Visualization of Integrated Model of Construction Projects

Y. Su[a], S. Isaac[b], and G. Lucko[a]

[a]Department of Civil Engineering, Catholic University of America, USA
[b]Department of Structural Engineering, Ben-Gurion University of the Negev, Israel
E-mail: isaacsh@bgu.ac.il, 61su@cua.edu, lucko@cua.edu

Abstract –

Efforts to integrate construction planning have thus far focused on the interoperability of models and computer applications. However, merely linking different models, e.g. design and schedule, while useful, necessarily restricts planners to an iterative improvement of plans. This research defines a single integrated mathematical model with singularity functions to directly represent the different aspects that need to be considered in planning construction projects, such as their design, schedule, budget, site layout, etc. This allows all relevant aspects to be taken into account simultaneously when optimizing the construction plan. The new model can be used to visualize different aspects of a project, allowing the planner to interact with the model and manipulate it. Examples are provided for the intuitive visualization of the time-space that building components, site objects and construction activities occupy, of the dynamic relations and buffers between objects and activities, and of the impact of changes in the plan.

Keywords –

Scheduling, Singularity Function, Site Layout, Cranes

1 Introduction

The efficient and safe planning and management of construction projects requires an integrated modeling of their different aspects (or 'dimensions'). Much effort, centering on applying Building Information Modeling (BIM), has consequently been made in the domain of integrated project modeling (e.g. [1], [2]). But as the following section will explain, currently available multi-dimensional models generally consist only of partially interlinked models. While this allows making certain manual and iterative improvements in the construction plan, it does not fully support its efficient optimization.

Therefore we propose that a truly integrated model of construction projects should be based upon a single mathematical model that is capable of representing all their relevant aspects. This would allow construction plans to be continuously modified and optimized according to evolving project objectives and constraints. These constraints (e.g. scope, time, cost, and resources) are typically interrelated, yet competing with each other. Also, effective means for an integrated visualization of these different aspects of a project should be developed to provide planners and managers with their complex information in a way that will be easily understood. It will also allow them to directly manipulate the model, in accordance with the dynamically changing conditions.

This paper presents such a mathematical model for integrated modeling of construction projects, which addresses different elements, e.g. building components, construction activities, and temporary site objects, and the dimensions of time, cost and spatial location. The paper also presents a novel framework for visualizing the model, using three-dimensional (3D) diagrams that can represent any chosen triplet of aspects of the project.

2 Literature Review

Visualization is an emerging area, riding on a rapid increase in ubiquitous computing capabilities and access devices. Tools equip their users with powerful ways to communicate with near and far participants, understand complex interactions of aspects, and support decision-making. A list of 17 'grand challenges' for visualization, information modeling, and simulation includes needs to provide 'formats and mediums suited for construction", "format and interoperability to enable data sharing", and "[g]enerating models that adapt to real-world changes", plus calls to stronger connect academia with industry [3, p. 2] Currently only partially interconnected models of the aforementioned vital aspects of construction projects are available, which allow merely an iterative process of limited improvement, but no truly efficient optimization.

Sorting the literature by dimensionality of its models illustrates the relevant rather diverse body of knowledge:

- 2D: Linking work with time, linear, repetitive, and location-based schedules [4] represent productivity explicitly, not just duration in network schedules. Focusing on finances instead of product, cash flow diagrams can track cost and payments over time to determine cumulative balances throughout the life of a project [5]. Analogously, resource use over time can be optimized for even workflow (leveling) [6] or shortest project duration (allocation). Some models allow handling multiple resource types [7];

- 3D: Three-dimensional space dominates existing commercial computer-aided design (CAD) tools, whose dimensions are spatial. Suitable coloration and iconography can give information of planned versus actual progress [8]. But 3D can also offer spatial-temporal views, e.g. work paths across sites at different times [9]. It can apply logic operators between site objects [10]. Extending the traditional criticality and float concepts gives spatial-temporal schedules [11]. Even more varied representations of any permutation of three aspects generates all existing diagram types, plus various new ones [12];
- 4D: Four-dimensional (4D) models are dominated by animating 3D CAD images as short videos, e.g. to plan site operations with an easy-to-understand way. Alternatively, selected view may be shown as slices of a 'space-time cube' [13]. The entire 4D realm is closely related to BIM applications [14];
- 5D: While five aspects, namely technical, schedule, cost, context, and financing, have been presented jointly [15], they unfortunately remained limited to mere spider charts, which simply plotted empirical assessments of megaprojects on a 0-to-100 scale;
- nD: Further expansion require shifting to a purely mathematical model, where complete graphics are no longer feasible, but all aspects can be contained in an *n*-dimensional vector space. Three aspects of interest could always be selected for visualization.

