

Reflective Twins for Deep Excavation using Physics-Informed Machine Learning

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Abstract

Basement construction projects often experience low productivity, schedule delays, cost escalation, safety risks, and high embodied carbon. These challenges largely arise from uncertainty in ground conditions and construction-stage soil-structure interaction, which cannot be fully resolved during design. The observational method offers a structured means of managing this uncertainty by updating design assumptions using monitored performance during construction. However, its adoption remains limited due to the lack of digital tools that robustly integrate monitoring data with predictive and reflective models within construction timescales.

This paper proposes a digital twin framework for deep excavations that embeds the observational method within a layered architecture to enable stage-wise model updating and adaptive decision-making. This structure supports progressive automation from descriptive to prescriptive operation while clarifying interactions between monitoring, data handling, modelling, and decision support. The central contribution of the paper is the development of a reflective twin enabling back-analysis engine, based on physics-informed neural networks. The engine enables continuous updating of the virtual model using field observations, allowing physically consistent inference of loads and deformation profiles from sparse and noisy monitoring data. Validation using canonical cantilever retaining wall problems demonstrates robust reconciliation between observed and predicted behaviour. This capability enables rapid model updating and adaptive construction control, thereby operationalising the observational method within a digital twin environment.

Keywords –

digital twins; observational method; Physics-informed machine learning;

1 Introduction

Urban densification is driving a rapid increase in deep basement and underground construction, including station boxes, high-rise foundations, and transport or energy infrastructure. These projects are typically delivered within highly constrained urban environments and close to existing assets, where construction-stage performance control is critical. A persistent challenge in such works is uncertainty arising from limited site investigation data, complex soil behaviour, and modelling idealisations. To manage this uncertainty, designs are often conservative, leading to inefficient construction sequences, increased material use, and elevated embodied carbon.

The observational method in geotechnical engineering provides a structured framework for managing uncertainty during construction through a learn-as-you-go process [1]. It enables continuous refinement of design assumptions using monitored performance, supported by back-analysis and predefined contingency measures. In deep excavation projects, the observational method is used to control construction-stage risk, optimise temporary works and excavation sequencing, validate soil-structure interaction models, and adjust support systems based on observed behaviour. Its application has demonstrated tangible benefits in construction control, safety, and efficiency, in several complex underground works and high-profile infrastructure projects [2], [3].

Despite its strengths, the practical implementation of observational method is constrained by analytical and computational limitations. The method relies on repeated back-analysis of non-linear ground-structure interaction, often using sparse and indirect measurements such as wall deflections. This process is typically manual and time-intensive, limiting its ability to operate within construction timescales and preventing monitoring data from being fully leveraged in predictive models.

Digital twins offer a pathway to overcome these limitations by integrating monitoring data with

computational models to create continuously evolving representations of underground systems. While sensing, data platforms, and visualisation tools have advanced significantly, a robust back-analysis engine capable of translating sparse field observations into an updated, physically consistent state remains a key bottleneck. The current study addresses this gap by developing a physics-informed back-analysis framework that forms the reflective core of a digital twin for deep excavation. Beyond a conceptual link between digital twins and the observational method, it provides a practical workflow for converting sparse monitoring data into an updated, mechanically consistent system state for stage-wise decision-making.

The main contributions of this work are:

- A digital twin framework tailored to observational method in deep excavations.
- A parsimonious inverse physics-informed neural networks (PINNs) for physically consistent back-analysis from sparse and noisy data.
- Validation on canonical examples, including a retaining-wall idealisation, to demonstrate feasibility in a reduced-order setting.

2 Related work

2.1 Digital twins and observational method

Digital twins are virtual representations of physical systems that are continuously updated using field data to reflect current behaviour and support prediction and decision-making [4]. For underground construction, Babanagar et al. (2025) [5] proposed a layered digital twin architecture (Figure 1) with increasing levels of intelligence, ranging from reflective twins that update models in real time to prescriptive twins that actively support decision-making. In this framework, the physical system, including ground, structures, and sensors, feeds into a data layer that manages monitoring and simulation outputs. These data are linked to modelling and visualisation tools such as BIM and numerical models, while an application layer delivers functions including model updating, stability assessment, risk evaluation, and decision support. While this architecture establishes a robust conceptual structure, it also highlights the opportunity to further develop practical guidance for digital twin frameworks tailored to specific requirements.

