PROBABILISTIC PERFORMANCE RELIABILITY-COST TRADEOFF FOR MAINTENANCE STRATEGY

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ABSTRACT: Maintenance strategies are typically implemented by optimizing only the cost whilst the reliability of facility performance is neglected. This study proposes a novel algorithm using multi-objective particle swarm optimization (MOPSO) technique to evaluate the cost-reliability tradeoff in a flexible maintenance strategy based on non-dominant solutions. Moreover, a probabilistic model for regression parameters is employed to assess reliability-based performance. A numerical example of a highway pavement project is illustrated to demonstrate the efficacy of the proposed MOPSO algorithms. The analytical results show that the proposed approach can help decision makers to optimize roadway maintenance plans.

Keywords: Cost Management, Maintenance Strategy, Performance Reliability, Monte-Carlo Simulation, Multi-objective Optimization, Particle Swarm Algorithm

1. INTRODUCTION

To improve pavement quality, regular maintenance is essential. Unfortunately, allocation of resources for rehabilitating highway pavement is often inadequate for their heavy use. Therefore, maintenance activity must be timely and effective.

The literature on pavement maintenance tends to focus on only one objective, optimizing maintenance cost, while enforcing permissible limits on performance criteria so as to maintain pavement safety and serviceability. Notably, many criteria involve tradeoffs. Therefore, the decision making process involved in scheduling pavement maintenance requires consideration of multiple objectives, which are often incompatible.

Reliability is a measure of the probability of a pavement performing its intended function under a given set of conditions. Predicting the reliability of pavement performance requires consideration of many variables, including traffic load, degradation from environmental exposure, and pavement material properties. Nevertheless, most of the proposed methods of optimizing pavement maintenance planning are only based on deterministic reliability prediction models.

This study therefore proposes a multi-objective particle swarm optimization (PSO) method for pavement maintenance planning, which simultaneously considers maintenance cost and performance reliability throughout the pavement service life. A performance reliability model integrated with Monte Carlo simulation is also introduced to account for uncertain input parameters and the effect of maintenance action on pavement system reliability.

2. LITERATURE REVIEW

Neves et al. (2005) proposed an analytical model for comparing projected costs and improvements in safety and condition among several options for maintaining a deteriorating bridge [11]. The proposed models could account for differences in the course of deterioration associated with different maintenance schedules. Likewise Garbatov et al. (2001) optimized strategies for maintaining floating structures considering reliability and maintenance cost. The maintenance cost was then compared with those associated with various reliability-based strategies [5]. Another alternative proposed by Lapa et al. (2006) was to use a genetics algorithm to optimize the preventive maintenance planning for a nuclear power plant [10].
Nevertheless, studies on the reliability-cost trade-off in pavement engineering are sparse. Hong and Prozzi (2006) predicted pavement performance using Bayesian network. Their model incorporated the three dominant factors in pavement performance, i.e., structural properties, environmental effects, and traffic load [6]. Zhang (2009) proposed an analytical method for estimating the pavement performance of North-West China roadways by using linear regression technique [22]. The performance measure was based on collected data and expert evaluations. A reliability-cost optimization model for optimizing the design thickness of the asphalt layer of a pavement was developed by Sanchez-Silva et al. (2005) [14]. Although many studies have proposed pavement performance measures, the above works focused on optimization only at the design stage and not during the equally important maintenance planning stage.

3. RELIABILITY MODEL

3.1 Overview of Reliability Models

Two subject states, failure and non-failure (survival), are considered in reliability models. Notably, reliability models are only effective for forecasting pavement performance if the failure and non-failure modes are clearly defined. The reliability of pavement structures depends on the reliability of various layers in the pavement. Therefore, reliability of a pavement structure is calculated using a transfer and limit-state function \( G(x,t) \), which is the difference between the allowable and the demand load application of a given pavement section. Mathematically, a limit-state function can be calculated by the following equation [1].

\[
G(x,t) = q(x) - N(x,t)
\]

where \( q(.) \) denotes the function of number of allowable load application, and \( N(.) \) is the function of demanded load application; \( x \) is the vector of basic design variables, and \( t \) is the time.

