COMPARISON OF TWO WATER STORAGE FUNCTIONS OF SOIL ON PORE-WATER PRESSURE OF EARTH-FILLED DAM UNDER CHANGING ENVIRONMENT

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ABSTRACT: Seepage of water through earth fill dam involves both the saturated and unsaturated flows of water. Unsaturated flow of water is often neglected because of the complexity required in solving the non-linear partial differential equation involved. This paper presents a seepage analysis of water flows through the partially saturated Renyi-Tan earth fill dam in Taiwan. The governing non-linear partial differential flow equation together with equations representing the characteristics of dam materials was solved using a PDE solver. Two water storage functions, which were derived by differentiating the van Genuchten (1980) and Leong and Rahardjo (1997)'s SWCC functions, have been used, and the two sets of results obtained have been compared for their sensitivity in seepage analysis of the study dam.

Keywords: *Earth-fill Dam, Seepage, Unsaturated Soil, Water Storage Function, PDE*

1. INTRODUCTION

Seepage analysis is important in the assessment of long term stability of slope, underground excavation, dam, etc. For homogeneous and isotropic material, the flow of water in saturated zone is commonly estimated by solving the linear partial differential equation (PDE) through the use of the graphical flow net method (Thieu *et al.*, 2001). The flow of water in unsaturated soil is a very complicated problem because water flow and moisture content in unsaturated soil may vary both spatially and temporally as a result of time-dependent changes in environmental conditions, such as rainfall and rising water-table, and the storage capacity of soil (Lu and Likos, 2004). The environmental changes are normally considered by including them into the boundary conditions of the problem under consideration while the storage capacity of the soil is considered via the governing laws of flow, which is essentially represented by a non-linear partial differential equation.

Taiwan is receiving plenty of rainfall that was brought by typhoons between May and October every year. Not only could the stability of dam be affected by the sudden rise of the upstream water-table it could also be affected by the infiltration of the rain. Taking two different functions of water storage capacity of soil into consideration, this study aims at examining the generation and distribution of pore-water pressure of Renyi-Tan earth fill dam due to the rising upstream water-table and rainfall infiltration. The two sets of results, generated by the two water storage functions, are then evaluated and compared in terms of pore-water pressure contour generated in the dam.

2. THEORETICAL BACKGROUND

The governing law for transient water flow in soil under isothermal conditions can be derived by applying the principle of mass conservation, which is also called the continuity principle. It states that for a given elemental volume of soil, the rate of water loss or gain is conservative and is equal to the net flux of inflow and outflow, leading to

$$-\rho\left(\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}+\frac{\partial q_z}{\partial z}\right)=\frac{\partial(\rho\theta)}{\partial t} \quad (1)$$

where ρ is the density of water (kg/m^3), q_x, q_y and q_z are fluxes in the *x, y* and *z* directions, respectively (m/s). Equation (1) is the governing equation of unsteady state of flow and it can be applied in both saturated and unsaturated conditions. Using Darcy's law

$$q_x=-k_x\frac{\partial h}{\partial x},\quad q_y=-k_y\frac{\partial h}{\partial y},\ \ and\ \ q_z=-k_z\frac{\partial h}{\partial z} \quad (2)$$

where k_x = hydraulic conductivity and $\frac{\partial h}{\partial x}=$ hydraulic gradient in x-direction and so on, thus

$$\frac{\partial}{\partial x}\left[k_x\frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[k_y\frac{\partial h}{\partial y}\right]+\frac{\partial}{\partial z}\left[k_z\frac{\partial h}{\partial z}\right]=\frac{\partial\theta}{\partial t} \quad (3)$$

Darcy's law also applies to water flow through unsaturated soil (Richards, 1931). However, the hydraulic conductivity in the unsaturated soil cannot be assumed to be a constant, as in the case of saturate soil, but a function of soil suction *h* (Richards, 1931), hence

$$\frac{\partial}{\partial x}\left[k_x(h)\frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[k_y(h)\frac{\partial h}{\partial y}\right]+\frac{\partial}{\partial z}\left[k_z(h)\frac{\partial h}{\partial z}\right]=\frac{\partial\theta}{\partial t} \quad (4)$$

