On the Estimation of Resonance Frequencies of Hydraulically Actuated Systems

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Abstract –

A common problem when operating heavy hydraulic machines consists in low-frequency resonance phenomena that significantly limit the bandwidth of the closed loop position control. The interest in this topic is further motivated by the fact that the usage of traditional dynamic models of hydraulic actuators usually leads to the identification of very high-frequency resonances. This paper tries to explain the origin of low-frequency resonances by analysing the kinematic coupling between hydraulic actuators and structural links that can be found in typical hydraulically actuated machines. A novel formula for the identification of such resonance frequencies is derived. A realistic simulation environment is used to identify the resonance frequencies corresponding to different load masses and link lengths. Finally, the identified frequencies are compared to the results obtained using the two formulas, showing the superior accuracy of the newly proposed approach with respect to the traditional one.

Keywords –

Hydraulic systems, Hydraulic actuators, Motion Control

1 Introduction

Hydraulic actuation systems are characterized by multiple convenient features (namely high power density, reliability, and relatively low maintenance cost) that make them particularly fit to several application domains. Not by chance these actuators are the most widespread in the construction and earthworks machinery sector [1,2], in the mining industry [3], and also in the agriculture and forestry field [4]. In spite of the aforementioned advantages, hydraulic actuators are typically affected by low-frequency resonance phenomena. In addition, these resonances are further exacerbated by the large masses and inertias typically characterizing the components of a hydraulically actuated machine. Unfortunately, these resonance phenomena significantly limit the performance and the overall bandwidth of any possible closed loop control systems [5]. In order to address this problem, several control strategies have been proposed, that aim at suppressing the oscillations caused by these resonances [6,7,8]. Machine design solutions have also been proposed. For instance, in [9] the authors increase the resonance frequency (thus minimizing the resulting oscillations) by properly sizing the hydraulic cylinders and by computing the optimal distribution of the inertias along the kinematic chain of the machine.

Even though a significant effort has been spent trying to limit the effects of these low-frequency resonances, very few research contributions focusing on the explanation of these phenomena can be found. Some contributions suggest that the origin of these low-frequency resonances may lie in the variation of the oil compressibility [10], that in turn can be triggered by the presence of air contaminating the fluid [11].

Nonetheless, one of the most interesting aspects of these resonance phenomena consists in their incoherence with respect to the classical dynamic equations of the generic hydraulic servomechanism, whose manipulation usually results in the identification of very high resonance frequencies. Starting from this observation, our work tries to explain the low-frequency resonance phenomena from an alternative point of view. As a matter of fact, traditional formulations of the dynamic model of the hydraulic actuator represent the load as a point mass simply acting in either compression or traction on the actuator itself, while in a generic hydraulically powered machine (e.g. an excavator) hydraulic actuators are connected to the links they move

through rotational joints. By explicitly taking into account the geometry of the connection (and consequently, the configuration-dependent nature of the mechanical load), it is possible to refine the dynamic model of the hydraulic actuator and to develop a suitable formula for the estimation of the smallest resonance frequency. The proposed approach was tested in simulation on a 1-DoF system modelled using MATLAB's SimHydraulics Toolbox. Results confirm that the proposed formulation of the resonance frequency is coherent with respect to the behavior of the simulated system.

This paper is organized as follows. Section 2 describes the classical model of a hydraulic servo-mechanism, while Section 3 introduces a more realistic model, characterized by a non-trivial coupling between the actuator and the load. For both systems the transfer function between the valve control signal and a mechanical quantity (either the cylinder's stroke or its linear velocity) is provided, together with a formula for the computation of the smallest resonance frequency. Then, Section 4, shows a comparative analysis between the frequency values obtained with a realistic simulator and the ones computed using the aforementioned formulas. Finally, conclusions and future development are discussed in Section 5.

2 Hydraulic cylinder with a rigidly connected load

In this section the standard model of a symmetric hydraulic actuator connected to a load mass is briefly recalled (see Figure 1).
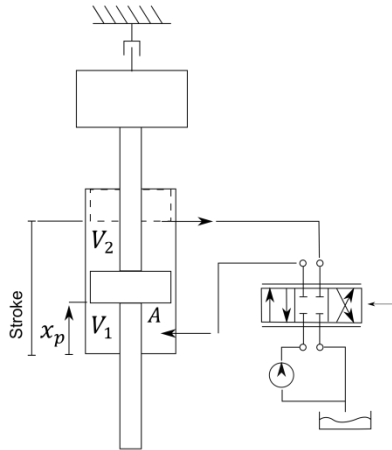

Figure 1. Hydraulic system with compressive load.

