Trajectory Planning of Forces and Arm Tips for Tumbling Operation by Two Arms

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Abstract -

Rock-wise objects made after natural disasters must be removed by construction machines which are maneuvered by human beings. In that case, from the viewpoint of preventing a secondary disaster, the construction machines are preferable to carry out the task autonomously or by remote operation. We take aim at autonomous tumbling operation by a mobile robot with dual arms to move a rock-wise object. In previous study, a path planning algorithm for an object in tumbling operations had been presented. The order of the selected ridge line determines the path of the object. Next, trajectory planning of forces and arm tips of the robot is necessary for autonomous motion. In this paper, we present a method for determining the combination of action points by analyzing the tumbling operation. First, we derive relational expressions of force for the rotational motion of the object. Conditional expressions are set for making stable operation. Then, the problem is solved as a nonlinear optimization problem by evaluation function of force. The constructed algorithm derives optimal trajectory planning of forces and arm tips. Finally, the usefulness of the algorithm is shown by a numerical example.

Keywords -

Tumbling operation; Nonlinear optimization; Trajectory planning; Algorithm;

1 Introduction

After the wide-scale landslides are caused by a natural disaster such as an earthquake or a concentrated downpour, rock-wise objects generated by the disaster must be removed by construction machines which are maneuvered by human beings. In that case, from the viewpoint of preventing a secondary disaster, the construction machines are preferable to carry out the task autonomously or by remote operation. In this paper, we describe automation of removal work. Grasping and lifting-up motion are a common handling to remove objects, and easy to move the object. However, if the object is too large or heavy for the machine to grasp and carry, removal work by this operation is difficult. On the other hand, graspless manipulation such as pushing, tumbling, and pivoting are effective methods to handle the object in the case mentioned above [1]. Since these operations are performed while the object is in contact with the environment, they are not necessary to support all the weight of the object, and their energy is less than the energy of pick-and-place operation. In addition, pushing, tumbling and pivoting operations have different characteristics, so by combining these operations it is possible to flexibly manipulate the object. Therefore, many studies have been aimed at assisting the grasping operation or the pick-and-place operation for the assembly work of parts [2] [3] .

On the other hand, we take aim at autonomous tumbling operation by a mobile robot with dual arms to move rock-wise objects. In previous study, a path planning algorithm for an object in tumbling operations had been proposed [4] . The study dealt with a problem to move a rock-wise object approximated as a convex polyhedron around a desired position by tumbling operation. And a method for planning semi-optimal path represented as a selection order of the edge for a tumbling operation to minimize a predefined performance index was presented. Next, trajectory planning of forces and arm tips of the robot is necessary for autonomous motion. In this paper, we present a method for determining the combination of action points by analyzing the tumbling operation. The above method is possible to obtain the optimum trajectory planning that suppresses the excessive internal forces on the object.

In the next chapter, related studies are described. In the chapter 3, the initial setting of the problem and the coordinate system are explained. Derivation of relational expressions of force for the rotational motion of the object, and conditional expressions for making stable operation are described in the chapter 4. Determination of the combination of action points is solved as a nonlinear optimization problem by the derived equation and evaluation function of force. Also, we reduce variables to solve the problem of multivariable. For all combinations of action points, a convergence analysis is performed, and an algorithm is shown for deriving the optimal trajectory planning of forces and arm tips in the chapter 5. In the chapter 6, the usefulness of the algorithm is shown by a numerical example, and the conclusion and the future work are described in the chapter 7. However, the trajectory planning doesn't consider the characteristics of the robot except for two arms.

