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THE DYNAMICS OF ROBOT MANIPULATORS WITH WHEELED BASE

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Abstract

A closed-form Newton-Euler dynamical model of a robot manipulator is derived. The model generalizes previous models in two respects:

- It allows for multiple degrees of freedom per robotic joint
- It allows for arbitrary placement of a local coordinate frame for each robotic link.

The importance of such a model is that it enables a complete force analysis of the robot. Forces and torques at arbitrary directions (not necessarily along or about the principal axes of motion) may be computed.

The generalized dynamic model is then applied to study the dynamics of a robot manipulator mounted on a wheeled platform. A particular example is analyzed in detail. That is the case of non-actuated three wheel configuration, two fixed wheels at the back of the platform and one free-spinning wheels at the front. The analysis includes:

1. Development of conditions for the vehicle stability
2. Development of the kinematic and dynamic constraints under the assumption of rolling motion.
3. Study of the coupling effects between the moving manipulator and the wheels.

The example is generic in the sense that it allows the formulation of a general computation procedure that checks the constraints of the system and evaluates motion trajectories of the vehicle as induced by the moving robot manipulator.

I. Introduction

This paper deals with the dynamics of a robot manipulator mounted on a non-actuated wheeled base that moves on a horizontal plane. A framework for analyzing vehicle stability, coupling effects between the moving manipulator and the moving base and vehicle path planning is proposed. The main analysis tool is a closed-form Newton-Euler dynamic model of a robot manipulator. The model allows for arbitrary placement of local cartesian coordinate frames on the manipulator links and joints with multiple degrees of freedom.

The paper organization is as follows: Section II outlines the derivation of the generalized Newton-Euler dynamic model for a manipulator with a stationary base. In Section III, the coupling between the manipulator and its moving base is discussed. Sections IV-VII deal with a particular, though generic, example -- that of a base with two fixed back wheels and a free spinning front wheel that roll on a horizontal plane.

II. Closed Form Generalized Newton-Euler Dynamical Model of a Robot Manipulator with Stationary Base

The robot dynamical model provides the relationship between the forces and torques acting on the manipulator links and the resulting joint accelerations at given joint positions and velocities. Let $N+1$ be the number of robot links numbered from 0 for the robot base towards link N to which the end-effector is attached.

Four assumptions are commonly made in deriving the dynamical model of an open kinematic chain. Only robot manipulators that are open kinematic chains will be considered in this paper.

(A1) The robot base is stationary. A coordinate frame attached to the base link may be viwed as a Newtonian (inertial) system.

(A2) Robot links are rigid. Thus, the relative position of any two points in a link is fixed and does not change while the robot moves.

(A3) The position and velocity of a point mass in link i ($i \in \{0,N\}$) are kinematically independent of the position and velocity of any point mass in a link j, where $j \geq i$.

(A4) The overall required force (torque) at joint i, that connects link $i-1$ to link i, is the sum of all forces (torques) that act on all mass elements of links i, $i+1,\ldots,N$ (including the end-effector and payload).

Note, assumptions (A3), (A4) are not independent of assumption (A2).

The model derived herein includes two nonessential, however useful, generalizations with respect to existing models in the robotic literature. First, the robot joints are not necessarily restricted to a one-degree-of-freedom motion. This is important if one wants to take into account clearance effects and other nonactuated motions due to imperfect joint design and implementation. Second, no fixed convention is followed in placing the local coordinate frame in each robotic link. To facilitate the robot force analysis the coordinate frames are placed at any location where computations of all the components of the forces and torques are of interest. For many applications the Denavit-Hartenberg kinematic formulation in which origins of the local coordinate frames lie on the respective joints axes of motion [1] is adequate but some applications may require a different treatment. In [2] and [3] for instance these origins are placed at the center of mass of each link which offers a potential increase in the efficiency of the dynamic computation. In this paper though, computational efficiency is of no concern. For a stationary base rigid link manipulator the force and torque components that are not along the principal axes of motion are cancelled out by the internal reaction forces and torques. For wheeled-base manipulators however such "side" torques and forces have an important impact on the vehicle stability and motion. This is the focus of this paper. Another example in which side components of the forces and torques are of interest involves the study of deflection and critical loading of a flexible arm [4]. Obviously since all force and torque components are of interest a Newton-Euler modeling approach is selected.