But a conceptual gap in the aforementioned spatial-temporal mathematical model remain is its lack of site objects. Refining it will create a comprehensive 3D tool as a foundation toward future generalization [12]. Two sequential objectives are established to support this goal:

1. Derive a mathematical model to handle any temporary site objects in spatial-temporal 3D space;
2. Create and validate a visualization of the cost of temporary site objects within spatial-cost 3D space.

3 Methodology

In this paper a mathematical model is presented. It can represent different aspects of a construction project, such as time, work, and cost, using singularity functions. It supports continuous modification and optimization of the construction plan by expressing physical elements:

- Building components and assemblies;
- Stationary and moving construction activities;
- Planned work spaces and the work paths within them, and required safety buffers around them;
- Temporary site objects: Cranes, stockpiles and their changing sizes, earthmoving equipment.

Elements are linked by Boolean operators of AND, OR, and XOR or the material implication (IF-THEN). Visualization as 3D diagrams will provides the planner with the complex information in a way that is easily understood to support manipulating the model directly.

3.1 Singularity Functions

Singularity functions were first used in structural engineering to calculate the effects of distributed loads at different sections of beams [16]. The term singularity indicates that the function behaves discontinuously at a point, but is defined for all values of the independent variable [17]. Managerial dimensions of construction engineering and management feature such independent-dependent variable pairs, e.g. work quantity and time (linear schedules), resource counts and time (resource leveling and allocation), and cost and time (cash flows). This implies that the underlying formulation of loads on beams can be swapped for such quantifiable aspects of projects, and indicates that the mathematics should work, as prior research has successfully shown [e.g. 6, 5, 12].

3.1.1 Point-Scalar Form

Equation 1 defines the basic term of any singularity functions within a two-dimensional coordinate system. Pointed brackets are a case distinction operator to select from two options: If the independent variable x is equal to or larger than the activation cutoff a, then brackets are treated like conventional round algebraic ones, else if x is smaller than a then the operator returns zero. Note that Equation 1 is right-continuous; Equation 2 provides an analogous left-continuous version. Changing strength s and exponent n generates customized behaviors of the function, whereby the former acts as a scalar to increase or decrease its magnitude and the latter steers its growth.

$$y(x) = s \cdot \langle x - a \rangle_{\mathrm{R}}^{n} = \begin{cases} 0 & if\ x < a \\ s \cdot (x-a)^n & if\ x \geq a \end{cases} \tag{1}$$

$$y(x) = s \cdot \langle x - a \rangle_{\mathrm{L}}^{n} = \begin{cases} 0 & if\ x \leq a \\ s \cdot (x-a)^n & if\ x > a \end{cases} \tag{2}$$

3.1.2 Point-Vector Form

Singularity functions can be extended to 3D [9, 10]. Vector $\boldsymbol{v}$ starts at point (x_{1S}, x_{2S}, z_S) to grows in direction (Δx_1, Δx_2, Δz) per Equation 3. An independent variable h_1 from 0 to 1 within the operator controls how $\boldsymbol{v}$ grows until it finishes at ($x_{1S} + \Delta x_1$, $x_{2S} + \Delta x_2$, $z_S + \Delta z$). Such a point-vector form can model any line segment from start to finish within a 3D space system. For spatial-temporal schedule modeling, the first two elements in the three coordinates (x_1, x_2, z) will represent its spatial dimension, while the third coordinate z denotes the time dimension.

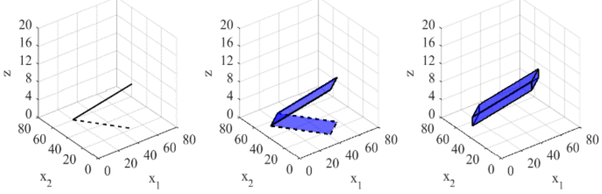
$$\mathbf{v}(h) = \begin{pmatrix} x_{1S} \\ x_{2S} \\ z_S \end{pmatrix}_{Start} + \begin{pmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta z \end{pmatrix} \cdot \left(\langle h_1 - 0 \rangle_R^1 - \langle h_1 - 1 \rangle_R^1 - \langle h_1 - 1 \rangle_L^0 \right) \tag{3}$$

3.2 Work Area Definition (Temporary Site Objects)

3.2.1 Earthmoving (Prism)

Many construction activities proceed linearly on a site, e.g. earthmoving or paving. Its equipment moves accordingly. The geometrical shape of that equipment in the 2D spatial plane is a rectangle, whose length is the moving distance (dashed line in Figure 1a) and width is a safety buffer for traveling equipment [10]. Adding the duration data for this earthmoving activity adds a slope in the 3D spatial-temporal coordinate system (solid area in Figure 1b) to the rectangle. Note that the projection of the sloped rectangular plane into the x_1-x_2 plane is the spatial work area of the equipment (dashed area in Figure 1b). Assume that a time buffer of 2 days must be maintained for an earthmoving activity. Then the sloped rectangular plane has a height of 2 days and expands the geometrical shape to a prismatic volume (Figure 1c).

