DECENTRALISED PREDICTIVE CONTROL WITH ENERGY DISSIPATION BOUNDS FOR WIRELESS STRUCTURAL CONTROL APPLICATIONS

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ABSTRACT: Decentralized control architectures for multi-storey buildings using wireless sensors and distributed embedding systems have been attracting more research recently. The designs for this application are however limited by the computational capacity embedded in each subsystem, the communication bandwidth and range of wireless channels, as well as the temporal constraints for real-time seismic protections. This paper presents a novel decentralized predictive control (DPC) strategy which can meet these design constraints. The control design takes into account the guaranteed closed-loop stability for the large-scale structure which is established from the dissipative systems theory. The proposed DPC strategy features two offline control laws to be switched-over online to minimize the computational and communication demands. By emulating the energy dissipation rate with a stabilizing constraint for the DPC optimization, the seismic protection performance of structural networked controls can be improved. The approach is designated with *energy-dissipation* DPC for structures using semiactive dampers. The approximated polynomial models of magneto-rheological (MR) dampers are adopted for linear systems in decentralized control designs. A numerical example is provided to illustrate the analytical developments.

Keywords: *Decentralized Structural Control, Predictive Control, Energy Dissipation Condition, Linear Matrix Inequality*

1. INTRODUCTION

Whilst becoming the most wide-spread used strategy in the process control field model predictive control (MPC) [1], with a finite horizon cost function and rolling (or receding horizon) computation for constrained problems, has failed to grow in structural control applications, notwithstanding its early considerations, see, e.g., [2] and references therein. A recent revisit of the MPC technique with offline calculations has, however, shown some promising results, especially in a decentralized scheme [3]. Decentralized control of large-scale structures, implemented with wireless sensors and distributed embedding systems, has attracted much research recently [4-6]. In this paper, we present an MPC strategy for large civil structures that is suitable for wireless applications using optimal control laws and online monitoring. We follow the definition of decentralized control in [8], which implies the non-communication version of non-centralized control strategies. The unconstrained control problem of structures is formulated with stability constraints to guarantee the closed-loop stability in a fully decentralized manner regardless of the length of predictive horizons and variations of interactive signals. This strategy eliminates the needs of exchanging data online at every updating time instant as in many other strategies [9], yet guarantees the stability of the large-scale structure. The stability constraint that emulates the energy dissipation is of a quadratic form developed from the dissipative system theory [10]. The dissipativity constraint was a major obstacle when combining with classical control methods due to BMI (bilinear matrix inequality) problems, as analyzed in [11]. We have presented an LMI-based (linear matrix inequality) constructive method for this type of dissipativity constraint for used with quadratic cost functions in MPC for systems with constraints [12]. Here, the performance of decentralized MPC is obtained by synthesizing the coefficients of the extended supply rate to satisfy energy dissipation conditions. By employing the energy dissipation rate as stability constraints in the interconnection stability condition, the closed-loop stability and control performance are achieved simultaneously. The two offline control laws are subsequently derived from the unconstrained optimality condition and the energy dissipation constraint. These offline control laws will be switched over on-the-fly based on the positively bounded condition of energy dissipation.

Albeit using nonlinear MR-damper, we employ a linear model (polynomial approximation) in our approach by using the results provided in [14]. From dissipativity theory perspective, it would be tempting to apply the mature results of port-controlled Hamiltonian systems for the nonlinear process of structures, such as the one in [18]. We, nonetheless, use linear system models here, as they are preferable in a decentralized control scheme that is limited by computation and communication capability of embedded systems and wireless sensors. The approach is, yet, extendable to nonlinear models without substantial modifications due to the maturity of the dissipative systems theory, especially for Hamiltonian processes [10]. Given a number of system models for decentralized (and distributed) structural control designs in the literature [4-6, 15, 16], we have chosen the decomposed model used in [6] which is closest to our developments. As a result, the *interaction-oriented model* for the large-scale structure is introduced in this paper for decentralized control designs. This type of model is standardized in the field of large-scale systems [17] and well-suited to the input and out property of interconnections in decentralized architectures. This choice of models is also viable for the clustered installations of dampers around the structure [16, 18].

Decentralized control has been one of the most popular design methods in the control engineering field since the 70's, see, e.g., [19, 20] and references therein. In such decentralized control schemes, a large diagonal controller provides a set of disparate local controllers for the associated subsystems which interact with each other. The drawbacks around the classical design methods such as static H2 or H∞ optimal control when applied to decentralized controllers lie with the non-convexity they may face, or the conservativeness of decoupled controllers. Different research has paid attention to the issues, such as the computations for achieving the highest decentralized H∞ performance based on the finite-horizon optimization [21], or the sufficient conditions upon which decentralized H2 optimal control problems become convex [22]. The improved solutions cannot still, however, avoid the conservative issues in static feedback designs if additional performance measures are not considered. The present DPC scheme offers an effective solution of incorporating the energy dissipation rate into the controller design using finite-horizon objective functions to eliminate such conservativeness.