The term $\frac{\partial\theta}{\partial t}$ can be written in terms of matric suction head

$$\frac{\partial\theta}{\partial t}=\frac{\partial\theta}{\partial h}\frac{\partial h}{\partial t} \quad (5)$$

where $\frac{\partial\theta}{\partial h}$ is the gradient of the soil-water characteristics curve (SWCC), plotted in terms of volumetric water content vs suction head, and is referred to the water storage function. Fredlund and Morgenstern (1976) suggested that the water phase constitutive relationship as

$$d\theta=m_1^w d(\sigma_{mean}-u_a)+m_2^w d(u_a-u_w) \quad (6a)$$

where σ_{mean} = mean stress, u_a = pore-air pressure, u_w = pore-water pressure, $(\sigma_{mean}-u_a)$ = mean net normal stress, (u_a-u_w) = matric suction, m_1^w = gradient of the water volume vs net normal stress curve when $d(u_a-u_w)$ is zero, m_2^w = gradient of the water volume vs matric suction curve when $d(\sigma_{mean}-u_a)$ is zero. For a non-deformable soil, it can be assumed that the total stress remains constant and the pore-air pressure remains at atmospheric conditions; hence, a change of volumetric water content can be related to a change in pore-water pressure as

$$d\theta=m_2^w d(-u_w) \quad (6b)$$

The PDE for water flow through an unsaturated soil is thus:

$$\frac{\partial}{\partial x}\left[k_x(h)\frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[k_y(h)\frac{\partial h}{\partial y}\right]+\frac{\partial}{\partial z}\left[k_z(h)\frac{\partial h}{\partial z}\right]=m_2^w\rho g\frac{\partial h}{\partial t} \quad (7)$$

To solve this PDE, three characteristic functions: (i) SWCC function, (ii) hydraulic conductivity function, and (iii) water storage function must first be defined. The SWCC is the relation between the volumetric water content and matric suction; the hydraulic conductivity is a function of the volumetric water content, which is in turn a function of matric suction; and the water storage, which is the gradient of the volumetric water content vs matric suction curve, is an indication of the amount of water entering or leaving the soil as a result of a change in the pore-water pressure or matric suction.

3. NUMERICAL MODELING

The solution to the above non-linear PDE can be obtained using a finite element (FE) partial differential solver FlexPDE (PDE Solution Inc., 2004). It is a scripted FE model builder and numerical solver for system of PDEs. It can solve systems of: non-linear or time-dependent, and 1st or 2nd order PDEs in two or three dimensional geometry. In FlexPDE, the non-linear system is solved by a modified Newton-Raphson iteration process while the time-dependent system is solved by a 2nd order implicit Backward Difference Formula (Gear method). It first tries to find a valid starting time step by comparing one-step and two-step solutions over a trial time step and reducing this trial time step until the one- and two-step solutions agree within a specified error tolerance. In short, the user writes the script of the PDE system and FlexPDE turns the script into a FE model and solve the system before presenting graphical output of the results. The program is capable of auto-mesh generation and refinement and, thus, simplifies the mesh creation process. It also allows the material properties to be input in equation form or a series of data points.

4. RENYI-TAN DAM AND INPUT PARAMETERS

4.1 Background of the Dam

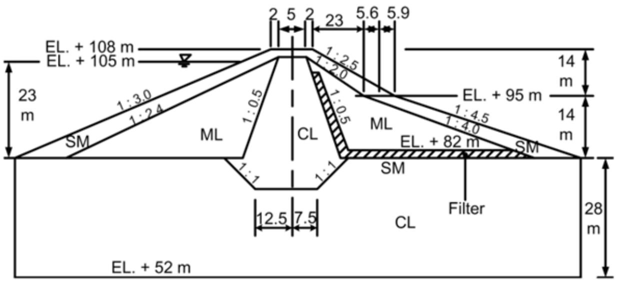


Fig. 1 Cross-sectional view of Renyi-Tan dam (in meter).

