Optimum Control of Vibratory Piling Process

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Abstract

A difficult task in densely constructed urban areas is to proceed to new extensions or to unavoidable repairs without creating damages to existing buildings. The method so far is to isolate the worked zone by sheet piles to maintain constant earth pressure conditions against existing foundations. But even if they are much less harmful than classical hammering methods, the vibratory methods in use to drive the sheet piles into the ground are still producing oscillations which propagate into the ground and may endanger the neighbouring buildings especially if a resonance phenomenon do occur. A control method is analysed in the following to keep the vibration amplitude below a fixed safety level while the sheet itself is piled in minimum time into its pre-assigned position.

Keywords: Vibratory Piling, Sheet Piling, Ground Vibrations

1. Introduction

Urban constructions are typically densely packed to accommodate high population density, and are very often located over relatively soft soils such alluvial ones, reminiscent of the early town origin close to water source. A consequence is that building renewing and/or repairing now requires to lay down very deep foundations to cast pillars in or to hit directly piles into the ground. This operation in turn creates serious difficulties as, even with vibratory units which are much less harmful for the environment than classical hammering methods, there is still generation of ground vibrations which propagates to neighbouring buildings and may seriously damage them especially if resonances are produced during the process, corresponding to critical speeds of the global pile-ground-building system [1]. Vibratory units in use are consisting of pair of shafts driven by electrical or fluid actuators to comply with the needed high power density, on which eccentric masses are mounted to excite a vertical harmonic force pushing the sheet or the pile into the ground. With a static driving force on top, the sheet penetrates with a speed depending on vibration amplitude and frequency, and the efficiency is increased by adding several shafts in parallel. With already two pairs, it is then possible to modulate the composition of individual shaft pair forces by adjusting both the phase angle between primary and secondary shafts and the rotation speed of the primary shaft. However, the interaction dynamics between the various sub-systems are becoming more complex, and in an industrial set-up, an automatic monitoring system has to be developed to assist the operator who cannot be left with full driving operation. Useful conditions are that the oscillation amplitudes produced by vibratory piling at fixed neighbouring locations are always below some pre-assigned threshold safety values, and also that the piling time is minimized for economic reasons. Because they are antagonistic, the previous conditions require first to set up the complete system dynamic equations. The corresponding optimum problem is analysed afterward, the properties of required controller are discussed and its analytic expression is given in specific case.

2. System Equations

For a two-shaft line system shown on fig.1, the degrees of freedom are the vertical position z of the vibratory unit of mass m, the position Z of the upper frame of mass M, and the angular positions φ_j of mass eccentricity m_j and inertia moments I_j ($j=1,2$). The dynamical equations then read in normalized form [2]

$$\ddot{Z} + \omega_M^2 (Z - z) = -\omega_g^2 - f_a$$
$$\ddot{z} + \omega_m^2 (z - Z) = -\omega_g^2 - f_s \,\mathrm{sgn}(\dot{z}) + f_p + \sum_{j=1,2} \mu_j [\dot{\varphi}_j^2 \sin(\varphi_j) - \ddot{\varphi}_j \cos(\varphi_j)]$$
$$\ddot{\varphi}_j \pm \omega_j^2 F(\varphi_1 - \varphi_2) = t_j - \mu_j (\omega_g^2 + \ddot{z}) \cos(\varphi_j) \qquad (1)$$

where here $z \to z/l$, $Z \to Z/l$, $\omega_M^2 = k/M$, $\omega_m^2 = k/m$, $\omega_j^2 = K/I_j$, $\omega_g^2 = g/l$, $\mu_j = (r/l).(m_j/m)$, $f_a = F_a/Ml$ $f_s = F_s/ml, f_p = F_p/ml$ are respectively the applied feeding force, the skin f riction and the point load of the piled object divided by the corresponding mass and the length l of piled object. $F(x)$ is the dead zone function

$$F(x) = x + \pi/2 \qquad \text{if} \qquad x < -\pi/2$$
$$F(x) = 0 \qquad \text{if} \qquad -\pi/2 \le x \le \pi/2 \qquad (2)$$
$$F(x) = x - \pi/2 \qquad \text{if} \qquad x > \pi/2$$