This paper is organized as follows. The system and control models are given in Section 2. The stability condition for decentralized structural control is addressed in Section 3. The MPC problem formulations and offline control laws are outlined in Section 4. A numerical example is provided in Section 5 to illustrate the analytical developments. Section 6 concludes our paper.

The notation in this paper is fairly standard. Matrix and vector variables are represented by bold typefaces where appropriate. $\boldsymbol{M}^T$ is the transpose of the matrix $\boldsymbol{M}$. $\boldsymbol{I}_n$ is identity matrix of dimension n. $\mathbf{0}_n$ is zero matrix of dimension n. In symmetric block matrices or long matrix expressions, we use $(*)$ as an ellipsis for terms that are induced by symmetry, e.g.,

$$\boldsymbol{K}\begin{pmatrix}\boldsymbol{R}+(*) & \boldsymbol{S}\\ * & \boldsymbol{Q}\end{pmatrix}(*)=\boldsymbol{K}\begin{pmatrix}\boldsymbol{R}+\boldsymbol{R}^T & \boldsymbol{S}\\ \boldsymbol{S}^T & \boldsymbol{Q}\end{pmatrix}\boldsymbol{K}^T$$

The block-diagonal matrix $\boldsymbol{M}$ consisting of N identical block-element matrices $\boldsymbol{M}_i$, $i=1\ldots N$ is written as $\boldsymbol{M}=[\boldsymbol{M}_i]_1^N$.

2. SYSTEM AND CONTROL MODELS

Decomposed subsystem models of the following form [6] are used in this paper:

$$\begin{gathered}\boldsymbol{M}_i\ddot{\boldsymbol{x}}_i(t)+\boldsymbol{C}_i\dot{\boldsymbol{x}}_i(t)+\boldsymbol{K}_i\boldsymbol{x}_i(t)=\boldsymbol{\tau}_i(t),\\ \boldsymbol{\tau}_i(t)=\boldsymbol{B}_{oi}\boldsymbol{u}_i(t)+\boldsymbol{\phi}_i(\boldsymbol{x}_j(t),\dot{\boldsymbol{x}}_j(t))+\boldsymbol{d}_i(t),\\ i=1\ldots N, j\neq i\end{gathered}\tag{1}$$

where $\boldsymbol{M}_i, \boldsymbol{C}_i$ and $\boldsymbol{K}_i$ are the mass, damping, and stiffness matrices of the decomposed subsystem i, respectively, $\boldsymbol{x}_i$ is the local floor displacement, $\boldsymbol{B}_{oi}$ is the decomposed control force location matrix, $\boldsymbol{u}_i$ is the local control force, $\boldsymbol{d}_i$ represents the seismic force, $\left(\boldsymbol{d}_i(t)=\boldsymbol{M}_i\ddot{\boldsymbol{x}}_g(t)\right)$, and $\boldsymbol{\phi}_i(\boldsymbol{x}_j(t),\dot{\boldsymbol{x}}_j(t))$ is the interactive vectors, representing the couplings (interconnections) among subsystems ($\boldsymbol{x}_j$ are the states of all neighbour subsystems). For a two story subsystem, the mass, damping, and stiffness matrices have the following forms:

$$\boldsymbol{M}_i=\begin{pmatrix}m_{i1} & 0\\ 0 & m_{i2}\end{pmatrix},\boldsymbol{C}_i=\begin{pmatrix}c_{i1}+c_{i2} & -c_{i2}\\ -c_{i2} & c_{i2}\end{pmatrix},\boldsymbol{K}_i=\begin{pmatrix}k_{i1}+k_{i2} & -k_{i2}\\ -k_{i2} & k_{i2}\end{pmatrix}.$$

By employing the empirical polynomial model of MR-damper developed in [14], the above system model remains unchanged. Once the force $\boldsymbol{u}_i$ is calculated from the control law, the required current feeding to the MR-damper will be determined from the polynomial model and the measured piston velocity ($\boldsymbol{v}_i$). The linearly-approximated relationship between the exerting force ($\boldsymbol{u}_i$) and supply current ($\boldsymbol{I}_i$) of the MR-damper (of order σ) is represented by

$$\boldsymbol{u}_i = \sum_{\theta=0}^{\sigma} \alpha_\theta \boldsymbol{v}_i{}^\theta = \sum_{\theta=0}^{\sigma} (\beta_\theta + \gamma_\theta \boldsymbol{I}_i)\boldsymbol{v}_i{}^\theta, \tag{2}$$

where the empirical coefficient α_θ is determined from the experimental data and curve fitting techniques, and β_θ and γ_θ can be determined from the least-square optimization method [14].

