deactivate it to prevent endless growth. This is accomplished by an inverse operator $-\langle r - R\rangle^1$. It shows how singularity functions incorporate constraints that otherwise must be written as inequalities (plus end term to subtract the radius outside the crane coverage). Together this gives $\Delta x_{1C}^2 + \Delta x_{2C}^2 = (x_1 - x_{1C})^2 + (x_2 - x_{2C})^2 = (\langle r - 0\rangle^1 - \langle r - R\rangle^1 - R \cdot \langle r - R\rangle^0)^2$ per Equation 4.

$$Area(x_1, x_2, r): \quad (x_1 - x_{1C})^2 + (x_2 - x_{2C})^2 = \left( \langle r - 0 \rangle^1 - \langle r - R \rangle^1 - R \cdot \langle r - R \rangle^0 \right)^2 \quad (4)$$

Standard geometry gives the relations between the offset distances and rotation angle $\varphi$ as $\Delta x_{1C} = r \cdot \cos \varphi$ and $\Delta x_{2C} = r \cdot \sin \varphi$ to convert from planar into polar coordinates. Such rotation will traverse a fraction of the circumference that simply becomes $2 \cdot \pi \cdot (\varphi / 360)$.

A spatial singularity function can model the area of crane coverage based on its mast location and radius as $x_2(x_1, r) = \pm \sqrt{[(\langle r - 0\rangle^1 - \langle r - 50\rangle^1 - 50 \cdot \langle r - 50\rangle^0)^2 - \langle x_1 - x_{1C}\rangle^2]} + x_{2C}$ and *vice versa* for $x_1(x_{2C}, r)$ and $r(x_{1C}, x_{2C})$; any two of the three variables must be known. Note that the circular crane rotation allows two valid solutions, one beyond the mast on the $x_2$ axis and one before it.

For example, for a mast at {65 m, 70 m} with an $r$ = 50 m jib for $r$ = 60 m the crane radius equation correctly gives $(\langle 60 - 0\rangle^1 - \langle 60 - 50\rangle^1 - 50 \cdot \langle 60 - 50\rangle^0)^2 = (60 - 10 - 50)^2 = 0$, because that location is outside the radius. Test input of $x_1 = 65\text{ m} + 50 \cdot [\sqrt{2}]/2\text{ m} = 65\text{ m} + 35.36\text{ m} = 100.36$ m for the crane coverage equation with a 50 m jib gives $x_2(100.36\text{ m}, 50\text{ m}) = \pm \sqrt{[(\langle 50 - 0\rangle^1 - \langle 50 - 50\rangle^1 - 50 \cdot \langle 50 - 50\rangle^0)^2 - \langle 100.36 - 65\rangle^2]} = \pm \sqrt{[(50 - 0 - 50 \cdot 1)^2 - 35.36^2]} = \pm 35.35$ m (having subtracted $x_{2C}$ = 70 m already), i.e. the crane rotates 45° (anti-clockwise or clockwise) for its two offsets to become identical. Moving the crane coverage area along the time direction with 8 units creates the solid of crane mathematically per Equation 5. Figure 2 visualizes its cylindrical solid.

$$Solid(h) = Area(x_1, x_2, r) + \begin{pmatrix} 0 \\ 0 \\ \Delta z \end{pmatrix} \cdot \left( \langle h - 0 \rangle_R^1 - \langle h - 1 \rangle_R^1 - \langle h - 1 \rangle_L^0 \right) \quad (5)$$

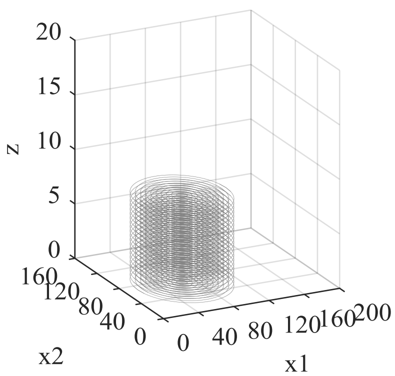


Figure 2. Cylindrical solid of crane

### 3.4 Other Equipment

Other types of equipment may be characterized by different shapes of its work within the construction site. These may be fixed locations (e.g. concrete plant silos, dewatering wells, material hoists), linear movements (e.g. utility pipe trenching, paving, tracked tower cranes and gantries) or work faces (e.g. masonry platforms), multiple linearized segments that compose work paths (e.g. delivery trucks, material handlers), and areas (e.g. storage and assembly yards, building section footprints).