More specifically, after the derivation of the dynamical equations, the transfer function between the piston velocity and the valve control signal is presented, together with a rough estimation of the first resonance frequency of the system.

2.1 Hydraulic model

In order to derive the piston equation, we assume that:

- the oil is characterized by a finite compressibility defined by the nominal Bulk Modulus β;
- the thermal expansion of the oil is considered negligible;
- the oil density ρ is the same in the two chambers;
- the fluid losses to the external environment are negligible;
- the internal leakage q_L is proportional to the chambers pressure drop: $q_L = C_L(p_1 - p_2)$;
- the valve electro-mechanical dynamic is considered negligible.

Under these assumptions and according to [12], ρ can be considered equal to its nominal value ρ_0:

$$\rho = \rho_0\left[1 + \frac{1}{\beta}(p_i - p_r)\right] \cong \rho_0 \tag{1}$$

(where p_i is the i-th chamber pressure and p_r is the initial pressure of the fluid). Therefore, the mass conservation equations for the cylinder chambers can be written as follows:

$$q_1 - q_L = \dot{V}_1 + \frac{V_1}{\beta}\dot{p}_1 \tag{2}$$

$$q_L - q_2 = \dot{V}_2 + \frac{V_2}{\beta}\dot{p}_2 \tag{3}$$

where p_i is the i-th chamber pressure and q_i is the i-th in/out volume flow rate. The two chamber volumes are given by:

$$V_1 = V_0 + Ax_p \tag{4}$$

$$V_2 = V_0 - Ax_p \tag{5}$$

where A is the piston area, x_p is the piston displacement and V_0 is the nominal chamber volume, computed when x_p corresponds to half of the cylinder's stroke.

Since the piston is symmetric the pressure drop

$\Delta p_m = p_1 - p_2$ can be considered as a state variable. Furthermore, by defining $q_m = q_1 = q_2$ and given the supply pressure p_p, the valve orifice equation can be written as follows:

$$q_m = \frac{K_v}{2}u\sqrt{\frac{p_p - p_m}{\rho_0}} \cong \frac{K_v}{2}u\sqrt{\frac{p_s}{\rho_0}} \tag{6}$$

where K_v is the discharge coefficient and u is the valve control signal. The overall hydraulic equation is then obtained as:

$$\frac{V_0}{\beta}\Delta \dot{p}_m = K_v u\sqrt{\frac{p_p}{\rho_0}} - 2A\dot{x}_p - 2C_L\Delta p_m \tag{7}$$

where C_L is the internal leakage coefficient. Finally, the resulting hydraulic force is given by:

$$F_p = A\Delta p_m \tag{8}$$

2.2 Mechanical equation

The mechanical equation is derived by considering a load mass rigidly connected to the piston rod:

$$(M_p + M_l)\ddot{x}_p + (D_p + D_l)\dot{x}_p + F_d = A\Delta p_m \tag{9}$$

where M_p (M_l) is the mass of the piston (load), D_p (D_l) is the damping coefficient of the piston (load), and F_d is a disturbance force (e.g. gravitational force).

2.3 Transfer function

By gathering the masses and the damping coefficients together in the terms M and D, respectively, it is possible to linearize equation (9) in a small neighborhood of the following equilibrium point:

$$x_p = 0 \quad \dot{x}_p = 0 \quad u = 0$$

This way, we obtain the following transfer function between the piston velocity and the control signal:

$$\frac{V_p(s)}{U(s)} =$$

$$= \frac{K_v\frac{\beta A}{V_0 M}\sqrt{\frac{p_p}{\rho_0}}}{s^2 + \left(2\frac{\beta}{V_0}C_L + \frac{D}{M}\right)s + 2\frac{\beta}{V_0 M}(A^2 + C_L D)} \tag{10}$$

Equation (10) represent a second order system whose resonance frequency can be roughly computed by simply neglecting the internal leakages ($C_L = 0$):

$$f_n = \frac{1}{2\pi}\sqrt{\frac{2\beta A^2}{MV_0}} \tag{11}$$

Notice that in case of a standard asymmetric piston the above relation slightly changes in:

$$f_n = \frac{1}{2\pi}\sqrt{\frac{2\beta}{M}\left(\frac{A_1^2}{V_{01}} + \frac{A_2^2}{V_{02}}\right)} \tag{12}$$

where A_1 and A_2 are the bottom-side and rod-side piston areas, respectively.