The point-vector form of Equation 3 captures a start point plus an offset; only the offset part has a singularity function term. Applying this offset concept to a line will generate a rectangular plane, and offsetting it will create a prism [10]. The flowchart of Figure 2 defines a prism in 3D space mathematically with three offsets functions per Equation 4 (n = 1, 2, 3). Note that a term '+ $Offset_n$' is applied to all points in the geometrical shape *before* the term (i.e. start point, start line, and start plane). For Example 1, an earthmoving activity starts at (20, 60, 0) and moves to its finish position at (50, 20, 10) with a duration of 10 days. The spatial safety buffer is 10 m to each side (perpendicular to the moving direction). Time buffer is 2 days. Thus the inputs are: (x_{1S}, x_{2S}, z_S) = (20, 60, 0), $Offset_1$ = (30, -40, 10), $Offset_2$ = (8, 6, 0), and $Offset_3$ = (0, 0, 2), respectively, as Figure 1 shows.



(a) Line (b) Plane (c) Prism

Figure 1. Process to Generate Prism

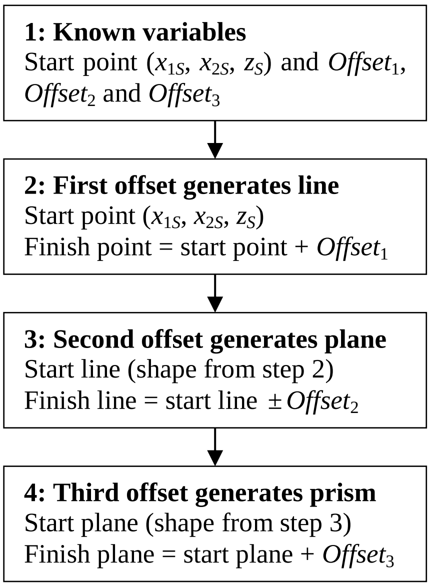


Figure 2. Flowchart to Define Prism

$$Offset_n = h_{nS} \cdot \langle h_n - h_{nS} \rangle_R^0 + \begin{pmatrix} \Delta_{nx_1} \\ \Delta_{nx_2} \\ \Delta_{nz} \end{pmatrix} \cdot \left(\langle h_n - h_{nS} \rangle_R^1 - \langle h_n - h_{nF} \rangle_R^1 - \langle h_n - h_{nF} \rangle_L^0 \right) \tag{4}$$

3.2.2 Crane (Prism Sector)

The geometric shape of the work area of a crane is a cylinder in the 3D spatial-temporal coordinate system, whose mast is located at the center around which its jib rotates with a certain radius [18]. If a crane may only rotate within an angle on a construction site, e.g. to not swing over a sidewalk for safety reasons, then its work area is a prism sector in the 3D space. To define a sector per Figure 3, $Offset_2$ in the flowchart will be a rotation and mathematically expressed as a rotation matrix [17]. Equation 5 is the rotation function, where θ is the angle of rotation. Note that the singularity function term $\langle \theta - 0 \rangle_R^0 \cdot \langle -\theta - (-2\pi) \rangle_R^0$ controls a range [0, 2π] of the angle. For Example 2, a crane mast is located at (70, 70, 0) and its jib radius is 60 m. The start position of the jib end point is (22, 34, 0) and the angle of rotation is $\pi/4$. The duration of the crane on the construction site is 10 days. Thus the inputs are: (x_{1S}, x_{2S}, z_S) = (70, 70, 0), $Offset_1$ = (-48, -36, 10), $Offset_2$ is the rotation matrix with θ is $\pi/4$, and $Offset_3$ = (0, 0, 10). Figures 4a-4c show how to generate the work area for this crane.