Recent developments in the observational method research have explored advanced sensing technologies and data-driven techniques to improve data processing and quantify uncertainty in monitoring data. However, in practice, the observational method still relies heavily on manual interpretation and expert judgement for model updating and construction-stage decisions, which limits responsiveness and consistency across projects.

Digital twins provide a natural platform to address these limitations by enabling automated data integration, real-time model updating, and continuous assessment of construction performance. Integrated analytics and visualisation allow alternative construction scenarios to be evaluated efficiently, supporting proactive risk management and cost control. The principles of observational method align closely with these capabilities. It requires continuous monitoring, comparison of observed and predicted behaviour, and timely adjustment of construction strategies when deviations arise. Digital twins directly support these steps through automated data pipelines, rapid model updating, and efficient evaluation of alternative responses [6]. Despite this strong conceptual alignment, the integration of observational method and digital twins have largely been unexplored in research [5]. Observational method studies typically treat monitoring, interpretation, and decision-making as loosely coupled processes, while digital twins research in underground construction has focused mainly on data integration, visualisation, and asset management, with limited attention to structured observational method workflows.

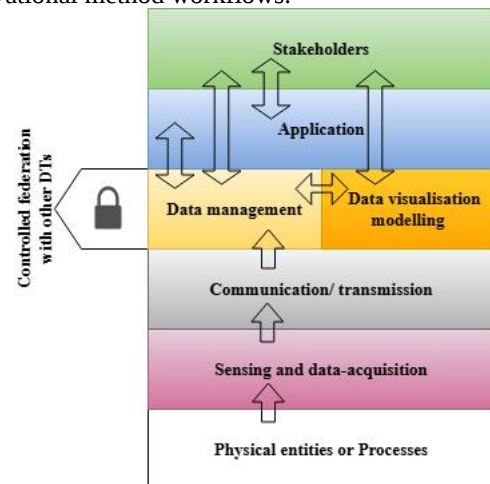


Figure 1: Layered architecture of digital twins for urban underground space.

A key barrier to effective integration is back-analysis, or system identification. Updating model parameters and load states from monitoring data is often iterative and computationally intensive, and typically requires specialist input, which limits its suitability for near real-time use. Without fast and reliable back-analysis, digital twins remain largely reflective and cannot fully support the adaptive decision-making that is central to the observational method. At the same time, the requirements of the observational method vary across underground construction types, including tunnels, deep basements, shafts, and caverns, due to differences in governing mechanisms, monitoring strategies, data characteristics, and decision timelines. This creates a

need for a tailored digital twin framework for the observational method that defines appropriate information flows, modelling and back-analysis techniques, and decision processes for each construction type, and that explicitly map these requirements across digital twin maturity levels, from reflective to predictive and prescriptive operation.

2.2 Back-analysis

Back-analysis of deep excavations has evolved from heuristic trial-and-error calibration and empirical case histories, through deterministic optimisation, towards probabilistic and hybrid data-driven approaches. Early practice relied on engineer judgement and empirical correlations between wall deflections and excavation geometry or ground conditions [2]. Formal optimisation-based back-analysis then coupled gradient-based or evolutionary algorithms with finite element models to minimise misfit between measured and simulated responses [7], improving objectivity but remaining computationally demanding and sensitive to local minima. Bayesian frameworks introduced probabilistic updating of soil parameters using staged monitoring data or related schemes, yet their cost can be prohibitive when combined with advanced constitutive models [8]. More recently, surrogate-assisted and deep learning-based methods have sought to reduce computational effort and exploit richer datasets, at the expense of reduced physical transparency [9]. In all these approaches, inferred parameters remain strongly dependent on the chosen constitutive soil model, so model-form error can be unwittingly hidden by best-fit parameters and propagated in future predictions. The evolution of back-analysis in the literature highlights a shift towards data-driven machine learning methods, while underlining that most existing techniques are either too costly for frequent updating or rely on black-box models that are difficult to trust in safety-critical decisions [10]. To address these issues, there is a need to adopt physics consistent methods such as physics-enhanced machine learning, which integrates domain knowledge and physical laws into data-driven models for greater transparency and reliability.