3 Hydraulically actuated mechanism

The above formulation clearly does not represent a system characterized by a non-trivial coupling between the actuator and the load. The case-study of this paper is indeed the 1-D.o.F. system shown in Figure 2, which is more similar to a real-world scenario (e.g. an excavator link).
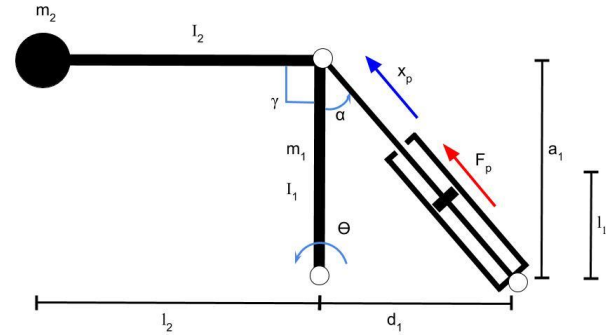

Figure 2. 1-D.o.F. hydraulically actuated mechanism

In this section we start by presenting the non-linear dynamical equations of the system, then we select a proper working point and, finally, we obtain the linearized model.

3.1 Kinematics

In order to retrieve the model as a function of the cylinder displacement, we can apply the cosine formula to the a_1-d_1-x_p triangle (see Figure 2):

$$\theta = \text{acos}\left(\frac{a_1^2 + d_1^2 - x_p^2}{2a_1 d_1}\right) = f_\theta(x_p) \qquad (14)$$

3.2 Mechanical dynamics

The mechanical balance of the system can be obtained by assuming the piston mass and inertia to be negligible. Given these assumptions, the mechanism is equivalent to a 2-Link planar robot, where angle γ corresponds to the second DoF. Consequently, the dynamic of the mechanism can be computed according to [13]. In particular, given all the geometrical parameters shown in Figure 2 and defining m_i and I_i as the mass and the inertia momentum of the i-th link, the following equation is obtained:

$$B_1 \ddot{\theta} + B_2 \ddot{\gamma} + C_1 \dot{\theta}\dot{\gamma} + C_2 \dot{\gamma}^2 + G = \tau_1 \qquad (15)$$

where:

$$B_1 = m_1 l_1^2 + I_1 + m_2 a_1^2 + m_2 l_2^2 \qquad (16)$$
$$+ 2m_2 a_1 l_2 \cos(\gamma) + I_2$$

$$B_2 = m_2 l_2^2 + m_2 a_1 l_2 \cos(\gamma) + I_2 \qquad (17)$$

$$C_1 = -2m_2 a_1 l_2 \sin(\gamma) \qquad (18)$$

$$C_2 = -m_2 a_1 l_2 \sin(\gamma) \qquad (19)$$

$$G = m_1 g l_1 \cos(\theta) + m_2 g\, a_1 \cos(\theta) \qquad (20)$$
$$+ m_2 g l_2 \cos(\gamma + \theta)$$

and τ_1 is the equivalent torque provided by the hydraulic actuator:

$$\tau_1 = d_1 \cos(\alpha)\, F_p = d_1 \cos(\alpha)\, A\Delta p_m \qquad (21)$$

A simpler formulation of the previous equations can be obtained by assuming:

- $I_1 = I_2 = 0$
- $m_1 = 0$
- $\gamma = {}^\pi/_2$ (fixed)

As a result, the dynamic equation reduces to:

$$m_2(a_1^2 + l_2^2)\ddot{\theta} + m_2 g a_1 \cos(\theta) - m_2 g l_2 \sin(\theta) \qquad (22)$$
$$= \tau_1$$

Finally, the hydraulic dynamics is obtained according to Equation (7).

3.3 Linearized model

To compute the linearized model, the working point defined by $\theta_0 = \pi/2$ is considered. It is also useful to define the following quantities:

$$\delta x_p = x_p - x_{p_0} \qquad (23)$$

$$\delta p_m = \Delta p_m - \Delta p_{m_0} \qquad (24)$$

$$\delta u = u - u_0 \qquad (25)$$

where:

$$x_{p_0} = \sqrt{a_1^2 + d_1^2} \qquad (26)$$

$$\Delta p_{m_0} = \frac{m_2 g l_2}{d_1 \cos(\alpha_0)\, A} \qquad (27)$$

$$u_0 = \frac{2C_L}{K_v}\sqrt{\frac{\rho_0}{p_p}}\,\Delta p_{m_0} \qquad (28)$$

$$\alpha_0 = \text{atan}\left(\frac{d_1}{a_1}\right) \qquad (29)$$

Moreover, the linear dependence of joint angle θ on piston displacement x_p can be defined as follows:

$$\theta = f_\theta\left(x_{p_0}\right) + \frac{df_\theta}{dx_p}\bigg|_{x_{p_0}} \delta x_p = \qquad (30)$$
$$= f_\theta\left(x_{p_0}\right) + n_\theta \delta x_p$$

$$\dot{\theta} = n_\theta \delta \dot{x}_p \qquad (31)$$

$$\ddot{\theta} = n_\theta \delta \ddot{x}_p \qquad (32)$$

Therefore, the linearized mechanical equation is given by:

$$m_2(a_1^2 + l_2^2)n_\theta \delta \ddot{x}_p - m_2 g l_2 n_\theta \delta x_p \qquad (33)$$
$$- m_2 g l_2 \text{f}\left(x_{p_0}\right) = A d_1 \cos(\alpha_0)\, \delta p_m$$

and the linear hydraulic equation is:

$$\frac{V_0}{\beta}\delta \dot{p}_m = K_v \sqrt{\frac{p_p}{\rho_0}}\, \delta u - 2A\delta \dot{x}_p - 2C_L \delta p_m \qquad (34)$$

3.4 Transfer function

Similarly to the previous case, it is possible to retrieve the transfer function of the linearized dynamics between the piston displacement and the valve control signal:

$$\frac{X_p(s)}{U(s)} = \frac{\mu}{Ks^3 + Fs^2 + Qs - H} \qquad (35)$$

where:

$$K = \frac{V_0}{\beta} m_2(a_1^2 + l_2^2)n_\vartheta \qquad (36)$$

$$F = 2C_L m_2(a_1^2 + l_2^2)n_\vartheta \qquad (37)$$

$$Q = 2d_1 \cos(\alpha_0) A^2 - m_2 g l_2 n_\vartheta \frac{V_0}{\beta} \qquad (38)$$

$$H = 2C_L m_2 g l_2 n_\vartheta \qquad (39)$$

$$\mu = d_1 \cos(\alpha) 2AK_v \sqrt{\frac{p_p}{\rho_0}} \qquad (40)$$

In addition, by neglecting the internal leakage ($C_L = 0$), it is possible to simplify the denominator, obtaining the following transfer function between the piston speed and the valve control signal:

$$\frac{V_p(s)}{U(s)} = \frac{\mu}{Ks^2 + Q} \qquad (41)$$

Therefore, the resonance frequency can be estimated by:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K}{Q}} =$$

$$= \frac{1}{2\pi} \sqrt{\frac{2d_1 \cos(\alpha) A^2 - m_2 g l_2 n_\vartheta \frac{V_0}{\beta}}{\frac{V_0}{\beta} m_2(a_1^2 + l_2^2)n_\vartheta}} \qquad (42)$$

4   Numerical Simulations

In order to better understand the effect of the joint connected load, several simulations have been performed. In this section, the results of these simulations are presented and compared to the results of the previously explained formulas.

For this purpose, a realistic simulator has been developed in MATLAB/Simulink environment, using the SimHydraulics toolbox [14]. More specifically, the parameters of an off-the-shelf double-acting asymmetric cylinder and the ones of a commercial 4/3 valve have been considered. In addition, the mechanical system shown in Figure 2 has been reproduced. Finally, the pump behavior was assumed to be ideal, meaning that the supply pressure p_p was always constant. The simulations have been performed by feeding the valve with a sine sweep-shaped control signal and varying the load mass m_2 and the link length l_2 for a total of 40

