Mathematical Optimisation of Rail Station Location and Route Design in Urban Regions through Minimising Noise Pollution

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Abstract

The number of cities that are implementing sustainable transportation programs has been recently on the rise. Reliance in large urban areas is on the use of public transportation systems such as busses and rapid transit lines (heavy rail) to act as the main mode of public transport in multi-modal transport systems. It should be noted however that operations of rapid transit lines are associated with heavy noise levels. When it comes to addressing the issue of noise pollution from rail lines, it is important to consider the design factors that affect the levels of noise disruption impacting the surrounding population. An important design factor that plays a significant role in determining the overall noise levels that propagate from the rail lines to the noise-sensitive residential zones is the choice of the rail route. As a result, careful consideration of the location of the stops of a rapid transit line is an important issue to address when attempting to curtail the noise pollution of rail operations. This paper proposes a novel mathematical formulation, based on a Binary Integer Programming model, to optimise the locations of stops for a typical rail line construction project. The focus in the proposed model is on minimising the total noise pollution levels associated with the operations of the transit line, given that noise is an important social and environmental factor that impacts the sustainability of the project. A case study is presented at the end to demonstrate the applicability of the proposed model.

Keywords –
Station Location Optimisation; Noise Pollution; Sustainable Rail Design, Binary Integer Programming; Location Theory

1 Introduction

Sustainable practice in the construction industry is based on balancing social, environmental and economic aspects when designing and planning projects. Designing sustainable infrastructure projects requires careful consideration of the resulting environmental impacts associated with the operation of the project [1]. One of the nuisances reported during and after infrastructure construction works is that related to noise pollution. The issue of noise pollution during the construction process can be handled through measures that target the noise source (i.e. noise dampeners installed at machines), noise receivers (noise insulation in wall buildings, or noise transmittance path (use of noise barriers around the construction site [2–4]. Once the rail line starts to operate then the issue of noise is harder to handle; as a result, enhanced decision making at the initial design stage requires a better understanding of factors that impact noise propagation when the rail line is operated.

Noise from rail tracks is a result of many elements, the most prominent of which is the impact between the wheels of the rolling stock and rough rail surface [5]. Other causes of noise from a rail system are the consequence of sharp turns of the vehicle due to large rail track [6], or the crossing of a vehicle over dipped weld within the track [2]. It is also important to note that there are certain factors which can exacerbate noise levels, and which need to be accounted for when assessing noise generated by a railway system; this includes the turbulent boundary created around the train and the Doppler Effect induced by high speed trains passing a particular point [7].

Noise pollution resulting from rail line operations is reported to be associated with a number of health
ailments. Exposure to rail noise during the night leads to sleep disturbances [8]. This in turn creates stress, triggering the sympathetic and endocrine system, and causing a change to blood pressure and heart rate in individuals [9]. There are studies that indicate a link between sleep quality depreciation and railway noise [10]. Others report that lower cognitive performance results following nocturnal railway noise exposure [11]. The cardiovascular system in sleeping subjects was also observed to be impacted by nocturnal railway noise [12]. In some instances, when compared to well documented noise from highways, annoyance of railway noise was found to be similar if not worse [13]. What the majority of studies agree on is the fact that noise disturbance from rail lines is present and requires practical regulatory policies for it to be tackled [3]. Factors that play into the perception of noise are wide and varied; some are even due to socio-cultural traits of a particular region [14]. An imperative factor discussed by [15] that is a strong determinant of noise annoyance reception is the distance between the receiving end to the railways. In addition, train timetable and schedule are thought to have a direct association on the level of noise generated from the rail lines [16]. Even though there are various studies that examine the location of rail stations in the literature [17] and its impact on land value [18,19], their focus has not been on minimising noise pollution from rails when it comes to the location of the stations.

It is thus essential to target more effective measures when addressing the issue of noise pollution of rail lines during the initial design phase. This paper attempts to achieve this through introducing a novel mathematical optimisation model for locating stations of a rapid transit line such that noise pollution is minimised.

2 Problem Description

The problem of locating a transit line or network is concerned with maximising riders’ accessibility while minimising rail line construction costs at the same time [20–23]. Accessibility of a transit line is modelled in two ways: in one of the approaches stations are positioned in a manner that ensures maximal service coverage to demand points surrounding the assigned transport corridor. This problem is known in the literature as the maximal coverage shortest path problem [24]. Another approach is to locate the stations so that total travel distance between demand regions and the nearest station located on a line is minimised; in this case the problem is known as the median shortest path problem [25].

For the majority of the work available in the literature the focus has been mainly on objective functions that concern the system’s patrons or operators. Little attention has been directed towards modelling the impact that the design of the proposed rail line is likely to cause on the neighboring population in terms of noise.