Shapes can be represented with existing singularity functions as follows: Fixed locations have a small non-zero footprint, which makes them conceptually identical to areas. Simple areas have a range of $\{\Delta x_{1S}$, $\Delta x_{1F}$ and $\Delta x_{2S}$, $\Delta x_{2F}\}$ if their rectangular shapes can be defined as a pair of coordinate ranges on the two spatial axes. Beyond this, Equation 2 can newly model the generalized areas that are non-aligned with major coordinate axes. Linear segments can use the model of Equation 2, but with zero (or nearly zero) width $w$ to emphasize that their primary direction is captured by the growth of $h$. More complex equipment travel paths can be composed from linear segments as a set of vectors. These definitions of different types of temporary site objects and their locations fulfill Research Objective 1.

## 4 Boolean Relations

A barely explored yet important issue within spatial scheduling has been how work areas of temporary site objects and the activities that occur there can be related explicitly by logical constraints. Traditional (non-spatial) network scheduling has been dominated by discrete end-point-links that express minimum constraints [17]. They function well for *If-Then* conditions in which one event triggers another, e.g. the predecessor finish that enables a successor start. But despite the popularity of strictly sequential relations, also due to their simplistic nature, real construction projects experience a profusion of concurrent activities that often at best compete for space or at worst interfere or conflict with one another.

Several Boolean algebra operators exist, which here will be used to establish if and how multiple temporary site objects can compete for the same location [16]:

- Two activities must occur sequentially (IF-THEN);
- Two activities must occur in parallel (AND);
- Two activities can (but need not) be parallel (OR);
- Two activities must never occur in parallel (XOR).

Applying them to temporal-spatial scheduling means considering the activity pair {A, B} in both time and space. Their truth values can be recorded per Table 1.

Table 1. Boolean Truth Values

| Operator | Activity A | Activity B | Result |
|---|---|---|---|
| IF-THEN | 1 | 1 | 1 |
| | 1 | 0 | 1 |
| | 0 | 1 | 0 |
| AND | 1 | 1 | 1 |
| | 1 | 0 | 0 |
| | 0 | 1 | 0 |
| OR | 1 | 1 | 1 |
| | 1 | 0 | 1 |
| | 0 | 1 | 1 |
| XOR | 1 | 1 | 0 |
| | 1 | 0 | 1 |
| | 0 | 1 | 1 |

Defining Boolean relations between work and crane solids will use a signal function per Equation 6 whose value is 1 if a coordinate point is inside the solid and 0 if it is outside. This concept can generate the Boolean operations AND, OR, and XOR in Equations 7, 8, and 9. The IF-THEN relation is not considered further, because it already exists in form of normal ‘finish-to-start’ links.

$$Signal(x_1, x_2, z) = \begin{cases} 0 & if \quad (x_1, x_2, z) \notin Solid \\ 1 & if \quad (x_1, x_2, z) \in Solid \end{cases} \tag{6}$$

$$AND_{Signal} = Signal_{Work} \times Signal_{Crane} \tag{7}$$

$$OR_{Signal} = Signal_{Work} + Signal_{Crane} - Signal_{Work} \times Signal_{Crane} \tag{8}$$

$$XOR_{Signal} = OR_{Signal} - AND_{Signal} \tag{9}$$

Such 3D signal functions can be handled like normal mathematical terms. This allows a seamless integration of the singularity functions for the work areas and crane ranges with these new Boolean relations between them.

Linking temporary site objects in their locations via such logical relationships fulfils Research Objective 2.

## 5 Validation

A validation example will test and demonstrate how temporary site objects and Boolean relations are used to more realistically model the constraints of construction projects. Assume the example of Figures 3 and 4 with areas and constraints as follows. The aforementioned tower crane that Section 3.3 has introduced is placed at the coordinates {65, 70} (all locations in meters) with a radius of 50 m. Zone I – familiar from Section 3.2 – is rotated relative to the $x_1$ and $x_2$ axes in Figure 1. Zone I cannot move in space. It occurs in the range from weeks 0 to 8 (all times in weeks) on the temporal axis $z$. Zone II is at {70, 100 and 55, 95} and can be moved to {130, 160} on axis $x_1$ (here called Solution I) or postponed along the time axis $z$ to weeks 10 to 16 (Solution II).