2 Related work

Sawazaki et al. analyzed three kinds of tumbling operations, and showed the feasibilities of them by carrying out the tumbling operation with multi-jointed robot fingers [5]. In contrast to the proposed method, which is an analysis based on a two-dimensional plane, this study extends the analysis to three dimensions in order to deal with the shape like a rock-wize object. Y. Maeda et al. proposed a method of motion planning of multiple robot fingertips for graspless manipulation including tumbling operations [6]. By considering whether each robot finger should be position-controlled or force-controlled, this method can obtain robust manipulation plans against external disturbances. Since large numbers of lattice points obtained by discretization of the configuration space are used as candidates of action points, it is theoretically possible to construct a plan for objects of any shape. However, in exchange for that, it takes a lot of calculation time. Therefor, in this paper, candidates of action point are narrowed down to cover a complicated shapes like a rock-wize object.

3 System initialization

3.1 Assumption

The assumptions in the tumbling operation are as follows.

- The operation is performed on a horizontal floor.
- The rotational motion of the object is slow and is regarded as quasi-static. Therefore, the inertia term is excluded for the motion of the object.
- The object is a convex polyhedron. Therefore, the object rotates around one ridge line on the bottom of the object.
- The friction coefficient generated at the contact portion is constant and it is possible to estimate approximate values from knowledge of the site.
- No-slip occur between the object and arm tips or a floor.
- The middle point of the ridge line is a position of action point. However, the middle point of the ridge line which is the bottom surface of the object is excluded from the action point.
- The initial state of the object is when the bottom surface and the floor surface are in contact with each other. Further, from Figure 1 the operation of rotating from the initial state to the other bottom surface and the floor surface around one ridge line is defined as "single tumbling operation".

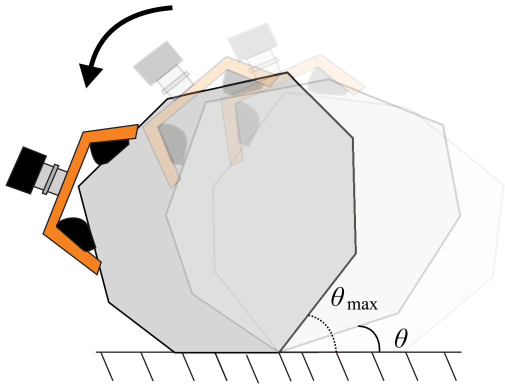

Figure 1. Single tumbling operation

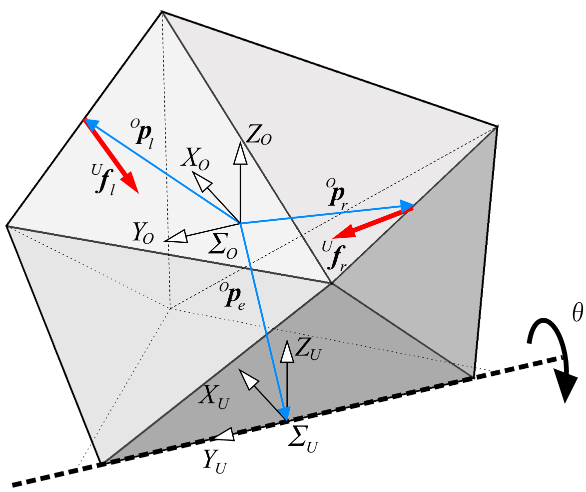

Figure 2. Coordinate system in object

3.2 Setting of the coordinate system

As shown in Figure 2, two coordinate systems, which are orthogonal and right-hand ones, are defined for calculation. The coordinate system Σ_O is fixed on the object, and its origin is located at the center of gravity of the object. The coordinate system $\Sigma_U(O_U - X_U Y_U Z_U)$ is defined with the middle point of the ridge line in the rotation axis as the origin. The Z_U axis is in the upper direction, and the X_U axis is in the advance direction of the robot. In the initial state of things, the postures of Σ_U and Σ_O are equal.