The cumulative demanded load application can be calculated by the following equation [2].

\[
N(t) = EASL_0 \cdot \left(1 + \frac{TGR}{1 - TGR}\right)
\]

where \( EASL_0 \) is the number of load applications at the initial year of pavement service life, and \( TGR \) is the annual traffic growth rate. If the limit state function is defined as above, the pavement failure at time \( t \) can be mathematically expressed by the following equation [16].

\[
F(t) = Pr\left[G(x,t) \leq 0\right]
\]

where \( F(t) = Pr[.] \) is the probability of an event, which can be calculated using the following equation.

\[
F(t) = \int_{G(x,t) \leq 0} f(x,t) dx
\]

where \( f(x,t) \) is the joint probability density function of basic random variables at time \( t \).

Finally, the overall reliability of pavement structure (\( R \)) can be expressed as follows [4]:

\[
R = 1 - \int_{G(x,t) \leq 0} f(x,t) dx \cdot N(t) dt
\]

where \( T \) is the service life of a pavement structure.

3.2 Failure Function

Functions are typically selected to obtain the best fit to fragile data, such as Weibull lifetime functions [12,19-21], exponential [21], and log-normal cumulative distribution function [7,9,13,15]. However, any given pavement structure has unique values for parameters of failure function. Consequently, the challenge is integrating the reliability model with the optimization process.

To solve this problem, this study applies the flexible pavement model proposed by Deshpande et al. (2010) [3]. The failure function is modeled by the following cumulative normal distribution function:

\[
F(t) = \Phi \left[ \frac{\log(N(t)) - \Psi(x)}{\zeta} \right]
\]

where \( \Phi[.] \) is the cumulative normal distribution function. Parameters \( \Psi \) and \( \zeta \) can be determined by fitting the lognormal distribution using historical data.

Further, to incorporate the design variables and distribution parameters, a polynomial function is used to model the relationship between design variables and distribution parameters.

\[
\Psi(x) = \Phi \cdot F(x)
\]

where \( \Phi \) is a vector of the regression parameters, \( F(x) \) (a column-vector having the same size with \( \Phi \) ) is the
regression variable, and \( \Psi(x) \) is the function of design variables.

When a maintenance action is performed, the pavement structure system changes. Thus the pavement failure function is recalculated to reflect the reconfigured pavement structure. The reconfiguration is mathematically expressed by the following equations:

\[
F(t) = \Phi \left[ \log(N(t)) - \Psi'(x) \right],
\]

(8)

\[
\Psi'(x) = \varphi \cdot F'(x),
\]

(9)

where \( \Psi'() \) and \( \zeta \) are the distribution parameters after the maintenance action is performed; \( x \) is the vector of the basic design variables at the time of the maintenance action; \( \varphi \) and \( F'() \) are post-maintenance regression parameters and functions of design variables, respectively.

3.3 Probabilistic Reliability Model

An effective maintenance process requires an accurate prediction of pavement performance. Several variables affect the reliability of the prediction model throughout the service life of the pavement. The three categories of these factors are [14] (1) uncertainties in traffic load and environmental condition, (2) uncertainties in material and structural behavior, and (3) uncertainties in the regression models. Most of the information related to materials, initial traffic data, and environmental condition can usually be determined while developing the maintenance program. Therefore, the primary uncertainty factors involving reliability of pavement rehabilitation are future traffic demand and the regression model.

This study proposes a probabilistic model of the reliability of flexible pavement so as to address the above issues by accounting for uncertainty in traffic growth and by including a failure function in the regression model. To model uncertainties, the regression parameters (\( \varphi \), \( \zeta \), \( \zeta' \)) and annual traffic growth rate (TGR) are considered random variables. Therefore, \( \Psi \), \( \Psi' \), \( F(t) \), \( F'(t) \) and \( R \) become random variables. This study calculates the distribution of the above random variables by Monte Carlo simulation.