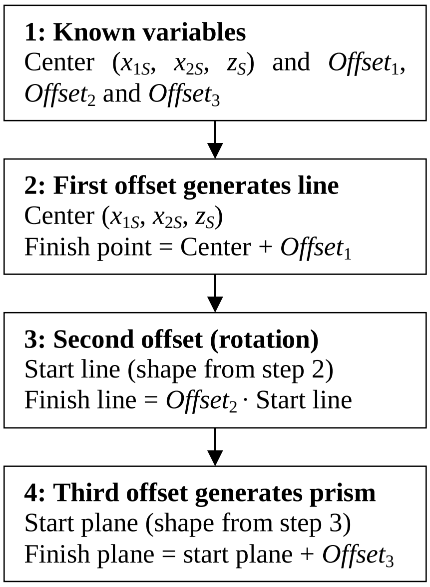
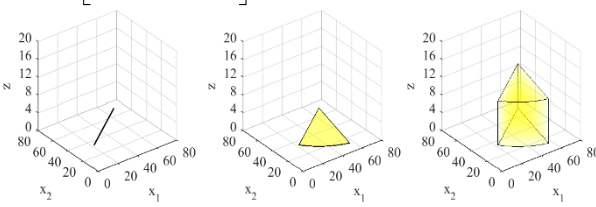


Figure 3. Flowchart to Define Sector

$$Offset_2 = R_z(\theta) \cdot \langle \theta - 0 \rangle_R^0 \cdot \langle -\theta - (-2\pi) \rangle_R^0 \tag{5}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \langle \theta - 0 \rangle_R^0 \cdot \langle -\theta - (-2\pi) \rangle_R^0$$



(a) Line (b) Sector (c) Prism

Figure 4. Process to Generate Sector

3.2.3 Material Stockpile (Cone)

Material waiting to be lifted by a crane is assumed to be a cone, so that the area of the material stockpile will decrease to zero. If an original pile with radius R will be exhausted in D days, then the radius Offset1 decreases from R to 0. Thus, the radius change rate Δr = R / D. Assuming that the pile is exhausted continuously, the offset in the time direction Offset3 for each layer of the pile at a time point also grows continuously. For this, the temporal dimension of Offset3 (Δz) has infinitesimal duration ε, and its integral is D ($\int_{z_S}^{z_F} \Delta z dz = D$).

Figure 5 shows the flowchart to define a cone. The difference between Figure 5 and Figure 3 is that there is an iteration loop for modeling a cone: If the radius $Offset_1$ is larger than zero and $\int_{z_S}^{z} \Delta z dz$ is smaller than D, then subtract the radius change rate Δr from the radius until the radius becomes zero (material exhausted) and $\int_{z_S}^{z} \Delta z dz$ is equal to D. For Example 3, the center of a pile is located at (40, 40, 0) with an initial radius of 20 m. The duration to exhaust the stockpile is 10 days. In the 3D spatial-temporal coordinate system this pile has a height (or duration) of 10 days. The radius change rate Δr is 2 meters / day. Thus the inputs are: (x_{1S}, x_{2S}, z_S) = (40, 40, 0), $Offset_1$ = (-20, 0, 0), $Offset_2$ is the rotation matrix with θ is 2π, and $Offset_3$ = (0, 0, ε). Figures 6a-6c shows how to create the work area of this pile.

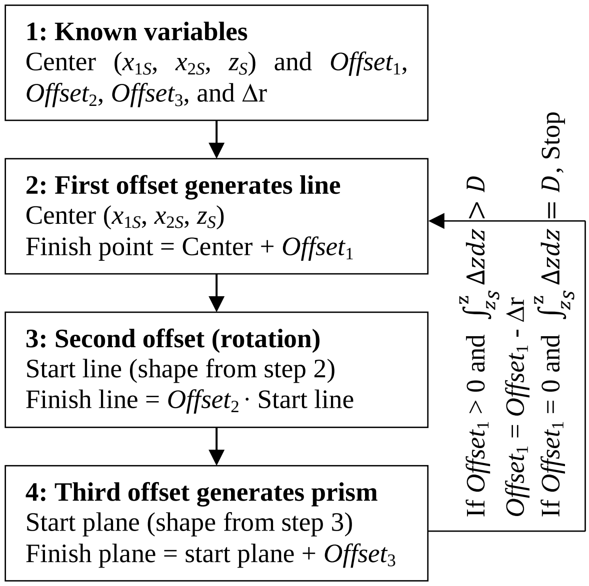
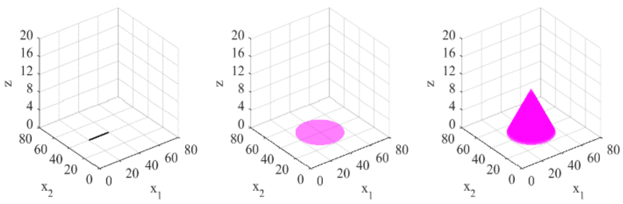