The state space model of a subsystem is thus obtained as follows:

$$\dot{\boldsymbol{z}}_i(t) = \boldsymbol{A}_i\boldsymbol{z}_i(t) + \boldsymbol{B}_i\boldsymbol{u}_i(t) + \boldsymbol{E}_i\boldsymbol{v}_i(t) + \boldsymbol{B}_{di}\boldsymbol{w}_i(t), \tag{3}$$

$$\boldsymbol{z}_i = \begin{pmatrix} \boldsymbol{x}_i \\ \dot{\boldsymbol{x}}_i \end{pmatrix}, \boldsymbol{A}_i = \begin{pmatrix} \boldsymbol{0}_{n_i} & \boldsymbol{I}_{n_i} \\ -\boldsymbol{M}_i{}^{-1}\boldsymbol{K}_i & -\boldsymbol{M}_i{}^{-1}\boldsymbol{C}_i \end{pmatrix},$$

$$\boldsymbol{B}_i = \boldsymbol{B}_i = \begin{pmatrix} \boldsymbol{0}_{n_i} \\ -\boldsymbol{M}_i{}^{-1}\boldsymbol{B}_{oi} \end{pmatrix}, \boldsymbol{B}_{di} = \begin{pmatrix} \boldsymbol{0}_{n_i} \\ \boldsymbol{D}_{n_i} \end{pmatrix}, \boldsymbol{E}_i = \begin{pmatrix} \boldsymbol{0}_{n_i} \\ \boldsymbol{G}_i \end{pmatrix},$$

$\boldsymbol{v}_i(t) = \boldsymbol{G}_i\boldsymbol{\phi}_i(t)$ is the normalized interactive vector, $\boldsymbol{w}_i(t)$ is the normalized vector of the seismic force $\boldsymbol{d}_i$. In this paper, the models are discretized for implementing with computerized control systems. For conciseness, we use the same notations for discrete-time models here. The time index $k \in \mathbb{Z}$ is thus used instead of the continuous time variable t. Without loss of generality, it is assumed that the normalized vector $\boldsymbol{w}_i$ is finite-horizon norm bounded:

$$\sum_{k=k_0}^{k_0+N_i} \|\boldsymbol{w}_i(k)\|^2 \le 1, \tag{4}$$

where N_i is the predictive horizon of the DPC.

The upper bounds of interactive vector $\boldsymbol{\phi}_i(t)$ are assumed to be known [6], and the controller designs are based on those bounds. In this paper, the upper bounds are implicitly described via the realization matrix $\boldsymbol{E}_i$ and the normalized interactive vector $\boldsymbol{v}_i(t)$ instead. This normalized vector $\boldsymbol{v}_i(t)$ is a function with respect to the state vectors of relevant neighbour subsystems. The interactions between all neighbours within the building are described via the global interconnection matrix $\mathbf{H}$ that contains $\boldsymbol{0}_{n_j}$ or $\delta_{ij}\boldsymbol{I}_{n_j}$ as its block elements. The constant δ_{ij} represents the interconnection effects of subsystem i to other subsystems j in the building, $j = 1 \ldots N, j \ne i$.

$$\boldsymbol{v} = \mathbf{H}\boldsymbol{z}, \tag{5}$$

where $\boldsymbol{v}^T = [\boldsymbol{v}_1^T \ldots \boldsymbol{v}_N^T]$ and $\boldsymbol{z}^T = [\boldsymbol{z}_1^T \ldots \boldsymbol{z}_N^T]$ are the global vectors.

By using uncertain realization matrices of the polytopic form $(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{E}_i) \in \Omega = \mathrm{Co}\{(\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{E}_{\Delta i} + \Delta\boldsymbol{E}_{i1}), \ldots, (\boldsymbol{A}_i, \boldsymbol{B}_i, \boldsymbol{E}_{\Delta i} + \Delta\boldsymbol{E}_{iL})\}$, where Co denotes the convex hull, similarly to a robust control formulation, the dissipativity constraints defined in later sections of the paper will guarantee the stability of the global system without conservatively restricting damping actuations from the assumed upper bounds.