3 Digital twins for the observational method in deep excavations

Retaining walls supporting deep excavations are subjected to complex, stage-dependent earth pressures that evolve with excavation progression, support installation, and groundwater fluctuations. Temporary props or struts are designed conservatively based on stage-wise numerical models. These models often predict higher wall deflections and lateral loads than observed in

the field, creating opportunities to optimise support systems, reduce prop requirements, and improve construction efficiency. Conversely, unforeseen variations in soil behaviour, groundwater conditions, excavation sequences, or environmental conditions can result in larger-than-anticipated wall movements, posing risks to wall stability and adjacent assets. Among these environmental factors, temperature fluctuations can have a significant influence on prop forces, as thermal expansion or contraction alters strut loads and can indirectly affect wall deflections. Accurate estimation of these effects is therefore essential to support adaptive construction decisions. Managing both opportunities and risks requires a system capable of continuously estimating the evolving structural state and forecasting future behaviour, which can be achieved by embedding the observational method within a digital twin framework tailored for deep excavation projects.

Practical application of the observational method in deep excavations is constrained by several factors. Monitoring networks are typically sparse and indirect, often comprising a few inclinometers, load cells, and piezometers, which cannot fully resolve the wall-soil-support system. Back-analysis is inherently ill-posed, as multiple combinations of lateral loads and support reactions may produce similar wall deflection patterns. Conventional approaches rely on iterative finite element calibration or empirical adjustments, which are often time-consuming, sensitive to model assumptions, and require dense instrumentation. In contrast, a physics-informed learning approach can directly incorporate governing mechanical laws, providing a data-efficient and physically consistent solution to the inverse problem. Moreover, repeatedly updating non-linear numerical models is computationally intensive, making stage-wise decision-making within construction timescales difficult. These limitations define the following specific requirements for a digital twin framework:

- (i) **data efficiency:** ability to reconstruct structural states from sparse and noisy measurements.
- (ii) **computational speed:** stage-wise updates and forecasts within construction timescales.
- (iii) **physical consistency:** mechanically realistic and interpretable state estimates.
- (iv) **decision alignment:** outputs directly mapped to pre-defined thresholds and construction controls.

3.1 Integrated layered architecture and maturity levels

The proposed digital twin framework for deep excavations is structured around four interdependent layers (Figure 2), enabling progressive levels of automation from descriptive to prescriptive operation. The physical layer captures the retaining wall, excavated soil, temporary supports, groundwater, and

instrumentation network, providing real-time measurements of structural response and environmental variables, including temperature, at each excavation stage. Measurements are acquired alongside construction metadata, such as excavation stage identifiers, prop configurations, and support layouts.

The data management layer consolidates measurements and metadata from multiple sources, performing data quality control, stage identification, time alignment, and contextual tagging. It acts as a central database where all project data are stored and enables federation of data with all stakeholders. The modelling and visualisation layer hosts numerical and surrogate models of the wall-soil-support system and maintains outputs from finite-element software (e.g., PLAXIS) and BIM-based structural representations. It integrates data from instrumentation dashboards, monitoring systems, and environmental sensors, ensuring that all observations, including temperature and water table variations are coherently visualised with the excavation sequence. Updated geometries, back-analysed lateral pressure distributions, and deflection profiles, are visualised through dashboards, providing interpretable insights for engineers. These outputs support assessment of wall performance against design criteria, trigger thresholds, and planned excavation stages. While these components are well established in conventional basement construction, they primarily provide descriptive or visual capabilities and do not inherently perform stage-wise back-analysis or forecasting.