Figure 5. Flowchart to Define Cone



(a) Line (b) Circle (c) Cone

Figure 6. Process to Generate Cone

3.3 Spatial and Temporal Relations (Boolean)

Boolean relations can be used to define constraints between multiple temporary site objects in the spatial-temporal 3D space [10]. Four types are considered: Material implication IF-THEN, conjunction AND, disjunction OR, and exclusive disjunction XOR. The IF-THEN is only defined on the time dimension (i.e. two activities must occur sequentially – if the predecessor is finished, then a successor can start). The other three relations are used to define spatial-temporal constraints: Multiple temporary site objects must occur concurrently (AND); multiple temporary site objects can (but need not) occur concurrently (OR); or multiple temporary site objects must never occur concurrently (XOR). Boolean operations between two objects can be calculated with signals of these objects. Per Equation 6, the signal value of an object is 1 if a coordinate point is inside the work area of the object and 0 if it is outside. Rules for signal functions to conduct these four Boolean operations are summarized in Table 1. Boolean operations with signal functions are then multiplied with work areas functions to model temporary site objects and their relations in spatial-temporal 3D space. This fulfils Objective 1.

$$Signal(x_1, x_2, z) = \begin{cases} 0 & if \quad (x_1, x_2, z) \notin Object \\ 1 & if \quad (x_1, x_2, z) \in Object \end{cases} \tag{6}$$

Table 1. Boolean Operations and Signal Functions (adapted from [10])

Type	Rule			Signal function
	A	B	result	
IF-THEN	1	1	1	
	1	0	1	N/A
	0	1	0	
AND	1	1	1	
	1	0	0	$Signal_A \times Signal_B$
	0	1	0	
OR	1	1	1	
	1	0	1	$Signal_A + Signal_B - Signal_A \times Signal_B$
	0	1	1	
XOR	1	1	0	
	1	0	1	$OR_{Signal} - AND_{Signal}$
	0	1	1	

3.4 Spatial-Cost Area Definition

In analogy to spatial-temporal work areas, spatial-cost can be modeled to visualize location and cost data, where the vertical axis is cost. This shows the total cost of different items on site and allows comparing options visually. To realize it in 3D, the assumption is: *Cost of a temporary site object is assumed as evenly distributed across its geometric shape of the spatial work area.*

The difference between a spatial-temporal work area and a spatial-cost area is that time grows continuously, whereas cost occurs discretely (e.g. daily, weekly, or once). For mathematical modeling, the temporary site object concept of Section 3.2 is used, but using the cost as the third dimension instead will return their functions for spatial-cost area, which is omitted here for brevity. From the general contractor's perspective, the shapes of spatial-cost areas of temporary site objects are:

1. Material: Cost occurs once when transporting it to the site. It is a cylinder with the height of its total cost;

2. Crane: Cost occurs periodically (e.g. weekly rental). Its shape are a multiple layers of a sector surface with the height of the cumulative total weekly cost;

3. Earthmoving: Cost gradually increases (daily). Its shape is a cuboid with a height of its total cost (or layers of a rectangular surface, whose number equals duration, with the height of the cumulative total cost for each day).

4. Indirect: Cost gradually increases (daily) and covers the whole project. Its shape is a solid with the height of its total cost (or layers of a whole site surface, for which the same rules will apply as for earthmoving).

Geometric shapes in 3D spatial-cost space are shown in the next section. Defining and visualizing cost data for temporary site objects fulfills Objective 2.

4 Application

The following example demonstrates how this new integrated model can analyze, visualize and improve the scheduling, cost and site layout planning of construction projects. In the example, two sequential activities (with a finish-to-start or IF-THEN relationship between them) are executed in a rectangular work area. Earthmoving equipment is used for Activity 1, whereas in Activity 2 a crane lifts material that is stored onsite. Consequently, AND relationships are defined between crane, material, and Activity 2. An XOR relationship is defined between the crane and Activity 1 to prevent safety hazards.

The model is applied to plan the activities so that their durations and costs are minimized. Two options will be compared. In Option 1, Activity 2 is carried out by one crew with a single crane. In Option 2, Activity 2 is carried out simultaneously by two crews with two cranes. The optimal option in terms of both duration and costs is identified by a least-cost scheduling approach, which reduces the duration of the activities in each option until their total cost has been minimized, without violating the previously described relationships.