A typical flexible pavement (Fig. 1) is used as an example of the proposed reliability model. The design variables (\( x \)) for each layer include thickness and elasticity modulus. A pavement structure is comprised of three components, the asphalt, the base, and the sub-base. The maintenance action is presumably to add a layer on top of the pavement structure. After the maintenance action is completed, the structure then has four layers. The pavement structure parameters before and after the maintenance action can be expressed by the following Eqs. 10 and 11, respectively.

\[
\Psi(h, h, h, E, E, E, E, E) = \varphi \cdot F \cdot h + \varphi \cdot E + \varphi \cdot E + \varphi \cdot E
\]

(10)

\[
\Psi(h, h, h, h, E, E, E, E, E) = \varphi \cdot F \cdot h + \varphi \cdot E + \varphi \cdot E + \varphi \cdot E + \varphi \cdot E + \varphi \cdot E
\]

(11)

Fig. 1 Typical section of a flexible pavement

The above parameters can be obtained by multiple regression analysis of historical field data. However, these parameters are inherent in the uncertainty factors due to the inaccuracy of proposed model. To account for uncertainties, the regression parameters are considered random variables. Since their distribution is assumedly normal, the mean and standard deviation of observed data are used as input parameters in a normal distribution given in [3].

The distribution of annual traffic growth rate is significantly influenced by the traffic condition of the region in which the project is located. The coefficient therefore fluctuates according to the regional condition that causes the uncertainty of this coefficient. Thus, modeling uncertainty in annual traffic growth rate requires a careful investigation. In this study, the details of this coefficient were derived by consulting civil engineering experts.

Pavement maintenance action can be classified as either preventive or essential. Preventive maintenance actions (e.g., painting, silane treatment, and cathodic protection) are defined as scheduled maintenance of functioning components. The justification for preventive maintenance action is to incur the present cost of preventive maintenance to minimize the cost of repairing damage in
the future. Essential maintenance actions are applied to components that have failed or are approaching failure. Since such components should be repaired or replaced as soon as possible, maintenance actions cannot be scheduled a priori. In this work, only overlay maintenance is considered essential since preventive action does not significantly affect the reliability of pavement structures. Without loss of generality, this study applied the approach suggested by Tsai et al. 2008 [17] for modeling the nonlinear accumulation of damage to the top asphalt layer under repetitive loading. Notably, this layer becomes the second layer after performing rehabilitative maintenance. The modulus of elasticity and the normalized modulus of elasticity relative to its initial value, which are typically used to quantify the fatigue damage in asphalt layers of a pavement, can be expressed as follows.

\[
E(N(t)) = E(N(t=0)) \cdot \exp(-\lambda(N(t))^\tau)
\]

where \( E \) is the modulus of elasticity of the asphalt layer under consideration, and \( \lambda \) and \( \tau \) are the scale and shape parameters, respectively. In the numerical illustration presented in the discussion below, these parameters are set to \( \lambda = 0.0024 \) and \( \tau = 0.4674 \) as suggested by Tsai et al. [17].

### 3.4 Monte Carlo simulation

Monte Carlo simulation (MCS) is a method of iteratively evaluating a deterministic model by using sets of random numbers as inputs. This method is often used when the model is complex, nonlinear, or involves several uncertain parameters. Because it involves repeated computation of random and pseudo-random numbers, MCS requires substantial computer processing power.

Assume \( g(X) \) is a function of random variable \( X = (X_1, X_2, ..., X_p) \), and \( f(X) \) is the probability density distribution function (PDF). The expectation of \( g(X) \) denoted by \( E[g(X)] \) can be calculated by the following equation:

\[
E[g(X)] = \int_{X \in M} g(X) \cdot f(X)d(X)
\]

where \( M \) is the space of \( X \). Unfortunately, the PDF is difficult to obtain in practice. Moreover, the integration is generally impossible to calculate by analytical method. The MCS is generally used to obtain a numerical solution via the above equation.