3. STABILITY CONDITION FOR STRUCTURES

The stability condition for the large-scale structure is given in this Section. Let us first introduce the dissipativity for the stand-alone subsystem i (3), where the interactive inputs $\boldsymbol{v}_i$ vanish. The stand-alone subsystem (3) is said to be *quadratically dissipative* with respect to the quadratic supply rate $\boldsymbol{\xi}_i(\boldsymbol{z}_i, \boldsymbol{u}_i)$ defined as,

$$\boldsymbol{\xi}_i(\boldsymbol{z}_i, \boldsymbol{u}_i) \triangleq \boldsymbol{z}_i{}^T\boldsymbol{Q}_i\boldsymbol{z}_i + 2\boldsymbol{z}_i{}^T\boldsymbol{S}_i\boldsymbol{u}_i + \boldsymbol{u}_i{}^T\boldsymbol{R}_i\boldsymbol{u}_i, \tag{6}$$

where $\boldsymbol{Q}_i, \boldsymbol{S}_i, \boldsymbol{R}_i$ are multiplier matrices, with $\boldsymbol{Q}_i$ and $\boldsymbol{R}_i$ symmetric, if there exists a nonnegative storage function $V_i(\boldsymbol{z}_i(k))$ such that for all $\boldsymbol{u}_i(k)$ and all $k \in \mathbb{Z}^+$, the following dissipation inequality is satisfied:

$$V_i(\boldsymbol{z}_i(k+1)) - V_i(\boldsymbol{z}_i(k)) \le \boldsymbol{\xi}_i(\boldsymbol{z}_i(k), \boldsymbol{u}_i(k)). \tag{7}$$

In this paper, the square storage function of the form $V_i(\boldsymbol{z}_i(k)) = \boldsymbol{z}_i(k)^T\boldsymbol{P}_i\boldsymbol{z}_i(k)$, $\boldsymbol{P}_i{}^T = \boldsymbol{P}_i > 0$ is considered. The stability condition is based on a combination of the structure being quadratically dissipative, and the controllers bounded by the dissipativity-based constraint. Defining a quadratic function $\boldsymbol{\xi}_{ic}$ by

$$\boldsymbol{\xi}_{ic}(\boldsymbol{u}_i, \boldsymbol{z}_i) \triangleq \begin{pmatrix} \boldsymbol{y}_i \\ \boldsymbol{u}_i \end{pmatrix}^T \begin{pmatrix} \boldsymbol{R}_{ic} & \boldsymbol{S}_{ic} \\ * & \boldsymbol{Q}_{ic} \end{pmatrix} \begin{pmatrix} \boldsymbol{y}_i \\ \boldsymbol{u}_i \end{pmatrix}, \tag{8}$$

the controller $\mathcal{C}_i$ is then said to satisfy the dissipativity-based constraint with respect to $\boldsymbol{\xi}_{ic}(\boldsymbol{u}_i, \boldsymbol{z}_i)$ (8) if the accumulation of $\boldsymbol{\xi}_{ic}(\boldsymbol{u}_i(k), \boldsymbol{z}_i(k))$ starting from a certain initial time, denoted as $\Xi_{ic}(k)$, is positive, i.e.

$$\Xi_{ic}(k) \triangleq \sum_{i=k_0}^{k} \boldsymbol{\xi}_{ic}(\boldsymbol{u}_i(k), \boldsymbol{z}_i(k)) > 0 \;\; \forall k \ge k_0. \tag{9}$$

This dissipativity-based constraint represents the *extended energy dissipation* of the system as described in [11]. For the exact energy dissipation, the multipliers will be such that $\boldsymbol{S}_{ic} = -\boldsymbol{I}$, $\boldsymbol{Q}_{ic} = \boldsymbol{R}_{ic} = \boldsymbol{0}$, i.e. only passive terms in the supply rate is considered. The choices of these matrices are crucial to the control performance as they will directly affect the transient responses and the actual energy dissipations. According to our previous work, the exact energy dissipation rate may become conservative for decentralized architectures [23]. Moreover, in the discrete-time domain, such passive systems do not exist [24]. A less conservative choice of $\boldsymbol{S}_{ic} = -\boldsymbol{I}, \boldsymbol{Q}_{ic} = -\epsilon_i\boldsymbol{I}, \boldsymbol{R}_{ic} = \boldsymbol{0}$, with small positive scalar ϵ_i, is more realistic and desirable. Further details on these multiplier matrices are provided the next section.

By imposing this constraint on every local controller, the energy dissipation processes are incorporated into the controller designs.