Figure 7 shows temporal and spatial dimensions of activities and site objects in Option 1 with 3D time-space diagrams. In them, the locations of the objects on the site are indicated on the horizontal axes, whereas the duration of their presence on site is indicated on the vertical axis. This allows the user to easily change their location and/or timing while exploring options. Figure 8 analogously shows cost and spatial dimensions in the cost-space diagrams. In them the vertical axis indicates the cost of each object. This provides the user with an immediate understanding of cost implications of any change in the plan. Time-space diagrams and cost-space diagrams for Option 2 are shown in Figures 9 and 10.

The resultant site layout plan for Option 1 is shown in Figure 11, and its final schedule and costs in Tables 4 and 5. Figure 12 shows the site layout plan for Option 2. The final schedule and costs of this Option are shown in Tables 2 and 3. A comparison shows that Option 2 has a shorter duration (6 days versus to 9 for Option 1) and a lower cost ($305,000 compared to $327,500).

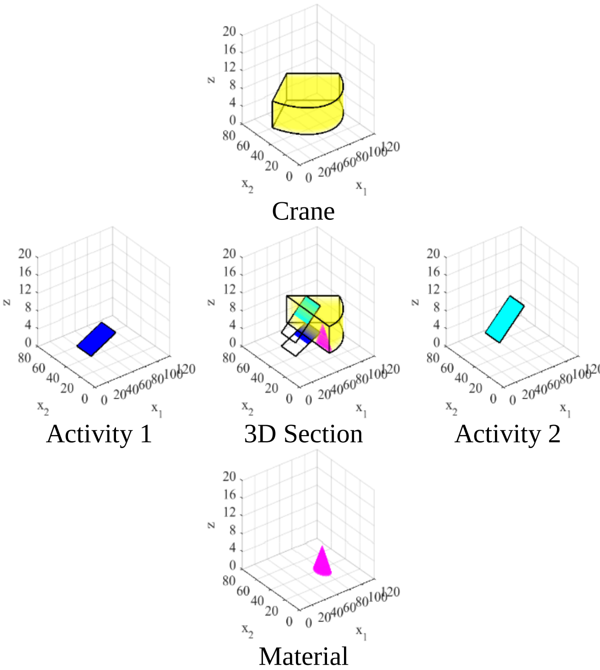


Figure 7. Time-Space Diagram (Option 1)

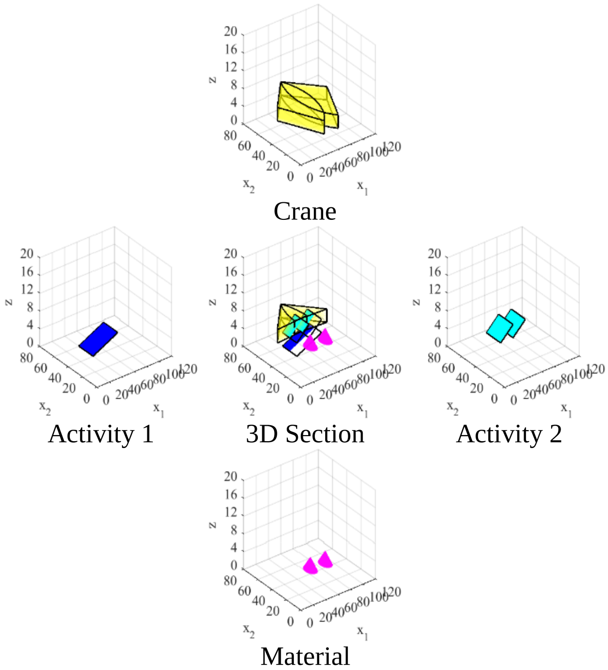


Figure 9. Time-Space Diagram (Option 2)

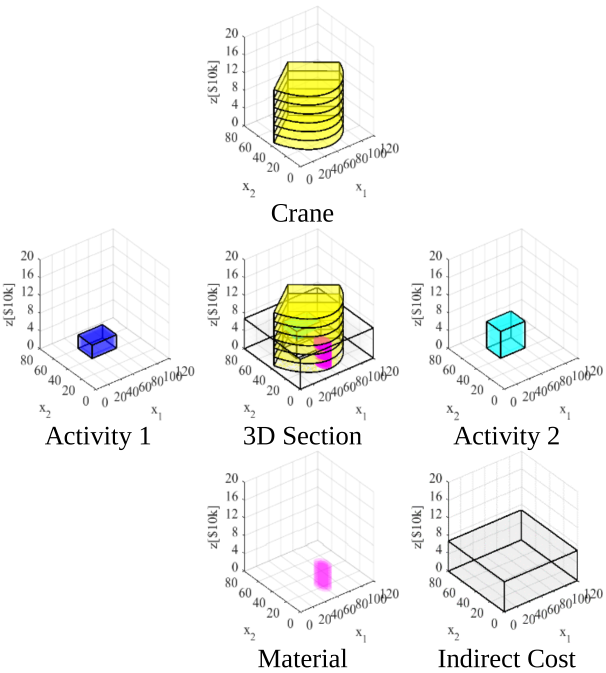


Figure 8. Cost-Space Diagram (Option 1)