The underlying concept of MCS is to draw an independent and identically distributed (I.I.D) set of samples \( \{x(1), ..., x(M)\} \) from a target density function \( f(X) \) defined in a high-dimensional space \( M \). Based on the \( M \) samples, the following empirical point mass function can then be used to approximate the target PDF.

\[
f_M(X) = \frac{1}{M} \sum_{i=1}^{M} \delta_{x(i)}(X)
\]

where \( \delta_{x(i)} \) denotes the delta-Dirac mass located at \( x(i) \), and \( M \) is the number of realizations. The integrals obtained by Eq. 14 can then be approximated by finding the tractable sums \( E_s(g) \) that converge as follows.

\[
E_s(g) = \frac{1}{M} \sum_{i=1}^{M} g(x(i)) - E[g(X)] = \int_{X \in M} g(X) \cdot f(X)d(X)
\]

where \( \rightarrow \) denotes the convergence in distribution.

### 4. PROBLEM FORMULATION

The maintenance cost is calculated as follows.

\[
C = \sum_{i=1}^{M} C_i \cdot T_i
\]

where \( C_i \) is the maintenance cost \( i \), \( T_i \) is the time that the maintenance action is applied, and \( q \) is the discount interest rate.

The subsequent case study analyzed the example of a maintenance action in which maintenance was assumedly proportional to the thickness of the overlay layer. Therefore, Eq. 16 can be simplified as

\[
C = \frac{CU \cdot h_\ast \cdot T}{(1+q)^{T_\ast}}
\]

where \( CU \) is the unit cost of maintenance cost, and \( T_\ast, h_\ast \) are timing and thickness of overlay course, respectively. Optimizing the reliability-cost tradeoff requires allocation of maintenance efforts in order to minimize the maintenance cost needed to achieve acceptable reliability of the pavement structure. This problem can be concisely stated as follows.

**Objective function:**

- Maximizing the mean performance reliability \( R \) of the pavement
- Minimizing the maintenance cost \( C \)
Decision variables: Time when the maintenance action is applied ($T_i$) and the thickness of overlay layer ($h_i$) obtained by the maintenance action.

Constraints:
- $h_{\text{min}} \leq h_i \leq h_{\text{max}}$
- $0 < T_i \leq T$
- $T_i$ are integer

5. MULTI-OBJECTIVE PSO FOR OPTIMAL MAINTENANCE PLANNING

5.1 Basics of PSO
A major advance in swarm theory was the particle swarm algorithm, which was designed to mimic the social behavior of animals such as birds in flocks or fish in schools [8]. The initial population consisting of random individuals or particles flies in the search space to search for the optimal solution. Each particle is identified by its velocity and location. The velocity of each particle is determined by its local best ($p_{\text{best}}$), which is the best solution in its search history, and by global best ($g_{\text{best}}$), which is the best solution achieved by the entire swarm.

Suppose that the search space is D-dimension and that there are N particles in the swarm. Particle $i$ is located at position $X_i = (X_{i1}, X_{i2}, ..., X_{iD})$ and has velocity $V_i = (V_{i1}, V_{i2}, ..., V_{iD})$, where $i = 1, 2, ..., N$. For each particle $r$ and dimension $s$, the new velocity $V_{rs}$ and position $X_{rs}$ can be calculated by the following equations.

$$V_{rs}^{(i+1)} = w \cdot V_{rs}^{(i)} + c_1 \cdot \text{rand}_1 \cdot (p_{\text{best}}^{rs}(i) - X_{rs}^{(i)}) + c_2 \cdot \text{rand}_2 \cdot (g_{\text{best}}^{rs} - X_{rs}^{(i)})$$

$$X_{rs}^{(i+1)} = X_{rs}^{(i)} + V_{rs}^{(i+1)}$$

Inertial weight $w$ controls exploration and exploitation. A large $w$ value maintains a high particle velocity and thus prevents the particles from being confined to the local optima. A small $w$ value, however, keeps the particles moving at a low velocity, which enables them to exploit the same search area. The constant $c_1$ and $c_2$ are acceleration coefficients that determine whether particles tend to move closer to $p_{\text{best}}$ or $g_{\text{best}}$. The $\text{rand}_1$ and $\text{rand}_2$ are both independent random variables between zero and one.