The model derivation follows the lines of [2]-[3]. First, Newton's law is invoked for an arbitrary point mass on an arbitrary robot link. Second, the total force is found by integration over all mass elements. Finally, the total torque is computed.

Let ${}_0\underline{f}$ be the force, with respect to an inertial coordinate frame F_0 attached to the robot base, acting on an arbitrary mass element m at link $j<N$ of the robot located at a position ${}_0\underline{r}^j$ with respect to that coordinate frame. Then by assumption (A1):

$$ {}_0\underline{f} = m\,{}_0\ddot{\underline{r}}^j \tag{2.1}$$

where (refer to Figure 2.1)

$$ {}_0\underline{r}^j = {}_0\underline{d}^1 + {}_0A^1\,{}_1\underline{r}^j \tag{2.2}$$

${}_0\underline{d}^1$ is the position of frame F_1 (attached to link 1) origin with respect to F_0, and ${}_0A^1$ is the orientation matrix of F_1 with respect to F_0. Joint 1 may have in general multiple degrees of freedom. These and the geometry of the links and the joint are built into the model of ${}_0\underline{d}^1$ and ${}_0A^1$.

Let ${}_0\underline{u}^1$ and ${}_0\underline{a}^1$ be the linear velocity and acceleration respectively and ${}_0\underline{\omega}^1$ and ${}_0\dot{\underline{\omega}}^1$ be the angular velocity and acceleration respectively of F_1 with respect to F_0. Then:

$$ {}_0\dot{\underline{r}}^j = {}_0\underline{u}^1 + {}_0\underline{\omega}^1 \times {}_0\underline{r}^j + {}_0A^1\,{}_1\dot{\underline{r}}^j \tag{2.3}$$

$$ {}_0\ddot{\underline{r}}^j = {}_0\underline{a}^1 + {}_0\dot{\underline{\omega}}^1 \times {}_0\underline{r}^j + {}_0\underline{\omega}^1 \times ({}_0\underline{\omega}^1 \times {}_0\underline{r}^j) + 2\,{}_0\underline{\omega}^1 \times {}_0\underline{u}^1 + 2\,{}_0\underline{\omega}^1 \times {}_0A^1\,{}_1\dot{\underline{r}}^j + {}_0A^1\,{}_1\ddot{\underline{r}}^j \tag{2.4}$$

As in (2.3)-(2.4) one can now express ${}_k\dot{\underline{r}}^j$ and ${}_k\ddot{\underline{r}}^j$ in terms of ${}_{k+1}\dot{\underline{r}}^j$ and ${}_{k+1}\ddot{\underline{r}}^j$, wherever $k<j<N-1$. For $k=j-1$, by assumptions (A2) and (A3):

$$ {}_{j-1}\dot{\underline{r}}^j = {}_{j-1}\underline{u}^j + {}_{j-1}\underline{\omega}^j \times {}_{j-1}\underline{r}^j \tag{2.5}$$

$$ {}_{j-1}\ddot{\underline{r}}^j = {}_{j-1}\underline{a}^j + {}_{j-1}\dot{\underline{\omega}}^j \times {}_{j-1}\underline{r}^j + {}_{j-1}\underline{\omega}^j \times ({}_{j-1}\underline{\omega}^j \times {}_{j-1}\underline{r}^j) + 2\,{}_{j-1}\underline{\omega}^j \times {}_{j-1}\underline{u}^j \tag{2.6}$$