The application layer forms the operational core and addresses the critical bottleneck of real-time system identification. It incorporates the inverse modelling or system identification engine, which performs reflective back-analysis to reconstruct a low-dimensional structural state vector describing wall deflections, lateral pressures, prop reactions, and environmental influences such as temperature-induced load variations. Forward simulation modules then use this state to predict system responses under planned excavation sequences, support adjustments, and environmental variations. Forecast envelopes, stage-wise load estimates, and trigger indicators guide adaptive construction decisions specific to deep excavation, including:

- Selective adjustment or removal of props when measured deflections and temperature-influenced loads remain within safe limits, optimising material use and schedule.
- Modification of excavation sequences, including stage height, duration, or order, based on current wall-soil system behaviour and anticipated temperature effects.
- Reconfiguration of temporary supports, such as strut repositioning or force redistribution by adjusting the hydraulic jacks in the props, to maintain stability

under evolving loads and thermal variations.

- Groundwater control interventions, including controlling dewatering rate in response to changing hydrogeological conditions.
- Trigger-based contingency actions, such as temporary support installation, monitoring intensification, or staged excavation pauses, when forecasted deflections or temperature-influenced prop forces approach thresholds.

This layered organisation is aligned with digital twin maturity levels. At the descriptive level, monitoring data, dashboards, and PLAXIS/BIM visualisations allow engineers to track system behaviour and environmental variations. At the reflective level, the inverse engine reconciles observed behaviour with the virtual model to produce a physically consistent structural state. The predictive level propagates this state to forecast responses for subsequent excavation stages, supporting risk-aware planning under evolving conditions. At the prescriptive level, outputs guide adaptive decisions on excavation sequencing, prop management, support modifications, and groundwater interventions, enabling safe, efficient, and optimised construction.

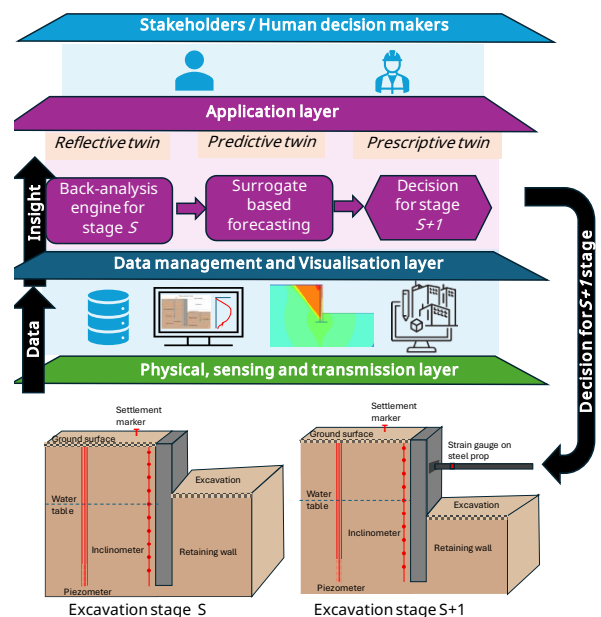


Figure 2: Layered architecture for digital twins of deep excavation to enable observational method.

As a first step in operationalising the framework, the back analysis or the inverse modelling engine serves as the critical component of the reflective twin, translating sparse monitoring data into a physically consistent representation of the wall-soil-support system. This structural state forms the foundation for both predictive forecasting and prescriptive decision-making within the

digital twin. To realise this capability efficiently and at scale, the framework employs PINNs, which provide a computationally effective and physically constrained approach for back-analysis in retaining-wall systems.

4 PINNs for back-analysis

The aim of the physics-informed neural networks is to perform inverse analysis i.e., to infer the lateral loads acting on retaining walls from sparse measurements of wall deflections. This provides the critical information required for reflective updating of the digital twin and supports stage-wise forecasting and adaptive decision-making in deep excavation projects.