5.2 Mapping of Decision Variable to Particles
Because overlay layer thickness ($h_i$) is a continuous variable, its value can be directly assigned to particle location. However, since the year ($T_i$) when the maintenance action is applied is discrete, particle locations must be discretized in the following steps: (1) calculate the distance from the particle location to the discrete points, (2) invert these distances, (3) evaluate the probability of selecting each discrete point, (4) generate a normally distributed random number as a probability, and (5) determine the discrete point.

5.3 Multi-objective PSO Algorithm
In the above mapping process, the pseudo code for multi-objective PSO is as follows.

- **Randomly initialize the first swarm**
- Initialize particle velocity
- Map the particle locations to decision variables
- Evaluate maintenance cost
- Simulate reliability
- Calculate the mean of reliability
- Initialize the first $p_{\text{best}}$ and $g_{\text{best}}$

WHILE the terminal condition has not been reached

For each particle DO

- Update the velocity
- Update the location
- Map the particle locations to decision variables
- Evaluate the maintenance cost
- Simulate the reliability
- Calculate the mean of reliability
- Update $p_{\text{best}}$
- Update archive
- Update $g_{\text{best}}$

Reset the archive to a non-dominant sort solution (Pareto front)

Increase the loop counter until the convergence of Pareto front

End WHILE

**Initialize first swarm randomly**: The location of each particle is generated with real numbers within the lower and upper bounds.
Initialize particle velocity: The initial particle velocity is randomly generated from the minimum and maximum values.

Evaluate the maintenance cost \( (C) \): The maintenance cost \( (C) \) is evaluated using Eq. 17.

Simulate the reliability \( (R) \): The reliability is simulated according to the procedure introduced in Section 3.

Calculate the mean of \( R \): Calculate the mean simulated reliability and use it as an objective function.

Update the velocity: Equation 18 is used to update particle velocity. To avoid vicious oscillation, velocity \( (V_r) \) is constrained within a specified bound.

\[
V_r^{*+1} = \frac{V_r^{*+1} + v_{\text{max}}}{V_r^{*+1} + v_{\text{max}}} \quad \text{if} \quad |V_r^{*+1} + v_{\text{max}}| > v_{\text{max}} \tag{20}
\]

where velocity bound \( v_{\text{max}} \) is generally lower than the search space.

Update the location: The location of each particle is calculated using Eq. 19. A particle that exceeds the search space is automatically dispatched to a feasible region. In addition to enhancing exploration capacity, the new particle location is randomly selected from a previous location to the boundary. The bouncing routine is expressed mathematically as follows.

\[
X^* + 1 = X^*_{\text{max}} - \text{rand} \cdot (X^*_{\text{max}} - X^*(i)) \quad \text{if} \quad X^*(i) > X^*_{\text{max}} \tag{21}
\]

\[
X^*(i+1) = X^*_{\text{min}} + \text{rand} \cdot (X^*(i) - X^*_{\text{min}}) \quad \text{if} \quad X^*(i+1) < X^*_{\text{min}} \tag{22}
\]

where \( X^*_{\text{min}}, X^*_{\text{max}} \) are the lower and upper bound of dimension \( s \), respectively, and \( \text{rand} \) is a random number uniformly distributed between zero and one.

Update pbest: The personal best of each particle is updated by replacing the current pbest by the current location if the current location dominates the current pbest.

Update archive: The archive is updated in two steps: (1) insert all feasible solutions into the archive to form a temporary archive; (2) eliminate the dominant archive in the template.

Update gbest: The global best is selected from the archive member that dominates the fewest feasible particles in the current iteration.

6. NUMERICAL EXAMPLE

6.1 Case Profile

The project described in V.M.O.T (2001) specification book was used as a case illustration in this study [18]. The number of ESAL at the initial year of service life was 189,292. The cost unit \( CU \) for the maintenance cost function was set to 20 units. The design service life for the pavement system was 15 years, and, according to the design specifications, only overlay maintenance action was applied during its service life. The lower bound and upper bound of overlay thickness were 5 cm (2 inch) and 12.5 cm (4.92 inch), respectively. These values were initialized by specification and real construction condition. The modulus of overlay \( (E_o) \), asphalt \( (E_a) \), and base \( (E_b) \) layers were 2757920 kPa (400,000 psi), 2757920 kPa (400,000 psi), and 137896 kPa (20,000 psi) respectively. The asphalt layer thickness \( (hbs) \) was 14 cm (5.51 inch).