The study dam that formed part of an off-stream reservoir is called Renyi-Tan dam. It is a roller-compacted earth fill dam with layout shown in Fig. 1. The dam has a maximum height of 28 m and a 9 m wide by 1550 m long crest located at elevation EL. 108 m. The highest water-table is at EL. 105 m with a water surface covers an area of 3.66 square km; the total water-holding capacity is 29.11 million m^3 but the effective water-holding capacity is only 27.31 million m^3. The dam itself is situated at the upstream of the Ba-Zhang River, which is 2.1 km from the Lan-Tan dam. Construction of the dam started in 1980 and it began to store water in August 1987. Together with Lan-Tan dam, it supplies water to the community in Jia-Yi County, and the industry around the area. The dam was constructed using four materials. The shell of the dam was constructed by SM material, the core by CL material and the two semi-impermeable zones, located between the shell and the core, by ML material (Fig 1). The surface layer of the shell was covered by gravelly material.

4.2 Input Parameters

Equation (7) reveals that three characteristic functions (SWCC, hydraulic conductivity and water storage) are needed in this seepage analysis.

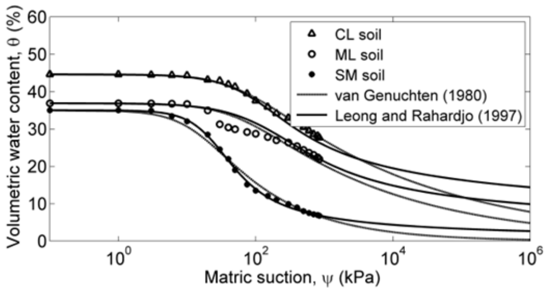


Fig. 2 Experimental and back-fitted SWCC functions.

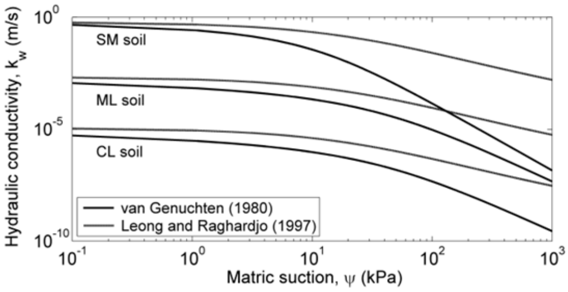


Fig. 3 Estimated hydraulic conductivity functions.

Table 1: Fitting parameters used in fitting SWCC and k_w.

Soil type	van Genuchten (1980) *a, n, m(=1-1/n)*	Leong and Rahardjo (1997) *a, n, m*
CL	0.01599, 1.180	70.54, 1.211, 0.4620
ML	0.01721, 1.209	104.0, 1.016, 0.5904
SM	0.06816, 1.411	21.42, 1.734, 0.8811

SWCC

Experimental data of the SWC of the three materials (CL, ML, and SM) have been obtained using the pressure plate extractor tests (Fig. 2). The figure shows that the saturated volumetric water content θ_s for CL, ML and SM materials was 45%, 37%, and 35%, respectively. These data were fitted, with the fitting parameters *a, n,* and *m* tabulated in Table 1, using the functions proposed by van Genuchten (1980) with residual volumetric water content θ_r=0:

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{[1+(a*\psi)^n]^m} \quad (8)$$

and Leong and Rahardjo (1997):

$$\theta = \frac{\theta_s}{\left\{ln\left[exp(1)+\left(\frac{\psi}{a}\right)^n\right]\right\}^m} \quad (9)$$

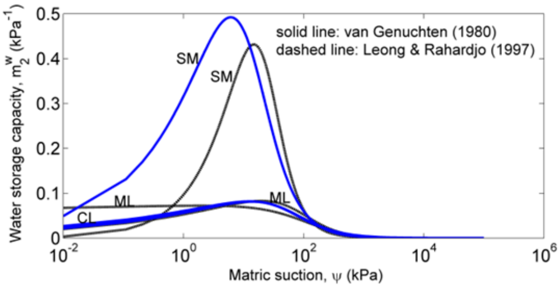


Fig. 4 Estimated water storage functions.