Furthermore the angular acceleration t_j produced by the hydraulic actuators are given by $t_j = \theta\,(p_j^+ - p_j^-)$, with

$$\dot{p}_j^+ = u_j \sqrt{p_s - p_j^+} - \lambda \dot{\varphi}_j$$
$$\dot{p}_j^- = u_j \sqrt{p_j^- - p_t} + \lambda \dot{\varphi}_j \qquad (3)$$

p_s , p_t the normalized source and tank pressures and θ, λ two normalizing parameters. Previous numerical analysis of eqns(1,2,3) for realistic industrial equipment [3] has shown the adequacy of both the model and the very fashionable used software to represent system dynamics. In a next step, the present complement paper is intended to study analytically global guideline properties for optimal control of the system in order to ease its numerical resolution. Along this view, it is first interesting to note that, though the physical control parameters are the u_j which are ultimately governing the inputs to the proportional control flow valves, the total system (1,2,3) could be split into the piled object dynamics from eqns(1,2) with inputs t_j , and the realization of the required inputs from fluid power eqns(3). The only difficulty is in the fact that the intermediate controls t_j satisfying the conditions may not be accessible from the set of possible physical controls u_j, in which case only an approximate optimum will be reached. Then the problem would amount to determine the inputs f_a and t_j so that the piled object reaches its final position inside the ground in the vibratory shortest time while the produced vibration amplitudes at assigned places are during this time always bounded above by a fixed threshold value.

3. Discussion

As just mentioned, the dynamic system (1,2,3) could naturally be split into the dynamic part of eqns(1,2) and the fluid power part of eqns(3). However another possible splitting is obtained by observing from the third equation in system (1) that the first two equations in system (1) are coupled to the set of the third of eqns(1) plus eqns(3) through the only term $\mu_j \ddot{z} \cos(\varphi_j)$, which can be a ''small'' one . So it is advisable to use this system property and to split instead the complete system in the following way

$$\ddot{Z} + \omega_M^2 (Z - z) = -\omega_g^2 - f_a$$
$$\ddot{z} + \omega_m^2 (z - Z) = -\omega_g^2 - f_s \,\mathrm{sgn}(\dot{z}) + f_p + U \qquad (5)$$

and

$$\ddot{\varphi}_j \pm \omega_j^2 F(\varphi_1 - \varphi_2) = \theta (p_j^+ - p_j^-) - \mu_j \omega_g^2 \cos(\varphi_j) + C_j \qquad (6)$$

$$\dot{p}_j^+ = u_j \sqrt{p_s - p_j^+} - \lambda \dot{\varphi}_j$$
$$\dot{p}_j^- = u_j \sqrt{p_j^- - p_t} + \lambda \dot{\varphi}_j \qquad (7)$$

where

$$C_j = \mu_j \ddot{z} \cos(\varphi_j) \qquad U = \sum_{j=1,2} \mu_j [\dot{\varphi}_j^2 \sin(\varphi_j) - \ddot{\varphi}_j \cos(\varphi_j)] \tag{8}$$

are respectively the coupling term between the two sets (5) and (6,7), and the input to the first set of eqns(1). This display clearly shows the important role of the rotation frequency of the masses in vibratory units, both with their actual values and with their variations. So in eqns(1) for a fixed applied force f_a the local input U plays the role of a control force which modulates the ground penetration of the piled object, especially when observing from experiments that the reaction force f_p from the ground is generally depending on vibratory unit rotation frequency and typically decays monotonically above some threshold value Ω_c .