The annual rate of growth in traffic is affected by several factors, including economic growth rate and regional condition. In this example, historical data were unavailable for annual TGR in the study region in which the pavement was located. Therefore, a panel of Vietnamese engineers was interviewed to estimate TGR. Annual traffic growth rate in the model was triangulated by using three parameters: min, mode, and max, which were 0.06, 0.07, and 0.08, respectively. In future works, annual traffic growth rate can be derived easily if traffic-related data are properly archived.

6.2 Optimization Results

The following PSO parameters were set in the trials and pilot study: (1) No. of particles in swarm: 20; (3) No. of simulation iterations: 1000, (4) initial weight: 0.5 (in accordance with Kenedy and Eberhat (1995), who suggested an initial weight \( w \) between zero and one [8]), (4) acceleration rates: \( c_1 = 2 \) and \( c_2 = 2 \); (5) velocity boundary \( (v^{\text{max}}) \): 1 and 4 for thickness of overlay and applied maintenance time, respectively. The proposed algorithm was coded in Matlab R2008 and run on a PC with a Windows 7 OS, a dual-core Pentium E7300 processor, and 2GB RAM.

Figure 2 illustrates an example of a Pareto front for a cost-reliability trade-off. A common problem during
analysis is choosing a particle solution from a Pareto front. If project managers (PM) are cost-sensitive, solution S₁ should be chosen because it optimizes reliability at the lowest cost. Nevertheless, if the pavement section in this example requires a high reliability regardless of cost, the solution S₈ should be selected because it gives the highest performance reliability. Moreover, the PM can also choose a solution by setting an expected cost level (e.g., 70 units) or reliability (e.g., 95%), which are indicated by vertical and horizontal dashed lines, respectively, in Fig. 2.

![Fig.2 Optimization results](image)

For example, if PM requires a maintenance strategy with a reliability level greater than 95%, solution S₄ is the best solution because it provides higher than expected reliability at the lowest cost, and vice versa when budget is the priority.

Further, one of the most effective ways to optimize the cost-reliability trade-off is to facilitate engineers in achieving a flexible solution when the expected level of objective (cost or reliability) varies. The Pareto front in Fig. 2 can be logically divided into two parts. Part I includes solutions that are less reliable than solution S₅, and Part II includes solutions that are more reliable than solution S₅. In Part I, reliability substantially increases as maintenance costs increase while in Part II, reliability slightly increases as maintenance costs increase. Specifically, in Part I, if engineers wish to increase the reliability level by more than 10% (from S₂ to S₃), the required investment is only 1.84 times the unit cost whereas, in the second part a 1% increase in reliability (from S₆ to S₇) requires a 30.29-fold increase in unit cost.

7. CONCLUSIONS AND FUTURE DIRECTIONS
This study optimized the maintenance planning for flexible pavement by using a probabilistic reliability-based approach integrated with a novel multi-objective PSO (MOPSO) technique. The maintenance cost and performance reliability of the pavement structure were considered simultaneously with the time of maintenance actions and the thickness of overlay in the optimization process. A probabilistic model for estimating the reliability of flexible pavement structure with and without maintenance action was proposed.

The cost-reliability trade-off results indicated that maintenance cost significantly and positively affects the reliability of pavement performance when reliability is lower than 97%. When reliability exceeds this percentage, the change in maintenance cost dramatically increases with only a negligible change in reliability. An approximate 1% increase in reliability requires an investment that is 0.184 times the unit cost whereas a reliability level greater than 97% requires an investment that is 30.29 times the unit cost (164.62 times the unit cost under previous conditions) to obtain a 1% increase in performance reliability. Future works can expand on this study by considering multiple maintenance actions concurrently.

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