4 Problem formulation

4.1 Relational Expression of force

The external force on the object is denoted as ${}^{U}\boldsymbol{f}_i = \left[f_{ix}, f_{iy}, f_{iz}\right]^T$ $(i = r, l)$, where r and l indicate right and left quantities in the robot. Also, M is the mass of the object, $\boldsymbol{g} = [0, 0, g_z]^T$ is the gravitational acceleration, and $\boldsymbol{N} = \left[f_{fx}, f_{fy}, N_z\right]^T$ is the reaction force given from the floor surface. When two action points are selected and a tumbling operation is performed around the Y_U axis, relational expression of force is expressed as follows.

$$ {}^{U}\boldsymbol{f}_r + {}^{U}\boldsymbol{f}_l + M\boldsymbol{g} + \boldsymbol{N} = 0 \quad (1) $$

Further, the position of the action point is ${}^{O}\boldsymbol{p}_i = \left[{}^{O}p_{ix}, {}^{O}p_{iy}, {}^{O}p_{iz}\right]^T$ $(i = r, l)$, the position of the center of gravity is ${}^{O}\boldsymbol{p}_e = \left[{}^{O}p_{ex}, {}^{O}p_{ey}, {}^{O}p_{ez}\right]^T$, and the minute torque is $\boldsymbol{d\tau} = \left[0, d\tau_y, 0\right]^T$. Moreover, denoting the action point expressed in Σ_U as ${}^{U}\boldsymbol{p}_i = \left[p_{ix}, p_{iy}, p_{iz}\right]^T$ $(i = r, l)$, and the rotational matrix from Σ_O to Σ_U as ${}^{U}\boldsymbol{R}_O$,the equation of the torque on the tumbling operation is expressed as follows.

$$ {}^{U}\boldsymbol{p}_r \times {}^{U}\boldsymbol{f}_r + {}^{U}\boldsymbol{p}_l \times {}^{U}\boldsymbol{f}_l - {}^{U}\boldsymbol{R}_O {}^{O}\boldsymbol{p}_e \times M\boldsymbol{g} = \boldsymbol{d\tau} \quad (2) $$

$$ {}^{U}\boldsymbol{p}_i = {}^{U}\boldsymbol{R}_O({}^{O}\boldsymbol{p}_i - {}^{O}\boldsymbol{p}_e) \quad (3) $$

$$ {}^{U}\boldsymbol{R}_O = \begin{bmatrix} cos(\theta) & 0 & sin(\theta) \\ 0 & 1 & 0 \\ sin(\theta) & 0 & cos(\theta) \end{bmatrix} \quad (4) $$

Forces are applied from the arm tip to the object as shown on the left side of Figure 3. In this case, we assume that the resultant force of ${}^{U}\boldsymbol{f}_a$ and ${}^{U}\boldsymbol{f}_b$ is ${}^{U}\boldsymbol{f}_i$. Also, it is assumed that the plane and the ridge line are in contact. Therefore, the action of force around the ridge line is expressed as shown on the right side of Figure 3. The unit direction vector of bisector of the inner angle in ${}^{U}\boldsymbol{p}_i$ is ${}^{U}\boldsymbol{n}_i = \left[n_{ix}, n_{iy}, n_{iz}\right]^T$ $(i = r, l)$, and the coefficient of friction between the object and the arm is μ_{rob}. From the angle α between $\boldsymbol{f}_i$ and $\boldsymbol{n}_i$, the condition for not causing slip between the object and the arm tip is expressed by the following Equation (5).

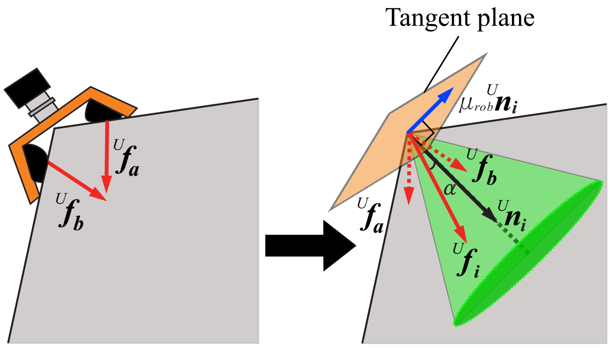

Figure 3. Relationship between force and friction cone at action point

$$ \frac{|{}^{U}\boldsymbol{n}_i||{}^{U}\boldsymbol{f}_i|}{\sqrt{1 + \mu_{rob}{}^2}} - {}^{U}\boldsymbol{n}_i \cdot {}^{U}\boldsymbol{f}_i \leq 0 \quad (5) $$