With these elements, a very simple optimum bang-bang type running operation would consist in turning the vibratory unit to its maximum rotation speed Ω_{max} to reduce f_p while the frequency shift is driven to the largest vibratory amplitude by fixing the φ_j at the maximum of the source term U, then insuring the fastest penetration in the ground with smallest optimal vibration amplitude at stationary regime. From eqn(8), and writing $\varphi_1 = \varphi$, $\varphi_2 = \varphi + \Delta\varphi$ for simplicity the maximum of U is obtained for $\Delta\varphi = 0$. This procedure would work as long as, during the piling operation, the resulting ground oscillation amplitude at specific sensitive location near by the piling place and propagated from it, is below a pre-determined threshold fixed by a risk of damage. This is not always guaranteed because one should ride over all frequencies in the interval $[0, \Omega_{max}]$ when departing the piling work from rest and one may cross some resonance or simply some sensitive frequency for which the ground response amplitude overpasses the threshold value. The problem is then to downgrade the previous optimum when approaching these frequencies in order to account for the imposed threshold constraint. Noting that $U = 0$ for $\Delta\varphi = \pi$, and that U is monotonic in the interval $[0,\pi]$, this is always possible by manipulating the phase shift between vibratory units. So a natural control strategy is to drive the phase shift so that, in accordance with predetermined amplitude condition all along the frequencies in the operation interval $[0, \Omega_{max}]$

$$A_G(\omega,\Delta\varphi) \leq A_{max}(\omega) \quad \text{for} \quad \omega \in [0, \Omega_{max}] \tag{9}$$

where $A_G(.,.)$ is the resulting ground amplitude at prescribed locations and $A_{max}(\omega)$ the local expression of the constraint for each frequency.

4. Oscillatory Source Amplitude

A first possibility to apply the strategy is to define by empirical observation a relationship between phase shift values and ground oscillation amplitude in interval $[0, \Omega_{max}] \times [0,\pi]$, ie to evaluate the function $A_G(\omega,\Delta\varphi)$, and to construct a set of simple fuzzy rules of car-driving type guaranteeing the satisfaction of amplitude condition in eqn(9). The procedure may be tedious as it has to be set up each time, and it is more convenient to construct directly the function $A_G(\omega,\Delta\varphi)$. This in turn splits into two problems : to calculate the oscillatory source at the piled object from eqns(5), and to determine the transmitted amplitude at interesting location from resolution of oscillation ground propagation. This last problem has already been investigated elsewhere[4], and only the first one will be analysed in the sequel from eqns(5). As mentioned earlier, this implies solving these equations with U as a source term and to use eqns(6,7,8) to determine the real input control $u(t)$ producing the source U. With $\Delta = Z - z$ and letting $\Delta_s = Z_s - z_s$ given by $\omega_M^2 \Delta_s = -\omega_g^2 - f_a$, eqns(5) reduces to

$$\begin{aligned} &\ddot{\overline{\Delta}} + \overline{\omega}^2 \overline{\Delta} = H + f_s \,\mathrm{sgn}(\dot{Z} - \dot{\Delta}) + U(t) \\ &\ddot{Z} = -\omega_M^2 \overline{\Delta} \end{aligned} \tag{10}$$

with $\overline{\omega} = (\omega_M^2 + \omega_m^2)^{1/2}$, $\overline{\Delta} = \Delta - \Delta_s$,

$$H = [1 + (\omega_m / \omega_M)^2]\,\omega_g^2 + (\omega_m / \omega_M)^2 f_a - f_p \tag{11}$$

and (formal) solution

$$\Delta(t) = \Delta_s - \frac{H \mp f_s}{\overline{\omega}^2} + \overline{\Delta}_0 \Delta_1(t) + \dot{\overline{\Delta}}_0 \Delta_2(t) + \int_0^t \Delta_2(t-t')U(t')dt' \quad (12)$$

where $\overline{\Delta}_0 = \Delta_0 - \Delta_s$, $\dot{\overline{\Delta}}_0 = \dot{\Delta}_0$, $\Delta_1(t) = \cos \overline{\omega}\, t$, $\Delta_2(t) = (\sin \overline{\omega}\, t)/ \overline{\omega}$. One then gets