4.1 Canonical cantilever beam

To develop and validate the core methodology, the PINNs-based inverse analysis is first applied to a canonical cantilever beam problem. This simplified model serves as a representative analogue of a retaining wall, capturing the essential mechanics of wall bending under lateral earth pressures. In this abstraction, the wall's flexural stiffness governs the deflection profile, allowing back-analysis to focus on inferring a small set of physically interpretable load parameters from sparse monitoring data. To simulate propped excavation conditions, temporary supports are modelled as discrete point loads applied at prop levels, representing measured prop reactions and further constraining the inverse problem. This approach maintains physical interpretability, computational efficiency, and practicality for stage-wise model updating in deep excavation projects.

In the canonical beam problem, the beam geometry, flexural stiffness EI , and boundary conditions were assumed to be known, while an unknown load distribution $q(x)$, is to be back-analysed from the sparse measured deflections, where x denoted the longitudinal coordinate along the beam axis. The transverse deflection $w(x)$ is governed by the Euler-Bernoulli beam equation which directly links the load distribution to the deflection profile and provides the core physical constraint in the PINN:

$$EI \cdot \frac{d^4 w(x)}{dx^4} - q(x) = 0 \quad (1)$$

A synthetic benchmark was constructed by solving the beam analytically under a combination of a uniform load q and a mid-span point load P , and then sampling sparse deflection values corrupted with Gaussian noise to emulate inclinometer data (Figure 3). The inverse task is to recover both the continuous deflection field $w(x)$ and the underlying load distribution $q(x)$ from these noisy

observations. Direct recovery of $q(x)$ by differentiating sparse, noisy deflections is ill-posed due to numerical instability of high-order derivatives, motivating a joint learning formulation where deflection and load are inferred simultaneously within a shared physics-informed network.

To regularise the problem and embed prior engineering knowledge, the distributed load is parameterised as a superposition of a uniform component and a localised (concentrated) component, representing lateral earth pressure and surcharge effects. In case of retaining walls, classical earth pressure formulations such as limit equilibrium method can be adopted as prior engineer knowledge.

The deflection $w(x)$ is represented by a feedforward neural network with two hidden layers of 30 neurons and Swish activation. Automatic differentiation is employed to compute the high-order derivatives required by Equation (1), enabling stable enforcement of the governing equation during training.

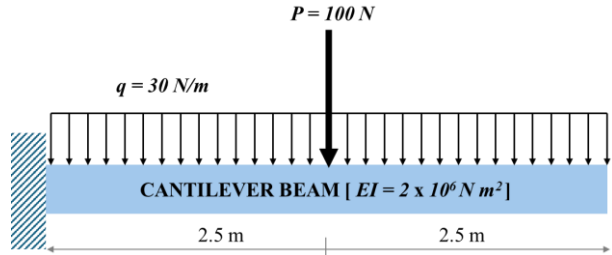


Figure 3: Canonical cantilever beam setup used for demonstrating PINNs framework.

The total training loss ($\mathcal{L}_{total}$) is defined as

$$\mathcal{L}_{total} = \lambda_d \cdot \mathcal{L}_d + \lambda_{phy} \cdot \mathcal{L}_{phy} + \lambda_{bc} \cdot (\mathcal{L}_{bc}) \quad (2)$$

$$\mathcal{L}_{bc} = w(0)^2 + w'(0)^2 + w''(L)^2 + w'''(L)^2 \quad (3)$$

Where $\mathcal{L}_d$ is the data loss enforcing agreement between predicted and observed deflections, $\mathcal{L}_{phy}$ is the physics loss penalising residuals of from Equation (1) at collocation points within the domain, $\mathcal{L}_{bc}$ is the boundary condition loss enforcing Dirichlet (deflection and slope at fixed end) and Neumann boundary conditions (Bending moment and Shear force at free end) of the beam (Equation (3)). The weights λ_d , λ_{phy} , and λ_{bc} control the relative influence of data fidelity, physics consistency, and boundary enforcement. Two Adam optimisers are used: one updates the network weights, and the other updates the trainable load parameters (including the point load), constrained to be non-negative via a soft plus transformation.