Further, the condition for not causing slip between the object and the floor surface is given by Equation (6) from the μ_{env} which is the friction coefficient between the object and the floor surface.

$$ \sqrt{f_{fx}{}^2 + f_{fy}{}^2} \leq |\mu_{env} N_z| \quad (6) $$

4.2 Formulation of problem to determine action points

In order to determine the optimal combination of action points, we define constraint equation and evaluation functions. The optimum solution in this paper means the combination of action points that can perform the tumbling operation with a force as small as possible. Therefore, the evaluation function on the magnitude of force is set as in Equation (7).

$$ f({}^{U}\boldsymbol{f}_r, {}^{U}\boldsymbol{f}_l) = |{}^{U}\boldsymbol{f}_r|^2 + |{}^{U}\boldsymbol{f}_l|^2 \quad (7) $$

By solving the nonlinear optimization problem shown in Equation (8), the problem of determining the action point is solved.

$$ \begin{array}{ll} \text{minimize} & f({}^{U}\boldsymbol{f}_r, {}^{U}\boldsymbol{f}_l) \\ \text{subject to} & h({}^{U}\boldsymbol{f}_r, {}^{U}\boldsymbol{f}_l) = 0 \\ & g({}^{U}\boldsymbol{f}_r, {}^{U}\boldsymbol{f}_l) \leq 0 \end{array} \quad (8) $$

$h({}^{U}\boldsymbol{f}_r, {}^{U}\boldsymbol{f}_l)$ and $g({}^{U}\boldsymbol{f}_r, {}^{U}\boldsymbol{f}_l)$ represent the equality constraints of Equation (1) and (2), respectively, or the inequality constraints of Equation (5) and (6).

However, it is desirable to improve computation efficiency as much as possible for multivariate analysis, so we will reduce variables. First, Equation (9) can be obtained by balancing the force in the Y_U axis direction of Equation (1) with only the external force related to the arm tips.

$$ f_{ly} = -f_{ry} \quad (9) $$

From Equation (9), one variable can be removed from the problem. Furthermore, there are five remaining variables, and there are three equations from Equation (2), so it is possible to reduce up to three variables.

For example, simultaneous equations for three variables are expressed as follows.

$$\begin{bmatrix} f_{rz} \\ f_{lx} \\ f_{lz} \end{bmatrix} = \boldsymbol{A}_0^{-1} \begin{bmatrix} -(p_{rz}-p_{lz})f_{ry} - Mp_{gy}g_z \\ -p_{rz}f_{rx} + Mp_{gx}g_z + d\tau_y \\ p_{ry}f_{rx} - (p_{rx}-p_{lx})f_{ry} \end{bmatrix} \tag{10}$$

Further, $\boldsymbol{A}_0$ is shown below.

$$\boldsymbol{A}_0 = \begin{bmatrix} p_{ry} & 0 & p_{ly} \\ -p_{rx} & p_{lz} & -p_{lx} \\ 0 & -p_{ly} & 0 \end{bmatrix}, |\det \boldsymbol{A}_0| \neq 0 \tag{11}$$

Three variables can be reduced by incorporating f_{rz}, f_{lx}, f_{lz} into the evaluation function or inequality constraint condition respectively. However, an inverse matrix of $\boldsymbol{A}_0$ is necessary for satisfying Equation (10). In a similar way, there are ten combinations when reducing three variables from five variables. Furthermore, determinants of $\boldsymbol{A}_{\mathrm{j}}(\mathrm{j}=0,\ldots,9)$ are as follows.