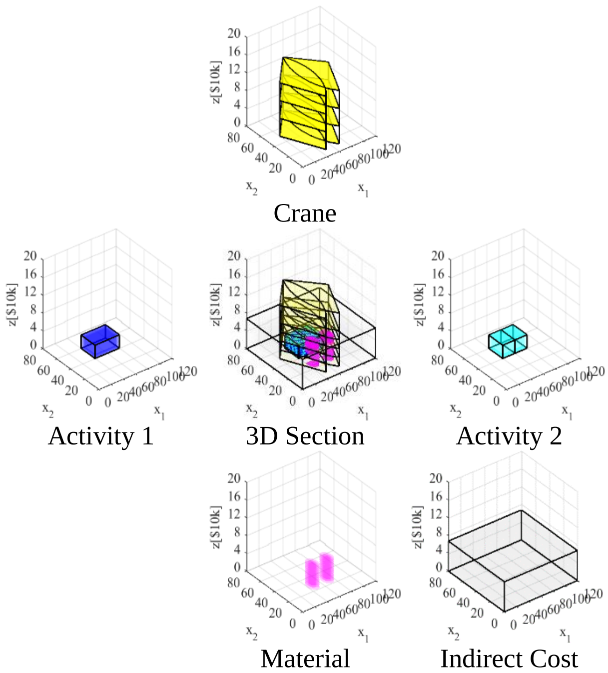


Figure 10. Cost-Space Diagram (Option 2)

Table 2. Schedule Data for Option 1

Activity	Duration (weeks)	Start (week)	Finish (week)
1	3	0	3
2	6	3	9

Table 4. Schedule Data for Option 2

Activity	Duration (weeks)	Start (week)	Finish (week)
1	3	0	3
2	3	3	6

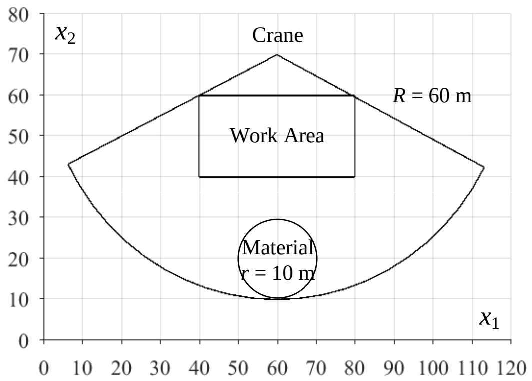


Figure 11. Site Layout for Option 1

Table 3. Cost Data for Option 1

Item	Cost	Subtotal
Manpower	$2,000 per day (entire project)	$90,000
Material	$50,000 (week 3)	$50,000
Crane	$20,000 per week (weeks 3-9)	$120,000
Indirect costs	$1,500 per day (for entire project)	$67,500
	Total cost	$327,500

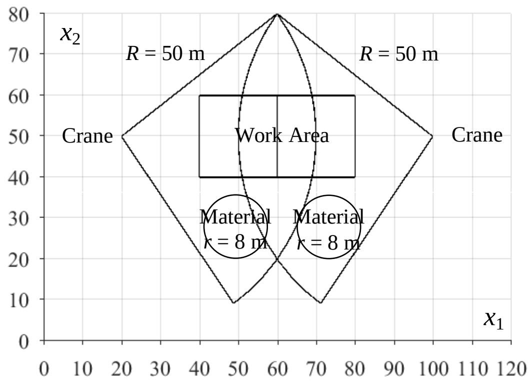


Figure 12. Site Layout for Option 2

Table 5. Cost Data for Option 2

Item	Cost	Total cost
Manpower	2,000 per day (weeks 1-3)	90,000
Material	50,000 (week 3)	50,000
Crane	20,000 per week (weeks 3-6)	120,000
Indirect costs	1,500 per day (for entire project)	45,000
	Overall cost	305,000

5 Conclusions

This paper has presented a novel integrated model to plan construction projects. As has been demonstrated, it can be used to maximize the efficiency of both the site layout plan and the execution of the planned activities, without compromising the safety. Options can be easily modified and compared through the new diagram types.