$$Z(t) = P^{\pm}(t) - \tilde{Z}(t) \equiv Z_0 + \dot{Z}_0 t - \frac{H \mp f_s}{2\overline{\omega}^2} t^2 - \tilde{Z}(t) \quad (13)$$

$$\tilde{Z}(t) = \overline{\Delta}_0 \Delta_1(t) + \dot{\overline{\Delta}}_0 \Delta_2(t) - \int_0^t dt' \int_0^{t'} U(t'')dt'' + \int_0^t \Delta_2(t-t')U(t')dt' \quad (14)$$

The expression of $Z(t)$ contains two different parts, the parabolic time varying one $P^{\pm}(t)$, depending on the sign, and the oscillatory one $\tilde{Z}(t)$. To improve the penetration into the ground, the sign of the coefficient of t^2 should be always negative and as large as possible in absolute value. This means from eqn(11) that the applied force f_a should be above a critical value

$$f^{+}_{a,crit} = \frac{\omega_M^2}{\omega_m^2} \left\{ f_p - \left[1 + \frac{\omega_m^2}{\omega_M^2} \right] \omega_g^2 + f_s \right\} \quad (15)$$

easier to reach as the ground reaction f_p is smaller, ie as vibratory unit frequency is larger. Then the trajectory goes monotonically downward along either of the two parabolas $P^{\pm}(t)$ defined in eqn(13), and goes up and down if $f^{-}_{a,crit} < f_a < f^{+}_{a,crit}$.

The other term $\tilde{Z}(t)$ fixes the amplitude and the frequencies of ground vibration oscillations, and depends itself on two terms, the mechanical mass oscillation with frequency $\overline{\omega}$ and the source term from input $U(t)$ which combines its own frequencies with $\overline{\omega}$. Its amplitude is typically given by

$$\left|\tilde{Z}(t)\right| = \left[\overline{\Delta}_0^2 + \dot{\overline{\Delta}}_0^2 + |U|^2 \right]^{1/2} \quad (16)$$

5. Determination of Source Term From Control Input

As indicated earlier, there remains to determine the apparent input to mechanical part $U(t)$ from its real input control source $u(t)$. This would rest upon solving the remaining system of eqns(6,7,8), but what is only needed is to reconstitute the input $u(t)$ which delivers the output $U(t)$ from eqns(6,7,8). A classical weaker form in control theory is to determine $u(t)$ in state space representation so that the output $U(t)$ asymptotically tracks a pre-assigned function $U_d(t)$ dictated by the research of smooth, monotonic and fast enough time response of the angular variables of the vibratory units[5]. Here elimination of the pressure variables p_j^+ and p_j^- from the subsystem with fixed j leads to the final general equation

$$\dot{Z}_j + 2A_j(\varphi_j, \varphi_1 - \varphi_2) - \frac{2\,\dot{\varphi}_j\, Z_j}{Z_j - B_j(\varphi_j)} = 0 \quad (17)$$

with $Z_j = \lambda A_j^2(\varphi_j)/u_j^2$, $\beta_j = \mu_j \omega_g^2/\lambda\theta$, $B_j = \overline{p}_j - \beta_j \cos\varphi_j - \ddot{\varphi}_j/\lambda\theta$, $\overline{p}_j = (p_{sj} - p_{tj})/\lambda$ and

$$A_j(\varphi_j, \varphi_1 - \varphi_2) = \ddot{\varphi}_j/\lambda\theta + +2\dot{\varphi}_j - \beta_j \sin\varphi_j.\dot{\varphi}_j + (-)^{j-1}\overline{\omega}_j^2[\Gamma(\varphi_1 - \varphi_2 - \pi/2) + \Gamma(-\pi/2 - \varphi_1 + \varphi_2)](\dot{\varphi}_1 - \dot{\varphi}_2) \quad (18)$$