$$\begin{cases} \det\boldsymbol{A}_0 &= p_{ly}(p_{rx}p_{ly} - p_{lx}p_{ry}) \\ \det\boldsymbol{A}_1 &= p_{ry}(p_{lx}p_{ry} - p_{rx}p_{ly}) \\ \det\boldsymbol{A}_2 &= (p_{rx}-p_{lx})(p_{rx}p_{ly} - p_{lx}p_{ry}) \\ \det\boldsymbol{A}_3 &= p_{ly}(p_{ry}p_{lz} - p_{ly}p_{rz}) \\ \det\boldsymbol{A}_4 &= p_{ry}(p_{ly}p_{rz} - p_{ry}p_{lz}) \\ \det\boldsymbol{A}_5 &= (p_{rz}-p_{lz})(p_{ly}p_{rz} - p_{ry}p_{lz}) \\ \det\boldsymbol{A}_6 &= p_{ly}(p_{lx}p_{rz} - p_{rx}p_{lz}) \\ \det\boldsymbol{A}_7 &= p_{ry}(p_{rx}p_{lz} - p_{lx}p_{rz}) \\ \det\boldsymbol{A}_8 &= (p_{rx}-p_{lx})p_{ry}p_{lz} - (p_{rz}-p_{lz})p_{rx}p_{ly} \\ \det\boldsymbol{A}_9 &= (p_{rx}-p_{lx})p_{ly}p_{rz} - (p_{rz}-p_{lz})p_{lx}p_{ry} \end{cases} \tag{12}$$

However, when neither p_{ry} nor p_{ly} has a value, in other words when both Y_U components of the action point are included on the origin of Σ_U, variables can not be reduced by the method like equation (11). At that time, if we substitute $p_{ry} = 0$ and $p_{ly} = 0$ for Equation (2), f_{ry} is expressed by Equation (13).

$$f_{ry} = \begin{cases} \frac{Mp_{gy}g_z}{p_{rz}-p_{lz}} & (p_{rz}-p_{lz} \neq 0) \\ 0 & (p_{rx}-p_{lx} \neq 0) \end{cases} \tag{13}$$

In addition, in order to determine one variable, it is divided into two cases as follows.

$$f_{rx} = \frac{p_{rx}f_{rz} - p_{lz}f_{lx} + p_{lx}f_{lz} + Mp_{gx}g_z + d\tau_y}{p_{rz}} \tag{14}$$

$$|p_{rz}| \geq |p_{lz}|, p_{rz} \neq 0$$

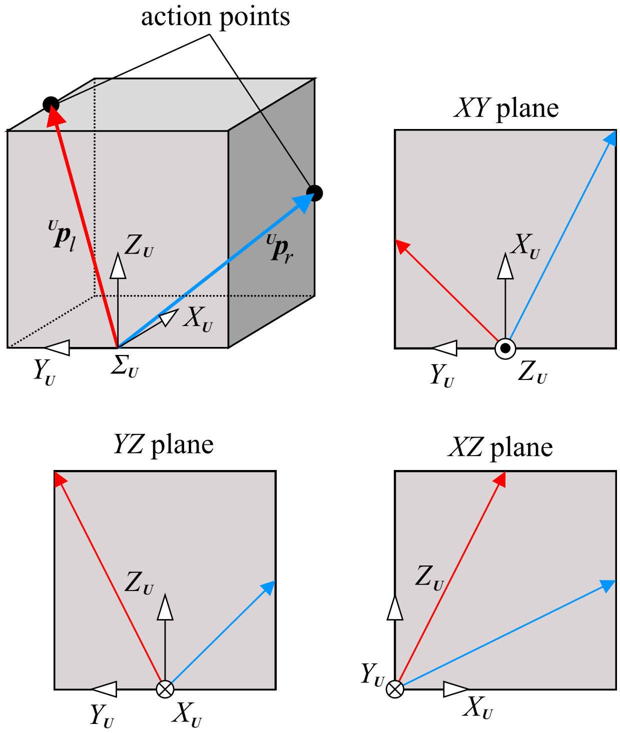

Figure 4. The positional relationship between the action point and the origin of Σ_U on each coordinate plane