The magnitude of the weights assigned to the data, physics, and boundary condition loss terms directly influences the accuracy of the inverse PINN solution. To identify the most suitable weighting scheme, a sensitivity analysis was performed in which 30 different weight

combinations were tested, and their results were compared. Solution accuracy was evaluated using two indicators: (i) the coefficient of determination R^2 (deflection) between the predicted deflection profile and the analytical reference, and (ii) the accuracy of the load distribution predictions. To enable balanced comparison, both deflection metric (R^2) and the load-accuracy metric were min-max normalised across all runs to place them on a common 0-1 scale. A single performance score was then defined as the arithmetic mean of the normalised values. Using only 16 sparse deflection measurements with 10% Gaussian noise, each model was trained for 8000 epochs. Figure 4 presents the sensitivity analysis results, from which the optimal weight combination $(\lambda_d, \lambda_{phy}, \lambda_{bc}) = (0.4, 0.4, 0.2)$ were selected based on the highest overall score, indicating a balanced trade-off between data fit, physics enforcement, and boundary satisfaction. All computations were performed on a laptop with an Intel Core Ultra 7 165H CPU and 32 GB RAM; for the canonical inverse-analysis setup trained for 8000 epochs, the runtime was approximately 63 s per case

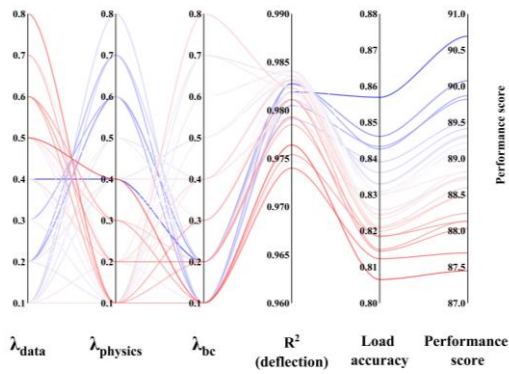


Figure 4: Sensitivity of PINN loss weights on predictive performance, evaluated by R^2 for deflections, load prediction error, and overall performance score.

Figures 5a and 5b present the predicted deflection profile and the inferred load distribution obtained using the best-performing weight combination identified in the sensitivity study. The deflection predicted by the PINN closely matches the analytical solution, achieving a coefficient of determination of $R^2 = 0.982$. Despite the sparse and noisy measurement inputs, the incorporation of governing beam mechanics through the physics loss constrains the solution space, preventing overfitting and ensuring mechanically consistent behaviour along the beam. The recovered load distribution $q(x)$ captures both the uniform and localised components of the applied loading with good agreement. The estimated uniform

load exhibits an error of approximately 14.9%, while the recovered point load shows an error of 14.6%.

For comparison, Figure 6 shows the results obtained using a purely data-driven neural network of identical architecture and trained for the same number of epochs, using deflection data only. In this case, the predicted deflections exhibit a lower agreement with the analytical solution, with $R^2 = 0.913$. This comparison highlights the benefit of embedding physical constraints within the learning process, particularly when measurements are sparse and noisy, as is typical in field monitoring of retaining structures.

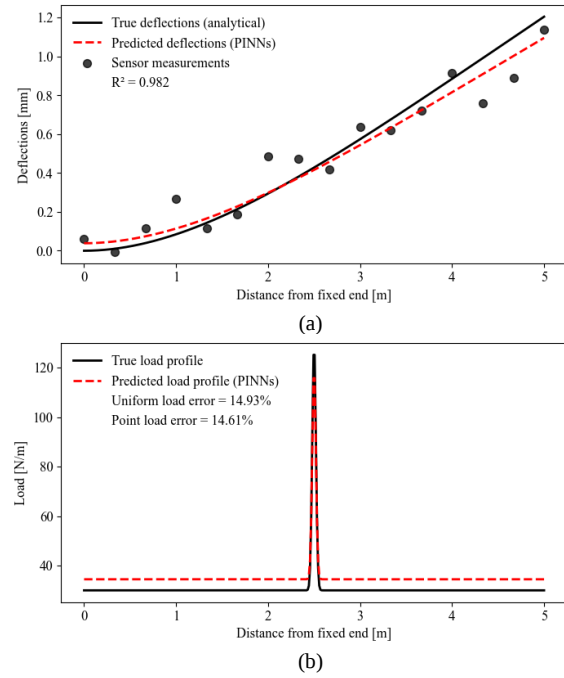


Figure 5: a) (a) Deflection and (b) Load profile - recovered from PINN framework compared with true values for cantilever beam canonical problem.