The problem examined in this paper is concerned with the location of the stations forming the rapid transit line. In order to find optimal locations for the network line, the problem is formulated to minimise the total noise levels reaching receiver points; these receiver points represent surrounding residential zones.

2.1 Measuring Noise Levels in Rail Lines

To assess the effect of rail noise exposure at residential buildings surrounding the rail system, some studies adopt the equivalent noise level assessed over a 24 hour period measure; this is calculated from the sound exposure levels and number of different train types [13]. It is, however, reported in the guidelines published by Environmental Protection Agency (EPA) in Australia that the continuous equivalent noise level ($L_{eq}$) measured over a certain period corresponding to daytime, night-time or the noisiest one hour duration, should be deployed [26].

When it comes to noise estimation from rail-lines, a number of steps needs to be followed. These are based on guidelines published by both EPA and the Department of Transportation [26,27]. The steps are summarised as follows. First, the railway needs to be divided into segments. For each segment, the number of receiver points that will be impacted by the noise need to be mapped. The angle of view of the receivers with respect to each track segment is then calculated.

The second step involves calculating the reference noise level (SEL) for each train that will be operating on tracks. The baseline SEL at a reference of 25m from the train (i.e noise source) is obtained from train manufactures. The baseline SEL will then need to be corrected to account for the number of vehicles in the train. A track correction also needs to be applied to account for the type of tracks and the associated ballast laid.

The third step involves accounting for the distance propagation impact. This will be determined based on the location of the reception point with respect to the tracks. The height of the track below/above the reception point needs to be considered. Ground propagation and air absorption are corrected for. The presence of any barriers around the tracks, separating the track line from the reception points, will also need to be considered in the distance propagation measure. The angle of view from the tracks with respect to the receiver points is also considered in the noise propagation calculations.

The fourth step involves consideration of noise reflection effects due to façade of the reception points (assuming the receiver is a building).

In the fifth step, the SEL is converted into the equivalent continuous noise measure $L_{eq,T}$ over the duration of the assessment ($T = 6$ hours usually for
nighttime and $T = 18$ hours for daytime) using the following equations, Eq. (1) – Eq. (2):

$$L_{Aeq,6} = SEL + 10\log_{10}Q_{\text{night}}$$  \hspace{1cm} (1)  

$$L_{Aeq,18h} = SEL - 48.1\log_{10}Q_{\text{day}}$$  \hspace{1cm} (2)  

where $Q_{\text{night}}$ and $Q_{\text{day}}$ represent the total number of trains predicted to pass the receiving points during the night and day time respectively. The unit of measurement for the equivalent noise level is the A-weighted decibel (dB(A)).

### 2.2 Mathematical Model

The mathematical optimisation model proposed is a Binary Integer Programming (BIP) model that attempts to find the most suitable rail line configurations associated with the least noise pollution, as measured at the surrounding population. The objective function and constraints formulated to address the problem are described next.

#### 2.2.1 Notation

The notation adopted in the proposed model is presented in Table 1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O$</td>
<td>Set of potential origin stations</td>
</tr>
<tr>
<td>$D$</td>
<td>Set of potential destination stations</td>
</tr>
<tr>
<td>$k \in O \cup D \subseteq K$</td>
<td>Set of all potential stations</td>
</tr>
<tr>
<td>$r \in R$</td>
<td>Set of noise-sensitive receivers</td>
</tr>
<tr>
<td>$P$</td>
<td>Maximum number of stations on a transit line</td>
</tr>
<tr>
<td>$N_{kr}$</td>
<td>Noise levels resulting at receiver point $r \in R$ due to train passing potential station $k \in K$</td>
</tr>
<tr>
<td>$z_k$</td>
<td>Binary variable, which equals 1 if station $k$ is selected, and 0 otherwise</td>
</tr>
<tr>
<td>$x_{ij}$</td>
<td>Binary variable, which equals 1 if potential stations $i$ and $k$ are connected, and zero otherwise</td>
</tr>
</tbody>
</table>

#### 2.2.2 Objective function

The objective function captures the maximum noise disruption that is caused at each potential station location and is formulated as shown in Eq. (3):

$$\text{minimise } \sum_{k \in K} \max_{r \in R} \left\{ N_{kr} \right\} z_k$$  \hspace{1cm} (3)  

In particular, the final equivalent continuous noise measure is mapped onto the noise parameter $N_{kr}$ (i.e. $L_{Aeq,6} \rightarrow N_{kr}$).