Hydraulic Conductivity

For hydraulic conductivity of unsaturated soil k_w, the difficult task of measuring this function directly can be overcome by estimating the function from either a measured or predicted SWCC function. It is unnecessary to specify the precise value of hydraulic conductivity when computing the distribution of pore-water pressure; however, if the quantity of flux is a concern, the function must be specified precisely (Thieu et al., 2001). For this study, two unsaturated hydraulic conductivity functions: van Genuchten (1980) and Leong and Rahardjo (1997) have been obtained and shown in Fig. 3 from the SWCC data shown in Fig. 2.

$$k_w = k_{sat}\left[\frac{[1-(a\psi)^{n-1}[1+(a\psi)^n]^{-m}]^2}{[1+(a\psi)^n]^{\frac{m}{2}}}\right] \quad (10)$$

$$k_w = \frac{k_{sat}}{\left\{ln\left[exp(1)+\left(\frac{\psi}{a}\right)^n\right]\right\}^m} \quad (11)$$

The laboratory measured saturated hydraulic conductivity k_{sat} for the SM, ML, and CL materials was 7.00E-6 m/s, 2.45E-8 m/s and 1.29E-10 m/s, respectively. The corresponding fitting parameters *a, n* and *m* used in the two functions have been shown in Table 1.

Water Storage

The water storage function m_2^w, as described earlier, is the gradient of the SWCC. Hence, after differentiating the SWCC functions of van Genuchten (1980) and Leong and Rahardjo (1997), respectively, we obtained the following two water storage functions:

$$m_2^w = \frac{d\theta}{d\psi} = \frac{-m*n*\theta_s*(a*\psi)^n}{\psi*[1+(a*\psi)^n]^{1+m}} \quad (12)$$

$$m_2^w = \frac{d\theta}{d\psi} = \frac{-m*n*\theta_s*\left(\frac{\psi}{a}\right)^{n-1}}{a*ln\left[\left(\frac{\psi}{a}\right)^n+exp(1)\right]^{m+1}*\left[\left(\frac{\psi}{a}\right)^n+exp(1)\right]} \quad (13)$$

The relationship between m_2^w and matric suction is shown in Fig. 4. The relatively sharp peak for the SM materials reflects that it has a relatively narrow pore size distribution (Lu and Likos, 2004).

4.3 Environmental Parameters

Rising water-table

From the record collected by Water Resources Agency it was observed that there was a continuous rise of upstream water-table between 21 May and 21 June, 2008 (Fig. 5a). The rise of the water-table was due to rainfall. Fig. The upstream water-table rose 7.09 m from EL. 96.54 m on 21 May to EL. 103.63 m on 9 June. The average lowest and highest water-table levels were determined at EL. 96.54 m and EL. 104.66 m.

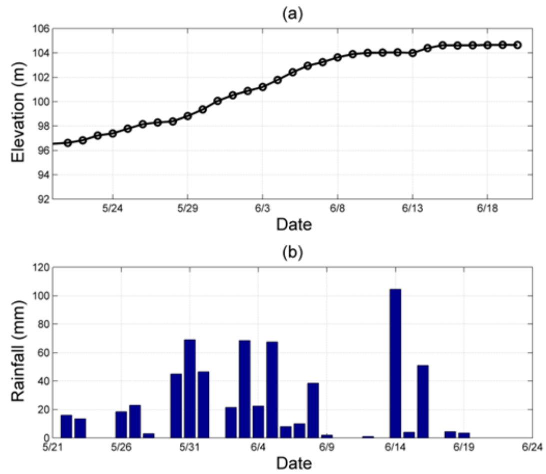


Fig. 5 (a) Water-table; and (b) rainfall between May 21 and June 21, 2008.