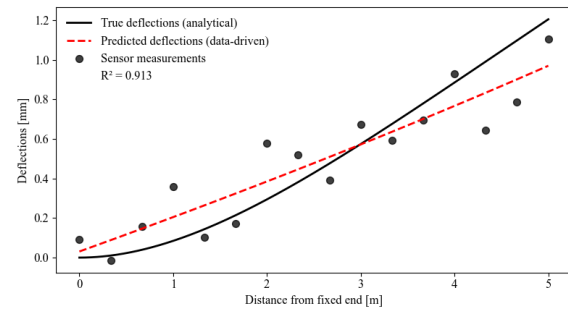


Figure 6: Deflections recovered from purely data-driven method compared with true deflections for cantilever beam canonical problem.

The extrapolation capability of the PINN was further assessed in an experiment where only eight sensors, each with 10% Gaussian noise, were assumed to be placed on the beam up to its mid-span. As shown in Figure 7a, the PINN still predicts the deflection profile with good accuracy ($R^2 = 0.939$), while Figure 7b shows that both the uniform load (13.1% error) and point load (22.8% error) are reasonably recovered. In contrast, the purely data-driven network (Figure 8) performs significantly worse, with a reduced accuracy ($R^2 = 0.850$) and clear signs of overfitting. This demonstrates that the PINN maintains stable and physically consistent inference even with sparse and spatially limited measurement.

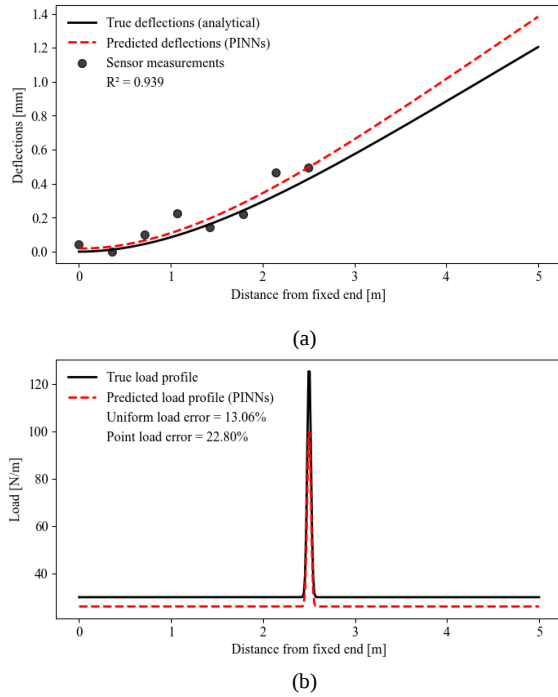


Figure 7: a) Deflection and (b) Load profile - recovered from PINN framework for extrapolation task on cantilever beam canonical problem.

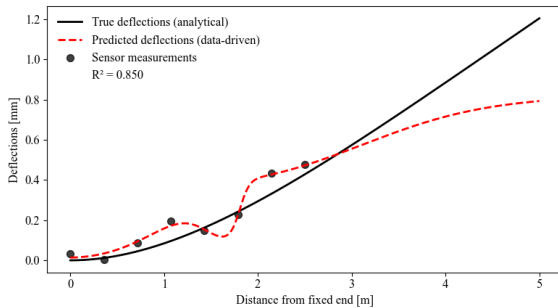


Figure 8: Deflections recovered from purely data-driven method for extrapolation task on cantilever beam canonical problem.