$$f_{lx} = \frac{p_{rx}f_{rz} - p_{rz}f_{rx} + p_{lx}f_{lz} + Mp_{gx}g_z + d\tau_y}{p_{lz}} \tag{15}$$

$$|p_{lz}| \geq |p_{lz}|, p_{rz} \neq 0$$

Next, the case where the Y_U component of one of the at least two action points does not exist on the origin of Σ_U will be described. In this case the determinants of equation (12) are narrowed down by using the geometric characteristics of the object. As an example, suppose two action points are selected in a cubic object as shown in Figure 4. Also, in Figure 4, the positional relationship between the action point and the origin of Σ_U is shown using the projection of each coordinate plane. It can be seen that a part representing the outer product is included in part of the Equation (12). When no outer product exists, the two vectors overlap each other, or they are in a parallel relationship. Since there is a possibility of obtaining a singular point of a determinant by a combination of action points, conditional branching is necessary to reduce variables. In this case, we select the determinant taking the maximum value and reduce the variables. Furthermore, based on geometrical characteristics, the condition that no outer product exists in the coordinate plane is only when the action point of the same point is selected, so the determinant to be calculated can be narrowed down.

Therefore, reduction of variables are determined only by the determinant shown in Equation (16).

$$\begin{cases} \det \boldsymbol{A}_1 &= p_{ry}(p_{lx}p_{ry} - p_{rx}p_{ly}) \\ \det \boldsymbol{A}_3 &= p_{ly}(p_{ry}p_{lz} - p_{ly}p_{rz}) \\ \det \boldsymbol{A}_5 &= (p_{rz} - p_{lz})(p_{ly}p_{rz} - p_{ry}p_{lz}) \\ \det \boldsymbol{A}_7 &= p_{ry}(p_{rx}p_{lz} - p_{lx}p_{rz}) \end{cases} \tag{16}$$

In the Equation (16), we select the determinant that has the largest value of determinant and reduce the variable.

From the above, we solve Equation (8) after reducing variables. Furthermore, variables to be reduced depend on the position of action points. When solving based on the above method, it means that there is no feasibility of the tumbling operation unless equation (8) converges.

5 Algorithm of trajectory planning

An algorithm was constructed to determine optimal combination of action points from the problems described in the previous chapter. The flowchart is shown in Figure 5, and one ridge line which is the rotation axis of the object is set beforehand.

First, convergence calculation of Equation (8) is performed for the selected two action points. At this time, if an executable solution of the tumbling operation is not obtained, another combination of action points is selected. On the other hand, when Equation (8) is satisfied, 1[deg] is added to the rotational angle θ, and the convergence calculation of Equation (8) is performed again. When θ which is the rotation angle of the object reaches θ_{max}, this means that single tumbling operation can be executed to the end.

Then, by using the value of the evaluation function of Equation (7), the combination of the optimum action points is determined. The optimal combination is determined by comparing the sum of the evaluation functions obtained after the convergence calculation. However, the trajectory planning obtained by this algorithm does not include the operation of changing the action point in the middle of single tumbling operation.