#### 2.2.3 Constraint Type 1: Origin & Destination Stations

The start and end of the transit line is determined by the choice of the origin and destination stations. Eq. (4) is defined to locate an origin station, while Eq. (5) defines the location of a destination station.

$$\sum_{k \in O} z_k = 1$$  \hspace{1cm} (4)  

$$\sum_{k \in D} z_k = 1$$  \hspace{1cm} (5)  

#### 2.2.4 Constraint Type 2: Connectivity

There needs to be a constraint defined to ensure that all selected stations are connected. Two types of equations are formulated: one is for stations that are either the origin or destination station, where these require to be connected to 1 other station. In particular, Eq. (6) is formulated for connecting the chosen origin station with another interim station on the line. Eq. (7) on the other hand is defined to link the chosen destination stations with other interim stations on the line.

The second type of connectivity constraints is associated with interim stations (not origin or destination stations), given that these require to be connected to 2 other stations. This is achieved through Eq. (8).

$$x_{ij} = z_i z_j \quad \forall i \in O, j \in K \cup O \cup D$$  \hspace{1cm} (6)  

$$x_{ij} = z_i z_j \quad \forall i \in K \cup O \cup D, \forall j \in D$$  \hspace{1cm} (7)
\[ \sum_{i \in k} x_{ik} + \sum_{j \in k} x_{kj} = 2z_k \quad \forall k \in K \setminus O \cup D \quad (8) \]

2.2.5 Constraint Type 3: Sub-tour elimination constraints

Only a single continuous tour, representing the route of the rapid transit, is to be produced as the final solution, as opposed to two or more disjoint sub-tours. The algorithm is prevented from creating sub-tours by introducing Eq. (9) in the model:

\[ \sum_{j \in K} \sum_{i \in j} x_{ij} \leq |S| - 1 \quad (9) \]

2.2.6 Domain of variables

The domain of the variables used in the model is given in Eq. (10) – Eq. (11):

\[ z_i \in \{0,1\} \quad \forall i \in K \quad (10) \]
\[ x_{ij} \in \{0,1\} \quad \forall i, j \in K : i < j \quad (11) \]

3 Case Study and Discussion

In order to display the applicability of the model, a realistic case study is examined. The case study, a simplified representation of a future rail project to take place in Doha, Qatar, comprises a single track railway. For the purpose of this study, only 1 type of train is assumed to traverse the rails. The section of the train to be designed is assumed to be a straight line segment. Average speed of the train across the rail way system is set at 150 km/h. A total of 179 train passes are assumed during the day, while the number drops to 17 passes during the night. The area between the track and the reception point is assumed to be flat. The region under consideration, with residential areas, and potential station locations, is highlighted in Fig. 1. In particular, the potential station locations for the origin and destination nodes of the rapid transit line to be constructed are highlighted by the circulated zones in Fig. 1. Table 2 displays the noise analysis conducted at each residential node.

![Fig. 1 Case study representation](image)

The solution yielded by the proposed model is contrasted with an approach that is based on selecting the route with the least overall cost; this can be achieved by minimising the travel distance and number of stations respectively. The results are displayed in Table 3. It is clear that some sort of trade-off exists between the two contrasted models. The minimum noise model produces a solution that is 22% less noise intensive than that of the minimum construction cost model. The cost of construction associated with the minimum cost model is however 39% higher. It is important to note that this is only case specific and there can be instances where the solution that minimises the noise pollution due to the operations of the rail line is also the one that minimises the construction cost. What the test highlights however is the need to consider the trade-off between other factors...
impacting the design of rapid transit lines in urban regions.

**Table 3** Noise level in dB (A) at each station

<table>
<thead>
<tr>
<th>Model</th>
<th>Average Noise Level (dB(A))</th>
<th>Monetary Cost ($ AUD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Noise Model</td>
<td>68</td>
<td>123,909</td>
</tr>
<tr>
<td>Minimum Construction Cost</td>
<td>87</td>
<td>75,444</td>
</tr>
</tbody>
</table>

4 Conclusion

In this study a novel mathematical optimisation model was presented, based on a binary integer programming approach, to minimise the noise pollution associated with the operation of an urban rapid transit line. The model optimises the locations of stations making up the rail line, through forming a continuous link between the selected stations, while minimising the noise levels measured at receiving points along the track. The proposed model can therefore act as a decision support tool to rail line designers at the initial planning phases of the project in order to reflect considerations for the sustainability of the rail track.

In order to test the applicability of the model, a realistic case example was examined. In addition, the solution yielded by the proposed model was also contrasted against one that was based on minimising the construction cost of the rail line. Results revealed that a drop of 22% can be achieved if noise is optimised, as opposed to construction cost. The solution produced by the cost minimisation model however yielded a track that was 39% cheaper. A trade-off was therefore realized between the two models for the case study considered.

The proposed model is formulated for new rail lines proposed in an urban region. With slight modification, the model can also be adopted to account for incorporation of one or more stations to an existing line. Future work will be focusing on developing a multi-objective optimisation model that accounts for social, environmental and economic aspects of rail line route design.

References