different configurations. Then, resonance frequencies have been identified by analyzing the spectrum of the resulting cylinder's speed signal. As far as the "rigidly connected load" case is concerned, f_n is computed according to equation (11). For the sake of clarity, we replaced mass m_2 with an equivalent mass, computed on the basis of the average static load "sensed" by the hydraulic cylinder. In this way, we were able to incorporate in a single inertial parameter the effects of both m_2 and l_2. Figure 3 shows how the resonance frequency changes with respect to different load masses. Moreover, for each load mass value, five different link lengths have been considered. On the other hand, Figure 4 shows the dependency of the resonance frequency with respect to different link lengths. In this case, eight different values of load mass have been considered for each link length value. In both cases the identified frequencies are displayed in red, while the computed ones are pictured in blue. As expected, given the same load mass (link length), the resonance frequency tends to decrease as the link length (load mass) increases.
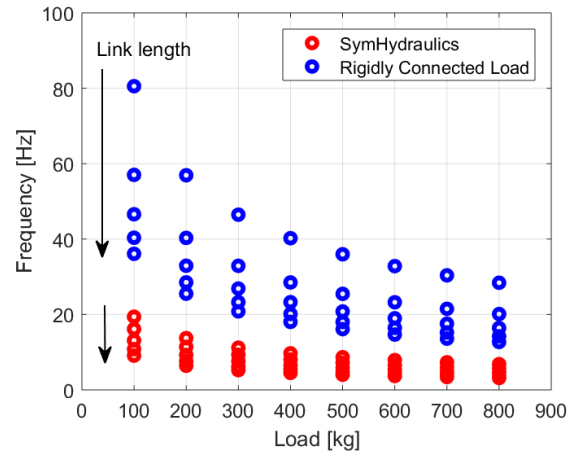

Figure 3. Resonance with varying load mass and rigidly connected load

Nevertheless, it is clear that the estimated resonance frequencies f_n computed according to equation (11) are significantly higher with respect to the corresponding identified frequencies. To this regard, Figure 5 and Figure 6 show the comparison among the identified frequencies and the ones computed according to equation (42). Clearly, the proposed formula is significantly more accurate than the traditional one. In addition, these results demonstrate how the kinematic coupling between the load and the actuator affects the natural frequencies of this system. Indeed, by means of Root Mean Square Error the following results are obtained:

$$RMSE_{rigid} = 23.4900 \, rad/s$$
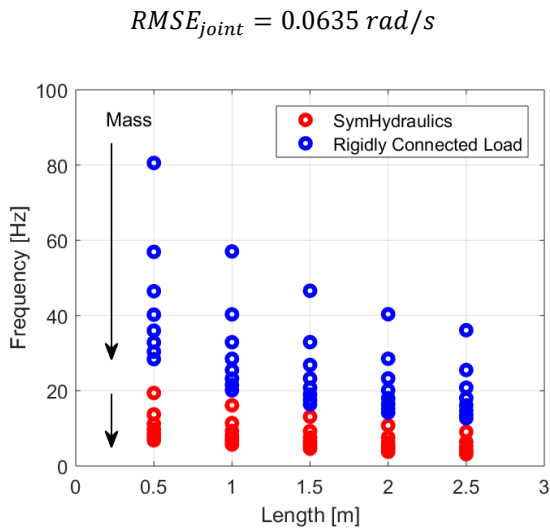
$$RMSE_{joint} = 0.0635 \; rad/s$$

Figure 4. Resonance with varying length and rigidly connected load
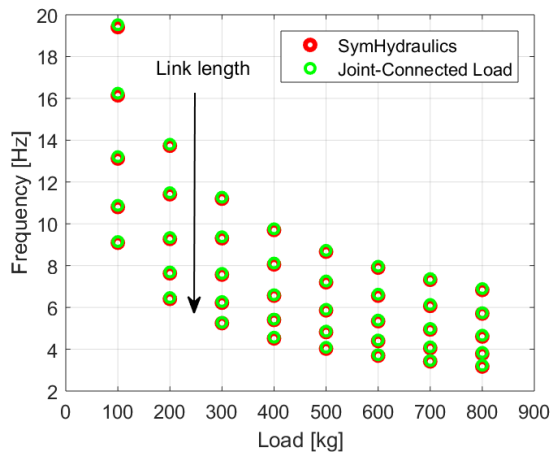

Figure 5. Resonance with varying mass and joint-connected load
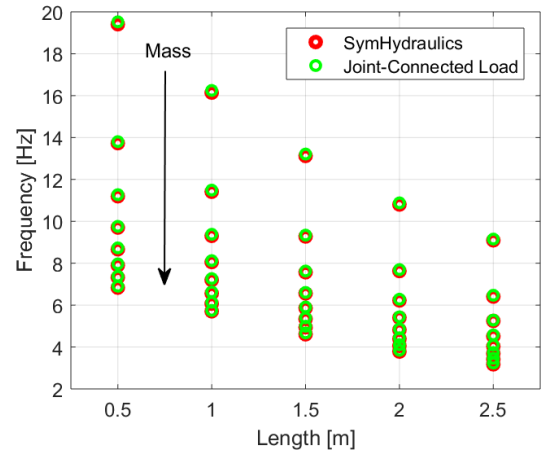