6 Numerical example

Our planning algorithm is implemented on Windows using C language. We used IPOPT (Interior Point OPTimizer) [7] , which is a library suitable for large-scale continuous optimization problems. For the convex polyhedron shown in Figure 6, the combination of action points in single tumbling operation is determined. The object used in the experiment is a wooden polyhedron with size 0.32 × 0.29 × 0.28[m], M= 2.0[kg], g =

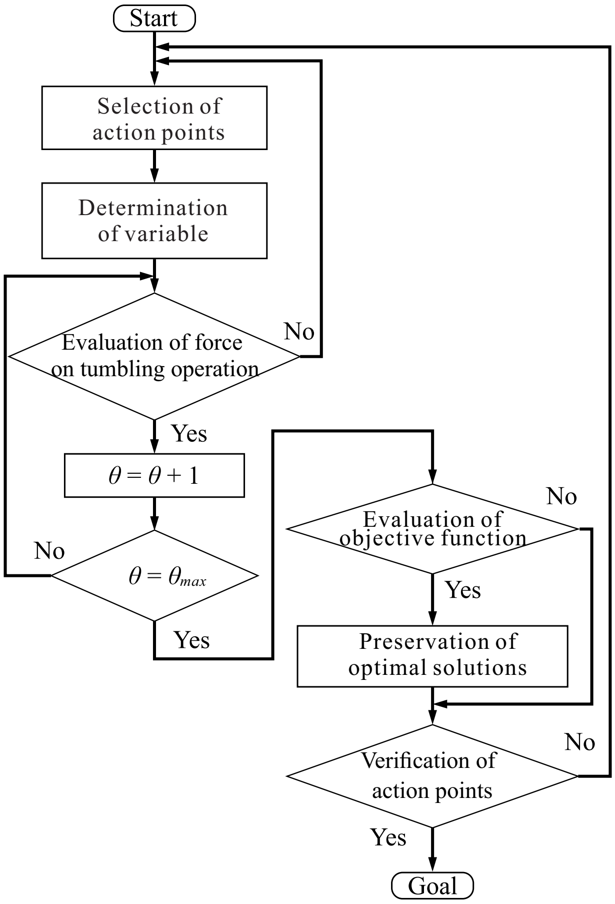

Figure 5. Flow chart for selection of action point

9.80665[m/s^2], θ_{max}=47.2[deg] and the friction coefficients are μ_{rob} = 0.5 and μ_{env} = 0.5, respectively. Figure 6 shows the object of a convex polyhedron. Since the object has twenty plane faces and the ridge line of the bottom surface is excluded, there are 27 action points. The combination of the action points obtained by the implemented algorithm is also shown in Figure 6.

Figure 7 shows the norms of force required for single tumbling operation for each θ, and Figure 8 is shown the planning trajectory of tumbling operation. In addition, as can be seen from Figure 7, when comparing the force acting on the object, the tumbling operation can manipulate the object with a smaller force than $Mg/2$ = 9.80665[N]. In the tumbling operation, it can be seen that the left and right forces fluctuate as θ increases.

We presented an impedance control of a mobile robot with dual arms for a three-dimensional tumbling operation [8] . Also, experimental verifications were conducted and its results were shown the effectiveness of the proposed algorithm. However, trajectory planning of this paper doesn't consider the characteristics of the mobile robot. Therefore, in the future, it is necessary to construct the