Rainfall

Taiwan faces high intensity rainfall between May and October every year, and the stability of dams could be compromised as a result of rainfall infiltration because the effective stress and, hence, the strength of the dam will decrease due to the increase of pore-water pressure caused by excessive rainfall. The daily rainfall data between 21 May and 21 June, 2008 is presented in Fig. 5(b).

4.4 Boundary Conditions

The earth fill dam considered is 28 m high and incorporates a filter under the downstream slope of the dam. The transient seepage problem requires that initial conditions of head be specified. To represent the low storage condition water-table level at 96.54 m, a head of 16.54 m was defined on the upstream face of the dam. The initial condition of the dam was obtained by first solving a steady state run of the problem with the 16.54 m head on the upstream face and a head of 0 m on the lower portion of the filter. All other boundaries were set to zero flux condition. The results from the steady state analysis were then used as the initial condition for the transient analysis.

5. RESULTS AND DISCUSSION

The above seepage analysis results are evaluated and plotted here in terms of pore-water pressure distribution.

5.1 Steady State: Initial condition

Fig. 6 shows the distribution of steady-state pore-water pressure distribution across the dam. The contour that corresponds to zero pressure (line 'd') represents the phreatic line in the dam. Soil that lies below this line is saturated with positive pore-water pressure and soil that lies above this line is unsaturated with negative pore-water pressure. At the crest, the value of the matric suction was 196 kPa.

In practice, dam design required the water to exit the dam before the toe of the downstream slope. If the water-table had extended to the toe of the dam, there would be concern that the toe of the dam would become unstable due to piping failure. As expected, with a filtering layer installed at the base of the downstream slope it is obvious that the pore-water pressure was unable to accumulate in the downstream semi-permeable zone (Fig. 6). In this zone, most of the pressure dissipated along the filter of the dam with zero pore-water pressure as shown by contour "d" in Fig. 6. Above the filtering layer, negative pore-water pressures (matric suction) exist. In general, matric suction would increase shear strength and, thus, the stability of the downstream slope in this case. In addition, the existence of the matric suction in the downstream slope would also prevent the occurrence of piping at the toe of the downstream slope.

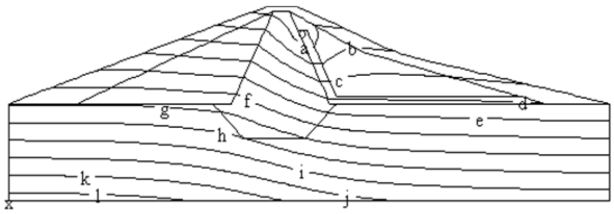
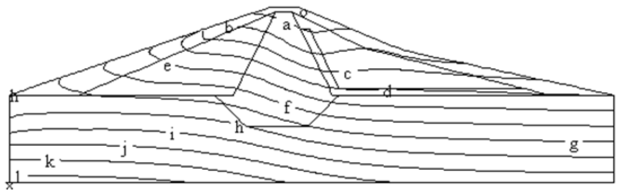


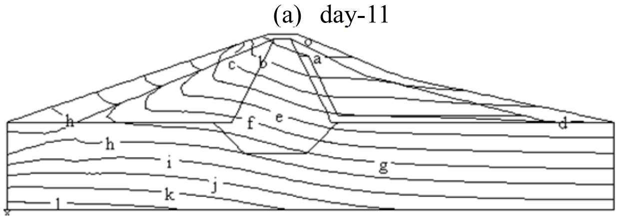
Fig. 6 Contour of pore-water pressure (kPa) from steady-state analysis with water-table at EL. 96.54 m [contour interval: 50 kPa; e.g. c=-50 kPa, d=0 kPa; e=50 kPa].