where $\overline{\omega}_j^2 = \omega_j^2/\lambda\theta$, $j = 1,2$ and $\Gamma(x) = 1$ for $x>0$ and 0 for $x<0$. Eqn(17) determines directly the control input $u_j(t)$ which produces the output $\varphi_j(t)$. So choosing $\varphi_j(t)$ in previous class, it is possible to verify that the corresponding $u_j(t)$ is doable. As an example one could first consider stationary situation where both angular variables are rotating with same frequency Ω_0 and have a constant phase difference $\Delta\varphi_0$. If furthermore $\beta_j << \mathrm{Min}[2, \overline{p}_j]$, A_j and B_j simplify to $A_j = 2\Omega_0$ and $B_j = \overline{p}_{j\ t}$ so that eqns(17) decouple and now become

$$\frac{dY_j}{d\tau} + 2 - \frac{Y_j}{Y_j - \overline{p}_j} = 0 \qquad (19)$$

with $Y_j = 4\lambda\Omega_0^2 X_j$, $\tau = 2\Omega_0 t$. Its explicit solution is given by

$$\frac{\tau}{\overline{p}_j} = \Phi(\chi_j) \equiv \frac{1}{\chi_{j0}^2} - \frac{1}{\chi_j^{\ 2}} - \log\frac{1-2\chi_j^{\ 2}}{1-2\chi_{j0}^2} \qquad (20)$$

from which the input control is $u_{j,st} = 2\lambda\Omega_0 (\lambda/\pi)^{1/2}\chi_j(\tau)$ with $\chi_{0j} \le 1/2$ and $\chi_j(\tau) = \Phi^{-1}(\tau/\overline{p}_j)$ from eqn(20). It represents a monotonically growing curve from χ_{0j} to $1/2$ when the time runs from 0 to ∞. So it is possible to maintain single frequency oscillation Ω_0 of vibrating unit for long time with this smooth control input provided the final (highest) value is reachable. It should be noticed that exactly the same equation (17) is found with now $Y_j = \lambda\dot{\varphi}_j^2 X_j$ and $\tau = \varphi_j(t)$ in the more general case where $\varphi_j(t)$ are arbitrary but satisfy the inequality $\ddot{\varphi}_j / \dot{\varphi}_j << \lambda\theta$ and the condition $\varphi_1(t) - \varphi_2(t) \in [-\pi/2, \pi/2]$. To move the phase shift from initial value $\Delta\varphi_{0j}$ to final one $\Delta\varphi_{fj}$ at time T in this case, one should just program the control inputs $u_j(t)$ so that they follow the same curve $\Phi^{-1}(\varphi_j(t))$ from eqn(20) but with a different timing fixed by $\varphi_j(t)$. For instance one possible choice is to take

$$\varphi_1(t) = \Omega_0 t \ ; \ \varphi_2(t) = \varphi_1(t) + \Delta\varphi(t) \qquad (21)$$

where $\Delta\varphi(t)$ is a smooth monotonic S-type function such that $\Delta\varphi(0) = \Delta\varphi_0$ and $\Delta\varphi(T) = \Delta\varphi_f$ where $\Delta\varphi_0$, $\Delta\varphi_f \in [-\pi/2, \pi/2]$. To properly scale the input, it will only be necessary to verify that its final required power level from normalising expression above is effectively available. With this type of functions it is possible to monitor the phase shift between the angular variables of vibratory units and to ultimately control the amplitude of the produced oscillation from eqn(16) with the use of eqn(8). One then gets with eqns(21)

$$U(\tau) = \Omega_0\left[\mu_1^2 + \mu_2^2 + 2\mu_1\mu_2\cos\Delta\varphi(\tau)\right]^{1/2} \qquad (22)$$

from which it is possible to satisfy eqn(9) by proper programming of $\Delta\varphi(t)$ in operation time interval. Finally, to satisfy the smoothness requirement $\ddot{\varphi}_j / \dot{\varphi}_j << \lambda\theta$ for which the analysis applies (here $0 << \lambda\theta$), one possible way is to raise the rotation of eccentric masses up to high enough value with $\Delta\varphi(t) = \pi$ before starting to pile, and to monitor after the phases as in eqns(21).