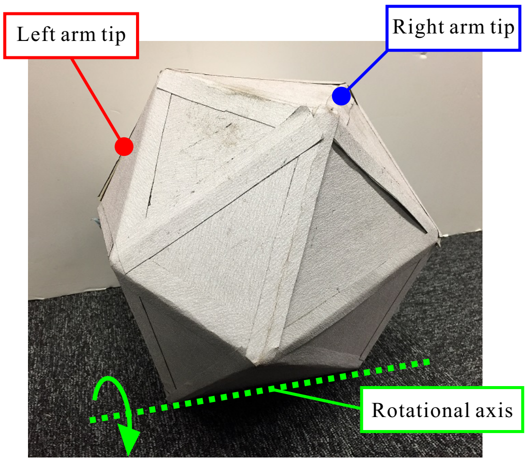

Figure 6. The appearance of the object

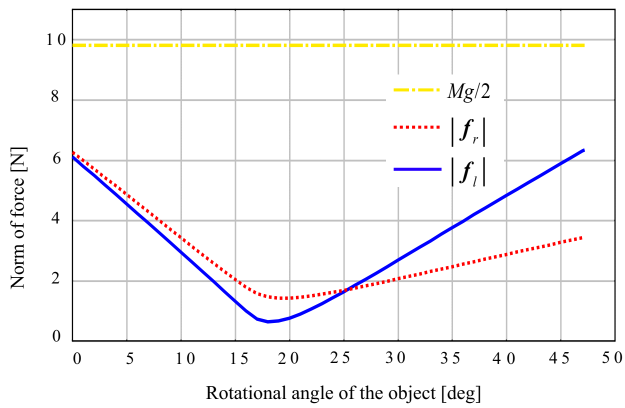

Figure 7. Result of tumbling operation and $Mg/2$

trajectory planning including characteristics of the robot.

7 Conclusion

In this paper, we presented the method for determining the combination of action points by analyzing the tumbling operation. We solved this problem as a nonlinear optimization problem by the evaluation function and the constraints on relational expression of force. From the characteristics of geometry, we showed effective reduction method of variable against multivariable problem. Finally, the usefulness of the algorithm was shown by a numerical example. As a future works, we will attempt to construct a trajectory planning that considers the robot's hand position and posture. Currently, we are trying verification experiments based on this algorithm by a mobile robot with dual arms.

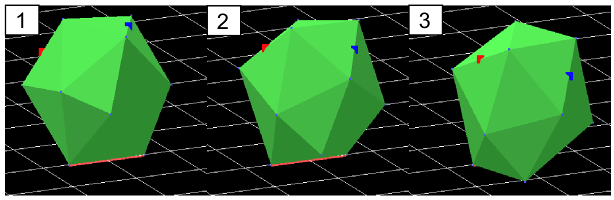

Figure 8. Planned tumbling operation

References

[1] Yasumichi Aiyama, Masayuki Inaba, and Hirochika Inoue. Pivoting: A new method of graspless manipulation of object by robot fingers. In *Intelligent Robots and Systems' 93, IROS'93. Proceedings of the 1993 IEEE/RSJ International Conference on*, volume 1, pages 136–143. IEEE, 1993.

[2] Atsushi Yamashita, Jun Ota, Kou Kawano, Masaki Fukuchi, Tamio Arai, and Hajime Asama. Motion planning for cooperative manipulation of an object by multiple mobile robots. *IEEJ Transactions on Electronics, Information and Systems*, 121(3):549–556, 2001.

[3] Y Aiyama. Environment-contacting task by position-controlled manipulator using free-joint structure. In *Advanced Intelligent Mechatronics, 2001. Proceedings. 2001 IEEE/ASME International Conference on*, volume 2, pages 816–821. IEEE, 2001.

[4] Masamitsu Kurisu, Kazuhiko Tsukada, and Koich Hanasaki. Path planning for an object in tumbling operation. In *Proceedings of the 17th Annual Conference of the Robots Society of Japan*, pages 1021–1022, Kanagawa, Japan, 1999.

[5] Naoyuki Sawasaki and Hirochika INOUE. Tumbling objects using a multi-fingered robot. *Journal of the Robotics Society of Japan*, 9(5):560–571, 1991.

[6] Yusuke Maeda, Tomohisa Nakamura, and Tamio Arai. Motion planning of robot fingertips for graspless manipulation. In *Robotics and Automation, 2004. Proceedings. ICRA'04. 2004 IEEE International Conference on*, volume 3, pages 2951–2956. IEEE, 2004.

[7] Andreas Wächter and Lorenz T. Biegler. On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming. *Mathematical Programming*, 106(1):25–57, Mar 2006.

[8] T. Kitazawa, M. Kurisu, and S. Takemasa. Impedance control of a mobile robot with dual arms for a tumbling operation. In *2017 11th Asian Control Conference (ASCC)*, pages 25–30, Dec 2017.