5.2 Transient State: Rising water-table

The process of filling up a reservoir is a transient process. The upstream water-table rose 7.09 m during a nineteen-day period between 21 May and 9 June (Fig. 5a). This is equal to an average rise of 0.373 m per day. Using the daily rise shown in Fig. 5(a) and setting the initial water-table at EL. 96.54 m, a 31-day seepage analysis has been carried out for this dam.



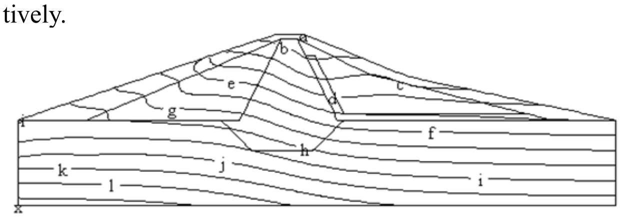
(a) day-11



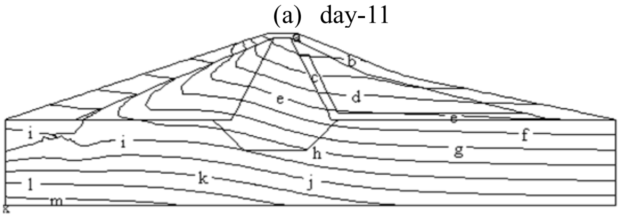
(b) day-31

Fig. 7 van Genuchten (1980)'s function: change of pore-water pressure due to rising water-table [contour interval=50 kPa; a=-150 kPa; d=0; l=400 kPa].

The distributions of the pore-water pressure in the dam, obtained through the functions of van Genuchten (1980) and Leong and Rahardjo (1997), for day-11 and day-31 are shown in Fig. 7 and 8, respectively. In all cases, the maximum pore-pressure of 426 kPa (11-day) and 428 kPa (31-day), compared to 424 kPa in the case of steady-state condition, occurred at the base of the upstream slope of the dam. No significant variation of pore-water pressure distribution is seen due to the 7.09 m rise of water-table. Using both the van Genuchten (Fig. 7) and Leong and Rahardjo (Fig. 8) functions: at day-11, the water flows mainly through the shell of the dam, which is shown by the curved pore-water pressure contour; at day-31, the water reached the semi-impermeable zone. The hydraulic conductivity of the core zone is about two orders lower than the hydraulic conductivity of the semi-permeable zones. This has in turn resulted in the slow movement of water in the core zone. The difference between the two functions was the value of the matric suction near the crest of the dam. van Genuchten's function gave a matric suction value of 192 and 191 kPa, respectively, for day 11 and 31; while Leong and Rahardjo gave a matric suction value of 201 and 195, respectively.



(a) day-11



(b) day-31

Fig. 8 Leong and Rahardjo (1997)'s function: change of pore-water pressure due to rising water-table [contour interval=50 kPa; a=-200 kPa; e=0; l=350 kPa].

5.3 Transient State: Rainfall infiltration

By setting the upstream water-table at EL. 96.54 m and using the daily rainfall pattern shown in Fig. 5(b), the distribution of pore-water pressure of the dam due to rainfall infiltration could be obtained. For both functions, except near the dam crest, no significant variation in pore-water pressure distribution was observed in the dam between day-11 and day-31 results. This was due to the low infiltration capability of the dam. There was a reduction of matric suction at the crest of the dam after a period of continuous rainfall. The magnitude of reduction was clearly seen in the van Genuchten's function while it was not so obvious in the Leong and Rahardjo's function. van Genuchten's function shows that the value decreased by about 27 kPa between day-1 and day-31.

CONCLUSIONS

Seepage analysis using two different water storage functions for a saturated-unsaturated earth fill dam has been performed and compared. Effects of rising water-table and rainfall were shown by the change of pore-water pressure distribution at the crest and shell regions of the dam. Preliminary result revealed that analysis using the water storage function derived by differentiating the van Genuchten's SWCC function is more sensitive in capturing changes of matric suction than the Leong and Rahardjo's function.

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