Figure 6. Resonance with varying link length and joint-connected load

5   Conclusions

This paper presents a study carried out on a hydraulic actuation system with the aim to achieve a better understanding of the causes of the low frequency resonances affecting the system. Starting from very simple models of the actuator and the load which it is connected to, the transfer function between the valve control signal and a mechanical quantity was computed and a formula to retrieve the resonance frequency was derived. Then, a more complex system characterized by a non-trivial coupling between the load and the hydraulic cylinder was considered. Once again, the model of the system was retrieved and, after linearization, a more complex formula for the estimation of the resonance frequency was computed.

Finally, a simulation environment of the joint-connected load was developed and several simulations were performed in order to obtain, via spectral analysis, an accurate estimation of the resonance frequencies corresponding to different load masses and link lengths. These estimated frequencies were compared to the results obtained using the two formulas, showing how critical is the impact of the kinematic coupling on the natural frequency of this kind of systems.

As far as future developments are concerned, the formula for the resonance estimation can be furtherly improved by taking into account the asymmetry of the hydraulic actuator and a more detailed description of the valve dynamics. To this regard, a promising solution could be represented by the adoption of object-oriented modelling strategies and tools [15], which allows to easily implement custom models such as the ones showed in [16].

References

[1] Hutter M., Leemann P., Stevsic S., Michel A., Jud D., Hoepflinger M., Siegwart R., Figi R., Caduff C., Loher M. and Tagmann S. "Towards optimal force distribution for walking excavators". In *International Conference on Advanced Robotics (ICAR)*, pages 295-301, Istanbul, Turkey, 2015.

[2] Tanzini M., Jacinto-Villegas J. M., Filippeschi A., Niccolini M., and Ragaglia M. "New interaction metaphors to control a hydraulic working machine's arm". In *IEEE Symposium of Safety and rescue Robotics (SSRR)*, pages 297-303, Lausanne, Switzerland, 2016.

[3] Corke P., Roberts J., Cunningham J. and Hainsworth D. "Mining Robotics". *Springer Handbook of Robotics*, Springer, Berlin, Heidelberg, 2008.

[4] Billingsley J., Visala A. and M. Dunn, "Robotics in Agriculture and Forestry" *Springer Handbook of Robotics*, Springer, Berlin, Heidelberg, 2008.

[5] Merritt H. E., *Hydraulic Control Systems*, John Wiley & Sons Inc, New York, 1967.

[6] Park J. Y. and Chang P. H. "Vibration control of a telescopic handler using time delay control and commandless input shaping technique" *Control Engineering Practice*, 12(6): 769-780, 2003.

[7] Singh S. "The state of the art in automation of earthmoving" *ASCE Journal of Aerospace Engineering*, 10, 1997.

[8] Singhose W. E., Porter L. J. and Seering W. P. "Input shaped control of a planar gantry crane with hoisting" In *Proceedings of American Control Conference*, pages 97-100, Albuquerque, NM, USA, 1997.

[9] Yoo S., Park C. G., Lim B., Lee K. I. and Park F.C. "Bandwidth Maximizing Design for Hydraulically Actuated Excavators," *Journal of Vibration and Control*, 16(14): 2109–2130, 2010.

[10] Ding G., Qian Z. and Pan S. "Active vibration control of excavator working equipment with ADAMS" In *International ADAMS User Conference*, 2000

[11] Sakama S., Tanaka Y. and H. Goto "Mathematical model for bulk modulus of hydraulic oil containing air bubbles" *Mechanical Engineering Journal*, 2(6), 2015.

[12] Manring N. D. *Hydraulic Control Systems*, John Wiley & Sons Inc, New York, 2005.

[13] Siciliano B., Sciavicco L., Villani L. and G. Oriolo, *Robotics: Modelling, Planning and Control*, Springer, London, 2009.

[14] Simscape Fluids™ (formerly SimHydraulics®), 2019, Online: https://www.mathworks.com/products/simhydraulics.html.

[15] Fritzson P., *Principles of Object-Oriented Modeling and Simulation with Modelica 2.1*, Wiley Press, 2004.

[16] Pencelli M., Villa R., Argiolas A., Ferretti G., Niccolini M., Ragaglia M., Rocco P. and A.M. Zanchettin, "Accurate dynamic modelling of hydraulic servomechanism," In *Design Automation and Test in Europe Conference*, 2019.