In decentralized architectures, each local controller is required to be constrained by the energy dissipation process (8) in association with the dissipativity of the large-scale diagonal system with respect to the supply rate of

$$\boldsymbol{\xi}(\boldsymbol{z},\boldsymbol{u}_d) \triangleq \boldsymbol{z}^T\boldsymbol{Q}\boldsymbol{z} + 2\boldsymbol{z}^T\boldsymbol{S}\boldsymbol{u}_d + \boldsymbol{u}_d{}^T\boldsymbol{R}\boldsymbol{u}_d,$$

where $\boldsymbol{u}_d{}^T = [\boldsymbol{u}^T\ \boldsymbol{d}^T\ \boldsymbol{v}^T]$ is the combined input vector, and the dissipativity matrices $\boldsymbol{Q}$, $\boldsymbol{S}$, $\boldsymbol{R}$ are block-structured diagonal of the form

$$\boldsymbol{R} = \begin{pmatrix} \begin{pmatrix} \boldsymbol{R}_{11u} & 0 \\ 0 & \boldsymbol{R}_{11d} \end{pmatrix} & 0 \\ 0 & \boldsymbol{R}_{22} \end{pmatrix},\ \boldsymbol{S} = \begin{pmatrix} \boldsymbol{S}_{11} & 0 \\ 0 & \boldsymbol{S}_{22} \end{pmatrix}, \tag{10}$$

in which each block-element is, in turn, a block-diagonal matrix formed by N corresponding subsystem matrices

$\boldsymbol{Q} = [\boldsymbol{Q}_i]_1^N, \boldsymbol{S}_{22} = [\boldsymbol{S}_{i22}]_{i=1}^N, \boldsymbol{R}_{22} = [\boldsymbol{R}_{i22}]_{i=1}^N, \boldsymbol{S}_{11} = [\boldsymbol{S}_{i11}]_{i=1}^N,$
$\boldsymbol{R}_{11u} = [\boldsymbol{R}_{i11u}]_{i=1}^N, \boldsymbol{R}_{11d} = [\boldsymbol{R}_{i11d}]_{i=1}^N,\ \boldsymbol{u}^T = [\boldsymbol{u}_1^T \dots \boldsymbol{u}_N^T].$

Now, by defining $\boldsymbol{A} = [\boldsymbol{A}_i]_1^N, \boldsymbol{B} = [\boldsymbol{B}_i\ \ \boldsymbol{B}_{di}]_{i=1}^N, \boldsymbol{E} = [\boldsymbol{E}_i]_1^N,$ $\boldsymbol{P} = [\boldsymbol{P}_i]_1^N, \boldsymbol{Q}_c = [\boldsymbol{Q}_{ic}]_{i=1}^N,\ \boldsymbol{S}_c = [\boldsymbol{S}_{ic}]_1^N,\ \boldsymbol{R}_c = [\boldsymbol{R}_{ic}]_1^N$, the stability condition is stated in the following Proposition:

Proposition 1 [12] – If the following LMIs are feasible in $\boldsymbol{P}, \boldsymbol{Q}, \boldsymbol{R}_{11u}, \boldsymbol{R}_{11d}, \boldsymbol{S}_{11}, \boldsymbol{S}_{22}, \boldsymbol{R}_{22}, \boldsymbol{R}_c, \boldsymbol{S}_c, \boldsymbol{Q}_c$:

$$\begin{pmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} \\ * & \boldsymbol{M}_{22} \end{pmatrix} \le 0, \tag{11a}$$

$$\begin{pmatrix} \boldsymbol{Q}+\boldsymbol{R}_c & \boldsymbol{S}_{11}+\boldsymbol{S}_c \\ * & \boldsymbol{R}_{11}+\boldsymbol{Q}_c \end{pmatrix} < 0,\ \boldsymbol{Q}_L < 0,\ \boldsymbol{Q}_c < 0, \tag{11b}$$

$$\boldsymbol{M}_{11} = \boldsymbol{A}^T\boldsymbol{P}\boldsymbol{A} - \boldsymbol{P} + \mathrm{H}^T(\boldsymbol{E}^T\boldsymbol{P}\boldsymbol{E} - \boldsymbol{R}_{22})\mathrm{H} + 2(\boldsymbol{A}^T\boldsymbol{P}\boldsymbol{E} - \boldsymbol{S}_{22})\mathrm{H}$$
$$\boldsymbol{M}_{12} = \boldsymbol{A}^T\boldsymbol{P}\boldsymbol{B} - \boldsymbol{S}_{11} + \mathrm{H}^T\boldsymbol{E}^T\boldsymbol{P}\boldsymbol{B}, \boldsymbol{M}_{22} = \boldsymbol{B}^T\boldsymbol{P}\boldsymbol{B} - \boldsymbol{R}_{11},$$
$$\boldsymbol{Q}_L = \boldsymbol{S}_{22}\mathrm{H} + \mathrm{H}^T\boldsymbol{S}_{22}{}^T + \mathrm{H}^T\boldsymbol{R}_{22}\mathrm{H},\ \ \boldsymbol{R}_{11} = \begin{pmatrix} \boldsymbol{R}_{11u} & 0 \\ 0 & \boldsymbol{R}_{11d} \end{pmatrix};$$

Then closed-loop large-scale structure is stabilized provided that the extended energy-dissipation condition (9) is satisfied.