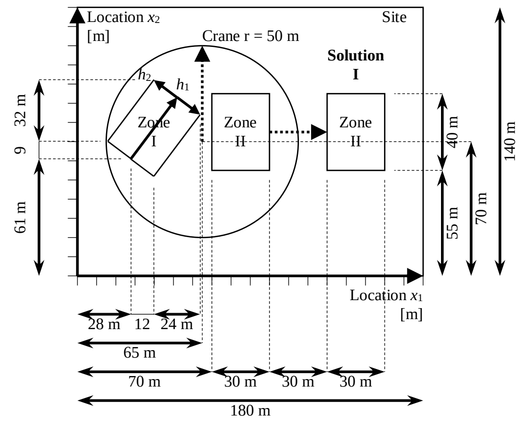


Figure 3. Validation example plan view of space

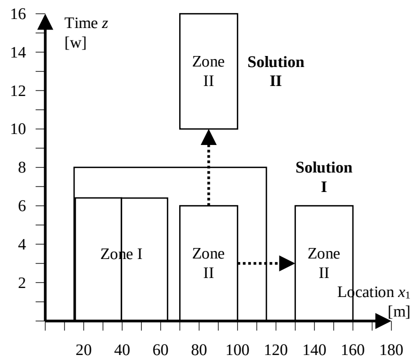


Figure 4. Validation example possible solutions

Applying Equation 5 gives Zone I as a solid of work as $Solid(h)_{I} = (28; 61; 0)_{Start} + (24; 32; 0) \cdot (\langle h_1 - 0\rangle^1_R - \langle h_1 - 1\rangle^1_R - \langle h_1 - 1\rangle^0_L) \pm 15/5 \cdot (-4, +3, \pm 0) \cdot (\langle h_2 - 0\rangle^1_R - \langle h_2 - 1\rangle^1_R - \langle h_2 - 1\rangle^0_L) + (0; 0; 8) \cdot (\langle h_3 - 0\rangle^1_R - \langle h_3 - 1\rangle^1_R - \langle h_3 - 1\rangle^0_L)$. Note that a solid could be spanned from any coordinate (with vector directions adjusted accordingly), but here a start close to the origin is chosen, so that most vector direction values will only have positive signs.

Work in Zone I requires assistance of the crane, so that it has a Boolean AND relation with the crane solid. Yet Zone II never needs the crane, so an XOR is applied between them. It is now possible to check whether the work area is fully within the crane coverage by using their signal functions. In the original plan of Figure 5, Zone II is given by $Solid(h)_{II} = (85; 55; 0)_{Start} + (0; 40; 0) \cdot (\langle h_1 - 0\rangle^1_R - \langle h_1 - 1\rangle^1_R - \langle h_1 - 1\rangle^0_L) \pm 30/2 \cdot (-1, 0, \pm 0) \cdot (\langle h_2 - 0\rangle^1_R - \langle h_2 - 1\rangle^1_R - \langle h_2 - 1\rangle^0_L) + (0; 0; 8) \cdot (\langle h_3 - 0\rangle^1_R - \langle h_3 - 1\rangle^1_R - \langle h_3 - 1\rangle^0_L)$. Boolean calculation of the signal space within Zone II gives $OR_{Signal} = Signal_{Crane} + Signal_{ZoneII} - Signal_{Crane} \times Signal_{ZoneII} = Signal_{ZoneII}$, while $AND_{Signal} = Signal_{Crane} \times Signal_{ZoneII} = Signal_{ZoneII}$, as a result $XOR_{Signal} = OR_{Signal} - AND_{Signal} = Signal_{ZoneII} - Signal_{ZoneII} = 0$. Only if the XOR signal is 1, the work in the Zone II may be conducted. Thus Zone II violates the XOR relation with the crane and is plotted in red.

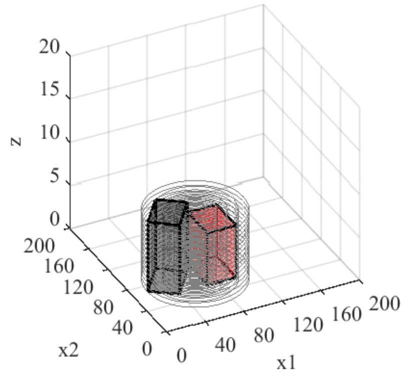


Figure 5. Validation example original plan

Figures 6 and 7 show relocating or postponing Zone II to resolve its conflict. Solution 1 relocates Zone II by 60 m into the $x_1$ direction to fulfill the XOR relation as can be mathematically confirmed. Solution 2 does not change its location, but postpones the start time of Zone by 4 weeks, which will also fulfill the XOR relation. A project planner can thus make an informed decision on how to position temporary site objects and work zones relative to one another to fulfill all constraints toward finding an optimum arrangement of such a schedule.