6. Conclusions

To determine the level of ground oscillation generated by vibrating piling units used in urban earth and building works, the dynamics of the complete system are needed in order to account for all elements of the chain from power source to observation point. So the equations describing these dynamics which include both the motion of eccentric masses generating the vibrations and the motion of piled object with all their interactions have been set first. They can be split into two subsystem concerning the piled object dynamics with a fictitious input created by the vibrating unit, and the vibrating unit dynamics themselves from which it should be verified that the fictitious input is realisable with input from real power source. It has been possible to solve these two subsystems in such a way that useful properties can be obtained. From the first one, the piled object trajectory is obtained as the sum of a smooth monotonic parabolic time depending motion and an oscillatory one with combination of mass and vibrating unit base frequencies. The first motion is possible if the constant pressing force is larger than an explicit threshold value expressed in terms of system parameters. The amplitude of the second oscillatory motion is also evaluated in terms of the fictitious input source, so that there remains from the second subsystem to calculate it in terms of the real control input. This has been explicitly done when assuming that the normalised mass ratio between the piled object and the eccentric vibrating unit is small enough, in which case the time dependence of the control input is expressed in term of the phase angle of eccentric rotating mass. For regular and smooth enough time functions, the behaviour of control input is itself a regular bounded and self-similar one in the sense that it is the same function of the phase angle, so it can be pre-programmed once for all.

Appendix

The analytic solution of eqns(5,6,7,8) is also obtained in the (realistic) case where the vibratory units frequency Ω_1, Ω_2 are very large compared to mechanical characteristic frequencies ω_m, ω_M, ω_g associated to mass displacements. In this case one can write $U = U(t/\varepsilon)$ and use appropriate formalism for resolution. With $X = (Z, \dot{Z}, z, \dot{z})$, eqns(5) take the form

$$\frac{dX}{dt} = F\left(X, t, \frac{t}{\varepsilon}\right) \qquad \text{(A1)}$$

with

$$F = col\begin{pmatrix} X_2, \omega_M^2 (X_3 - X_1) - \omega_g^2 - f_a, X_4, \\ \omega_M^2 (X_1 - X_3) - \omega_g^2 - f_s \,\mathrm{sgn}(X_4) + U(t/\varepsilon) \end{pmatrix}$$

Splitting now in its slow and fast component $X = [X] + \{X\}$, and observing that integral on fast time writes $< F > = \int F(.,.,t/\varepsilon)\, dt = \varepsilon \int F(.,.,u)\, du$, it is possible to develop eqn(10) order by order in ε. One then gets to first order

$$\frac{d[X]}{dt} = \left[F\left([X], t, \frac{t}{\varepsilon}\right)\right] \qquad \text{(A2)}$$

$$\{X\} = < \left\{F\left([X], t, \frac{t}{\varepsilon}\right)\right\} > \qquad \text{(A3)}$$

and to second one

$$\frac{d[X]}{dt} = \left[F\left([X], t, \frac{t}{\varepsilon}\right)\right] + \left[< \left\{F\left([X], t, \frac{t}{\varepsilon}\right)\right\} > .\nabla F\left([X], t, \frac{t}{\varepsilon}\right)\right] \qquad \text{(A4)}$$

$$\{X\} = < \left\{F\left([X], t, \frac{t}{\varepsilon}\right)\right\} > + < \left\{< \left\{F\left([X], t, \frac{t}{\varepsilon}\right)\right\} > .\nabla F\left([X], t, \frac{t}{\varepsilon}\right)\right\} > \qquad \text{(A5)}$$

which gives explicit expression for the solution Z, z when reporting $F(X,t,t/\varepsilon)$.

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