5. UNCONSTRAINED PREDICTIVE CONTROL SYNTHESIZED WITH ENERGY DISSIPATION BOUNDS

The quadratic objective function of predictive states and controls in association with adequately chosen weighting matrices $\boldsymbol{\mathcal{R}}_i$, $\boldsymbol{Q}_i$ and a predictive horizon N_i, is considered for every subsystem:

$$\mathcal{J}_i(k) = \sum_{j=1}^{N_i+1} \boldsymbol{z}_i(k+j)^{\mathrm{T}}\boldsymbol{\mathcal{R}}_i\boldsymbol{z}_i(k+j) + \sum_{j=0}^{N_i} \boldsymbol{u}_i(k+j)^{\mathrm{T}}\boldsymbol{Q}_i\boldsymbol{u}_i(k+j). \tag{12}$$

The problem of minimizing $\mathcal{J}_i(k)$ subject to the equality constraint of the system model (3) and the extended energy-dissipation constraint (9) up until the end of the horizon N_i can be expressed as follows [12]:

$$\begin{aligned} &\min_{\widehat{\boldsymbol{u}}_i}\ \ \widehat{\boldsymbol{u}}_i^T\Phi_i\widehat{\boldsymbol{u}}_i + 2\Upsilon_i\ \widehat{\boldsymbol{u}}_i + \delta_i \\ &\text{subject to}\ \ \widehat{\boldsymbol{u}}_i^T\Psi_i\widehat{\boldsymbol{u}}_i + 2\Upsilon_{ic}\ \widehat{\boldsymbol{u}}_i + \delta_{ic} > 0, \end{aligned} \tag{13}$$

where

$\Phi_i = \Gamma_i{}^T\boldsymbol{\mathcal{R}}_{iN}\Gamma_i + \boldsymbol{Q}_{iN}, \Upsilon_i = \Lambda_i^T\boldsymbol{\mathcal{R}}_{iN}\Gamma_i, \Lambda_i = \Theta_i\boldsymbol{z}_i(k),$
$\delta_i\ = \Lambda_i^T\boldsymbol{\mathcal{R}}_{iN}\Lambda_i, \Psi_i = \Gamma_i{}^T\boldsymbol{R}_{icN}\Gamma_i + 2\Gamma_i{}^T\boldsymbol{S}_{icN} + \boldsymbol{Q}_{icN},$
$\boldsymbol{Q}_{iN} = [\boldsymbol{Q}_i]_1^{N_i}, \boldsymbol{\mathcal{R}}_{iN} = [\boldsymbol{\mathcal{R}}_i]_1^{N_i}, \Upsilon_{ic} = \Lambda_i^T(\boldsymbol{R}_{icN}\Gamma_i + \boldsymbol{S}_{icN}),$
$\delta_{ic} = \Lambda_i^T\boldsymbol{R}_{icN}\Lambda_i + \boldsymbol{\Xi}_{ic}(k-1) - \|\boldsymbol{R}_{11d}\|,$
$\boldsymbol{Q}_{icN} = [\boldsymbol{Q}_{ic}]_1^{N_i}, \boldsymbol{R}_{icN} = [\boldsymbol{R}_{ic}]_1^{N_i}, \boldsymbol{S}_{icN} = [\boldsymbol{R}_{ic}]_1^{N_i},$

$$\Gamma_i \triangleq \begin{pmatrix} \boldsymbol{B}_i & 0 & \dots & 0 \\ \boldsymbol{A}_i\boldsymbol{B}_i & \boldsymbol{B}_i & \dots & 0 \\ \dots & \dots & \dots & \dots \\ \boldsymbol{A}_i{}^{N_i}\boldsymbol{B}_i & \dots & \boldsymbol{A}_i\boldsymbol{B}_i & \boldsymbol{B}_i \end{pmatrix}, \Theta_i \triangleq \begin{pmatrix} \boldsymbol{A}_i \\ \dots \\ \boldsymbol{A}_i{}^{N_i} \\ \boldsymbol{A}_i{}^{N_i+1} \end{pmatrix}.$$