4.2 Back-analysis on a retaining wall

Building on the cantilever beam example, the inverse PINN framework is extended to a canonical propped retaining wall representative of deep-excavation behaviour. To keep the inverse problem tractable while preserving physical interpretability, the lateral earth pressures are represented by classical linear depth-dependent active and passive pressure distributions scaled by mobilisation coefficients m_a and m_p . These coefficients, bounded as $m_a, m_p \in [0,1]$, act as lumped parameters capturing partial mobilisation and provide an inductive bias that stabilises inversion from sparse measurements.

Figure 9 illustrates the wall configuration, with total height H , retained height h , and a prop force P applied below ground level; wall deflections are observed at eight sensor locations along the depth. The inverse task is to recover m_a and m_p from these sparse deflection measurements, optionally incorporating measured prop force where available.

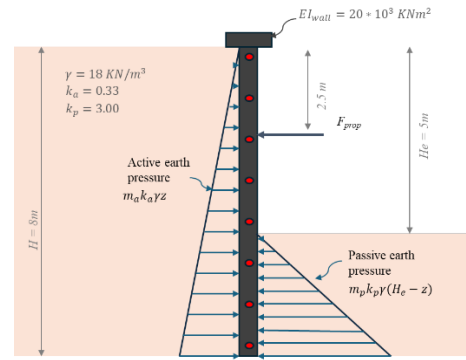


Figure 9: Illustration of the propped retaining wall canonical example.

Synthetic training data are generated by prescribing m_a , m_p , and P , solving the corresponding boundary-value problem, and perturbing the sampled deflections with 10% Gaussian noise. The network architecture and training strategy follow the cantilever-beam case, with depth as input, deflection as output, and automatic differentiation used to enforce the Euler-Bernoulli residual. Using the best-performing loss weights identified previously, the results in Figure 10 show that the PINN can recover excavation-relevant reduced-order state variables from sparse and noisy data. This retaining-wall example therefore provides a more application-relevant intermediate step between the beam analogue and future field deployment.

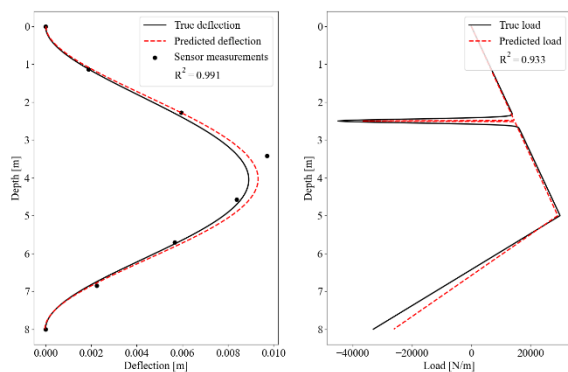


Figure 10: (a) Deflection and (b) load profiles recovered by PINN inverse analysis for the retaining wall.

5 Conclusions

This paper has presented a digital twin framework that embeds the observational method within a layered architecture for deep excavations, clarifying the roles of monitoring, data management, modelling, and application layers, and identifying key requirements including data efficiency, computational speed, physical consistency, scalability, and alignment with trigger-based decision-making.

Within this framework, a parsimonious inverse PINN was developed as the reflective engine of the twin. Validation on a canonical cantilever-beam analogue of a propped retaining wall showed that the method can recover deflection profiles and mobilised load distributions from sparse and noisy measurements, while outperforming a purely data-driven neural network and retaining mechanical interpretability and numerical stability.

Together, the framework and inverse PINN provide a basis for reflective digital twins that support stage-wise model updating and integration with construction monitoring and decision workflows. By enabling physically consistent interpretation of sparse monitoring data, the approach offers a pathway toward predictive and prescriptive digital-twin capabilities for deep excavation management.

Several important areas for future work include, validation using real excavation monitoring data, broader benchmarking against optimisation-based and probabilistic back-analysis approaches, and extension to more realistic retaining-wall idealisations and higher-fidelity 2D/3D soil-structure interaction models. Further work is also needed to assess robustness and computational performance for routine project deployment

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