5.1 Contributions

This paper has made the following contributions:

- A mathematical model has been presented that can represent different aspects of a project, (e.g. time, work, and cost), and its physical elements (e.g. building components, activities, work spaces, and temporary site objects);
- Boolean operators (AND, OR, and XOR or the material implication IF-THEN) can define the constraints between activities and site objects;
- Novel 3D diagrams provide planners with a visualization of complex information in a way that it can be easily understood and changed.

5.2 Recommendations for future research

Future research should expand the proposed model into a true multi-dimensional model of construction projects. Such *n*D model is envisioned to integrate and simultaneously handle multiple managerial dimensions like time, work, cost, resources, etc. Selections thereof could be visualized. The use of weighted constraints in addition to binary Boolean operators should also be inserted to enable representing not just hard constraints, but also 'soft logic' such as priorities and preferences of planners for their site layout based on their experience.

References

[1] Heesom, D., Mahdjoubi, L. Trends of 4D CAD applications for construction planning. *Construction Management and Economics*, 22(2), 171-182, 2004.

[2] Bansal, V. K. Integrated CAD and GIS–Based Framework to Support Construction Planning: Case Study. *Journal of Architectural Engineering*, 23(3), 05017005(13), 2017.

[3] Leite, F., Cho, Y., Behzadan, A. H., Lee, S. Choe, S., Fang, Y., Akhavian, R., Hwang, S. Visualization, Information Modeling, and Simulation: Grand Challenges in the Construction Industry. *Journal of Computing in Civil Engineering*, 30(6), 04016035(16), 2016.

[4] Lucko, G., Alves, T. da C. L., Angelim, V. L. Challenges and Opportunities for Productivity Studies in Linear, Repetitive, and Location-Based Scheduling. *Construction Management and Economics*, 32(6), 575-594, 2014.

[5] Lucko, G. Optimizing Cash Flows for Linear Schedules Modeled with Singularity Functions by Simulated Annealing. *Journal of Construction*

Engineering and Management, 137(7): 523-535, 2011.

[6] Lucko, G. Integrating Efficient Resource Optimization and Linear Schedule Analysis with Singularity Functions. *Journal of Construction Engineering and Management*, 137(1): 45-55, 2011.

[7] Jun, D. H., El-Rayes, K. A. Multiobjective Optimization of Resource Leveling and Allocation during Construction Scheduling. *Journal of Construction Engineering and Management*, 137(12), 1080-1088, 2011.

[8] Sacks, R., Treckmann, M., Rozenfeld, O. Visualization of Work Flow to Support Lean Construction. *Journal of Construction Engineering and Management*, 135(12), 1307-1315, 2009.

[9] Isaac, S., Su, Y., Lucko, G., Dagan, D. Work-Path Modeling and Spatial Scheduling with Singularity Functions. *Journal of Computing in Civil Engineering*, 31(4), 04017008(14), 2017.

[10] Su, Y., Isaac, S., Lucko, G. Integrated Temporal-Spatial Model for Construction Plans with Boolean Logic Operators. *Journal of Construction Engineering and Management*, 144(4), 04018009-(15), 2017.

[11] Said, H. M. M., Lucko, G. Float Types in Construction Spatial Scheduling." *Journal of Construction Engineering and Management*, 142(12): 04016077(12), 2016.

[12] Lucko, G., Su, Y. Singularity Functions as New Tool for Integrated Project Management." *Procedia Engineering*, 85: 339-350, 2014.

[13] Bach, B., Dragicevic, P., Archambault, D., Hurter, C., Carpendale, S. A Descriptive Framework for Temporal Data Visualizations Based on Generalized Space-Time Cubes. *Computer Graphics Forum*, 36(6), 36-61, 2017.

[14] Son, H., Kim, C., Cho, Y. K. Automated Schedule Updates Using As-Built Data and a 4D Building Information Model. *Journal of Management in Engineering*, 33(4), 04017012(13), 2017.

[15] Gransberg, D. D., Shane, J. S., Strong, K., Lopez del Puerto, C. Project Complexity Mapping in Five Dimensions for Complex Transportation Projects. *Journal of Management in Engineering*, 29(4), 316-326, 2013.

[16] Macaulay, W.H. Note on the deflection of beams. *Messenger of Mathematics*, 48(9), 129-130, 1919.

[17] Kreyszig, E. *Advanced engineering mathematics.* 10th ed., John Wiley and Sons, Hoboken, NJ, 2011.

[18] Isaac, S., Su, Y., Lucko, G. Integrated Activity and Site Layout Planning. *Proceedings of the 2017 34th International Symposium on Automation and Robotics in Construction*, Taipei, Taiwan, International Association for Automation and Robotics in Construction, 804-810, 2017.