Albeit a constrained problem, it is possible not to apply the KKT (Karush-Kuhn-Tucker) optimality condition [13] for the control laws here. From our analysis in previous sections, the purpose of having the inequality constraint is to ensure that subsystems actually dissipate energy. With the extended supply rate (8), the problem can be turned into a synthesis problem of finding the multiplier matrices $\boldsymbol{R}_{ic}$, $\boldsymbol{S}_{ic}$ and $\boldsymbol{Q}_{ic}$ in the supply rate such that the energy dissipation is positively bounded in the worst case scenario. A second control law will be employed if the dissipation condition cannot be met. The problem (13) then becomes unconstrained optimization. Applying the unconstrained optimality condition, the control law is obtained in the sequel:

$$\widehat{\boldsymbol{u}}_i^{\mathrm{o}} = G_i\boldsymbol{z}_i(k) = -\Phi_i^{-1}\Gamma_i{}^T\mathcal{R}_{iN}\Theta_i\boldsymbol{z}_i(k). \tag{14}$$

Only the first vector element $\boldsymbol{u}_i^{\mathrm{o}}(k)$ of the finite-horizon optimal control sequence $\widehat{\boldsymbol{u}}_i^{\mathrm{o}}$ is applied to the MR-damper model equation (2) to find the control current needs to be exerted at time step k.

Substituting this control law into the inequality constraint, it follows that the condition of energy dissipation is fulfilled by the feasibility of the following LMI in $\boldsymbol{Q}_{ic}$, $\boldsymbol{S}_{ic}$, if $\boldsymbol{R}_{ic} = 0$ and $\boldsymbol{\Xi}_{ic}(k-1) - \|\boldsymbol{R}_{11d}\| \ge 0$:

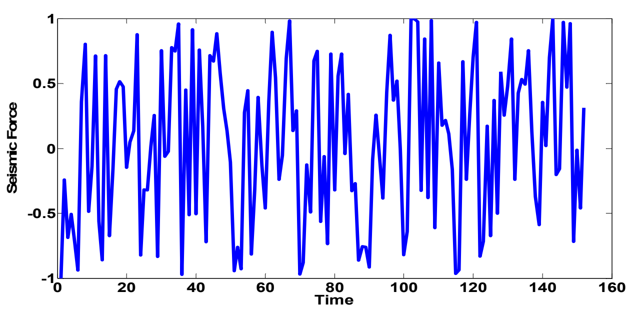
$$\begin{aligned} &\Theta_i{}^T\mathcal{R}_{iN}\Gamma_i\Phi_i^{-T}\Gamma_i{}^T(\boldsymbol{Q}_{icN} + 2\Gamma_i{}^T\boldsymbol{S}_{icN})\Phi_i^{-1}\Gamma_i{}^T\mathcal{R}_{iN}\Theta_i \\ &\qquad -2\Theta_i{}^T\boldsymbol{S}_{icN}\Phi_i^{-1}\Gamma_i{}^T\mathcal{R}_{iN}\Theta_i \succ 0. \end{aligned} \tag{15}$$

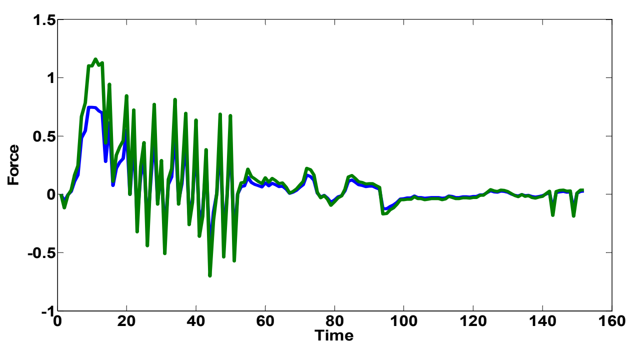
To emulate the actual energy dissipation rate, which is determined by $-\dot{\boldsymbol{x}}_i\boldsymbol{u}_i = \boldsymbol{z}_i\begin{pmatrix} 0 & 0 \\ 0 & -\boldsymbol{I} \end{pmatrix}\boldsymbol{u}_i$, due to $\boldsymbol{z}_i = \begin{pmatrix} \boldsymbol{x}_i \\ \dot{\boldsymbol{x}}_i \end{pmatrix}$, the multiplier matrices should have the form of $\boldsymbol{S}_{ic} = \begin{pmatrix} -\mu_i\boldsymbol{I} & 0 \\ 0 & -\boldsymbol{I} \end{pmatrix}, \boldsymbol{Q}_{ic} = -\epsilon_i\boldsymbol{I}, \epsilon_i > 0, \mu_i > 0.$ By using the resulting μ_i and ϵ_i coefficients from feasible solutions to the offline LMIs (15) for $i = 1 \dots N$ and (11) (Proposition 1) in the control law (14), the energy dissipation bounds will always be positively bounded provided that the subsystem dissipates energy at initial time steps. To guarantee $\boldsymbol{\Xi}_{ic}(k-1) - \|\boldsymbol{R}_{11d}\| \geq 0$ at those initial steps, it is necessary that the instant values of the supply rate $\xi_{ic}(k) = \boldsymbol{z}_i^T(k)\boldsymbol{Q}_{ic}\boldsymbol{z}_i(k) + 2\boldsymbol{z}_i^T(k)\boldsymbol{S}_{ic}\boldsymbol{u}_i(k)$ ($\boldsymbol{R}_{ic} = \boldsymbol{0}$ is considered here) at some initial steps are positive. A second control law is derived here to assure the occurrences. By rewriting $\boldsymbol{\xi}_{ic}(\boldsymbol{u}_i, \boldsymbol{z}_i) = (\boldsymbol{u}_i - \boldsymbol{Q}_{ic}^{-1}\boldsymbol{S}_{ic}\boldsymbol{z}_i)^T\boldsymbol{Q}_{ic}(\boldsymbol{u}_i - \boldsymbol{Q}_{ic}^{-1}\boldsymbol{S}_{ic}\boldsymbol{z}_i) - \boldsymbol{z}_i^T\boldsymbol{S}_{ic}^T\boldsymbol{Q}_{ic}^{-1}\boldsymbol{S}_{ic}\boldsymbol{z}_i$, $\boldsymbol{Q}_{ic} < 0$, it is obviously to see that, if