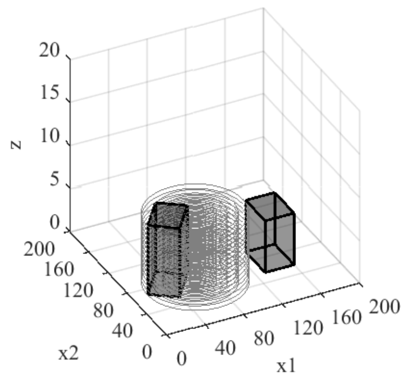


Figure 6. Validation example Solution I (relocate)

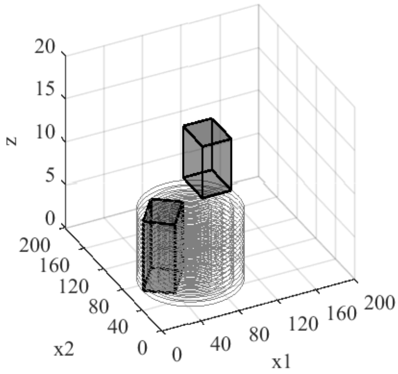


Figure 7. Validation example Solution II (postpone)

## 6 Conclusions

This research opens an avenue for integrating the domains of activity scheduling, workspace planning and site layout planning in a single mathematical model. Its model is sufficiently flexible to allow representing the spatial and temporal dimensions of both construction activities and site objects. It can thus enhance the safe and efficient planning of construction project execution.

### 6.1 Contributions

This paper makes three contributions to knowledge:

- It explores how spatial singularity functions model work areas and work paths and also temporary site objects, within a 3D space-time coordinate system;
- Work areas and temporary site objects are linked by logical relationships such as AND, OR and XOR, in addition to existing sequential relationships, to fulfill applicable efficiency and safety requirements;
- An example has validated that the planned activities and temporary site objects like a tower crane, which constitutes a vital resource on site, can be optimally planned in accordance with their defined relations.

### 6.2 Recommendations

- Since the structural moment of any given crane load depends on its distance between the trolley and mast, the mathematical formulation should be refined to decreasing load capacity based on radius;
- While this research has assumed that cranes have a circular radius, the need for stabilizing outriggers gives mobile cranes a potentially non-circular radius of equal lifting capacity. The mathematical formulation should be refined with an equation of such radius-and-rotation-dependent lifting capacity;
- Purely temporal interval relations have first been described by Allen [18], who considered that activity durations can be equal or unequal and listed all possible constellations of two activities in time, including those with overlap, or concurrency. This idea should be generalized to temporal-spatial scheduling to categorize all possible constellations of temporal site objects and their locations in time;
- Areas that change their shape or size over time (e.g. a crane that is not allowed to swing over an adjacent property, this operating as a circle sector or building with smaller footprints on upper floors) should be modeled as time-dependent functions;
- Rotations could also be modeled via a rotation matrix, which will be explored in future research;
- Complex work paths, e.g. for earthmoving, need to be explored further. They could be combined with capacity calculations, e.g. the curves of mass haul diagrams that calculate cut and fill quantities from integration of volume curves in cross-sections and longitudinal sections of topographical profiles;
- Developing a computer tool for project planners could automatically perform analyses like the example of Section 5 to directly identify conflicts, offer options for solutions, and visualize potential impacts of spatial or temporal changes in plans;
- Full integration with optimization methods should be explored, especially which type of evolutionary algorithm and what parameters perform best for representative sets of real-world construction sites.

## Nomenclature

Table 2. List of Symbols

| Symbol | Unit | Definition |
|---|---|---|
| $x$ | m | Spatial variable |
| $z$ | w | Time variable |
| $s$ | w/m | Strength factor |
| $a$ | m | Activation cutoff |
| $n$ | - | Behavior exponent |
| $R$ | - | Right continuous |
| $L$ | - | Left continuous |
| $l$ | m | Vector length |
| $w$ | m | Half width of area |
| $r$ | m | Radius of interest |
| $h$ | - | Vector direction |
| $R$ | m | Crane jib radius |
| $C$ | - | Crane index |
| $S$ | m | Start coordinate |
| $F$ | m | Finish coordinate |
| $\Delta$ | m | Offset distance |
| $\varphi$ | ° | Rotation angle |
| IF-THEN | - | Sequential relation |
| AND | - | Parallel relation |
| OR | - | Inclusive relation |
| XOR | - | Exclusive relation |

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