$$\boldsymbol{u}_i(k) = Q_{ic}^{-1}S_{ic}\boldsymbol{z}_i(k), \tag{16}$$

we always have $\boldsymbol{\xi}_{ic}(\boldsymbol{u}_i, \boldsymbol{z}_i) \geq 0$. From the two offline control laws (14) and (16), the control algorithm is summarized as follows:

- The multiplier matrices $\boldsymbol{Q}_{ic}$, $\boldsymbol{S}_{ic}, \boldsymbol{R}_{ic}$ and dissipativity matrices $\boldsymbol{Q}$, $\boldsymbol{S}$, $\boldsymbol{R}$ are determined offline from LMIs (11) and (15).
- The control law (16) is applied to control the structure in the first N_i steps, or until $\boldsymbol{\Xi}_{ic}(k) - \|\boldsymbol{R}_{11d}\| \geq 0$, to ensure that the energy dissipation condition is initially delivered.
- The optimal control law (14) is then applied to control the structure whilst calculating and monitoring the accumulation $\boldsymbol{\Xi}_{ic}(k)$ online. Whenever $\boldsymbol{\Xi}_{ic}(k) - \|\boldsymbol{R}_{11d}\| < 0$, switch over to the control law (16), or retain (14) otherwise.





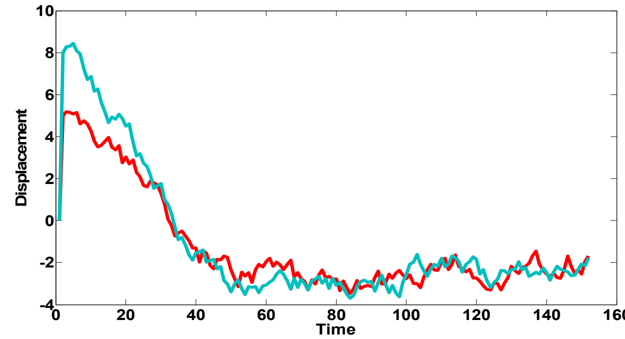


Fig. 1 – The Time Responses of Control Forces and Displacements of Subsystem 2.

6. NUMERICAL EXAMPLE

Decomposed models for two storey subsystems from a six-storey building are used in this numerical example. The mass, stiffness, and the damping coefficient of each floor are 345,600kg, 340,400kN/m, and 100,000kg/s, respectively [25]. The weighting coefficients of the cost function are chosen as $\mathcal{R}_i = [1.0\ \ 1.0\ \ 1.0\ \ 1.0]$, $\mathcal{Q}_i = [0.5\ \ 0.5]$. The predictive horizons are chosen as $N_i = 10$. The sampling time is $T_s = 20$ms. It is assumed that the couplings between three floors are evenly distributed. A random signal is generated by a Matlab function to simulate the disturbance force input. The output and control trends of subsystem 2 produced by the online control algorithm in Section 5 are provided in Fig. 1.

7. CONCLUSION

The extended energy-dissipation condition is proposed for the synthesis problem of decentralized predictive control (DPC) in this paper. The control laws are derived from the unstrained optimality condition of given finite-horizon cost functions and the energy dissipation bounds. Theoretical developments and illustrative simulations are presented. A benchmark test is underway to verify the efficacy of the proposed DPC strategy.

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