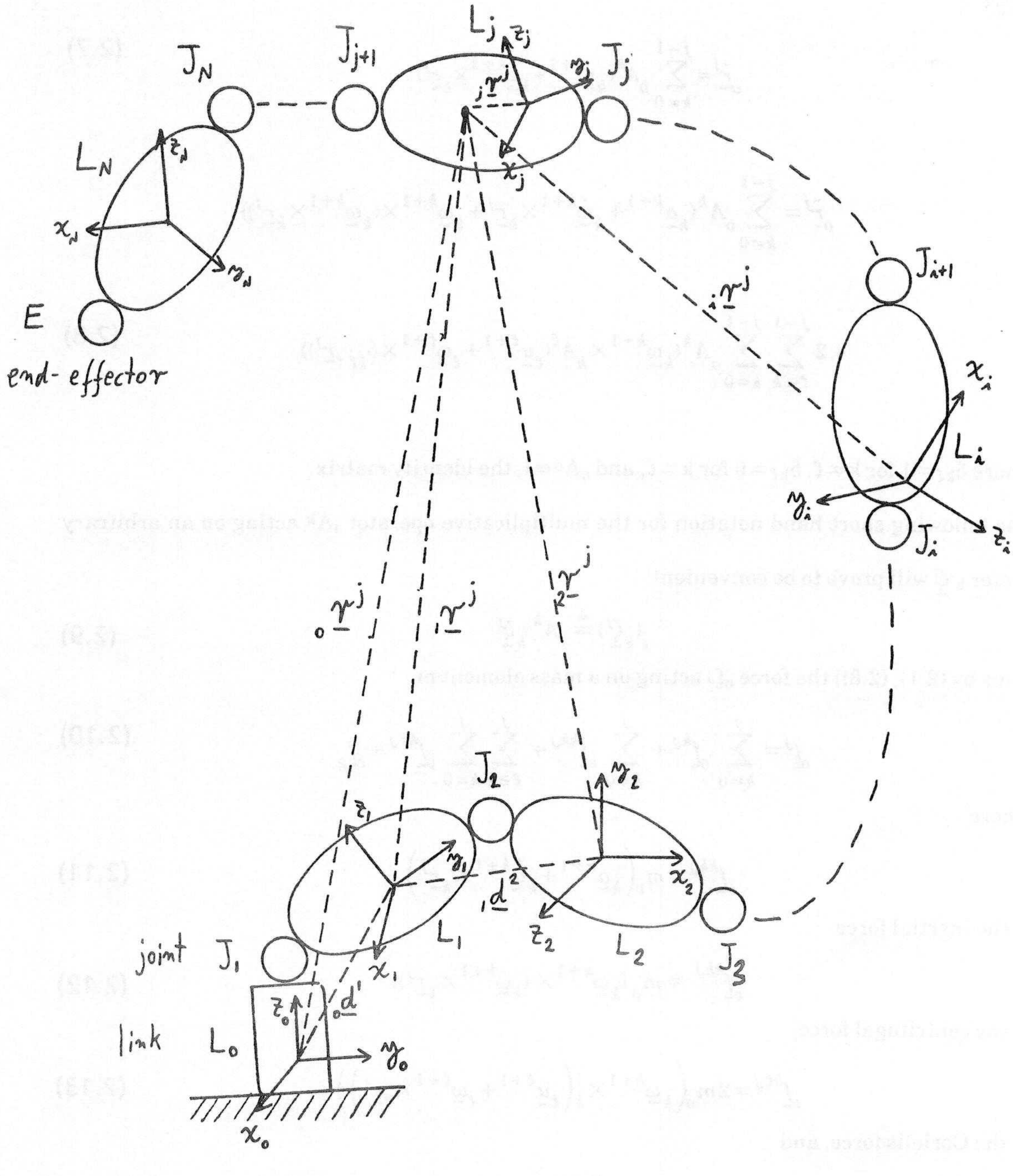

Figure 2.1. N-link Manipulator with Arbitrary Placement of Link Coordinate Frames.

Repeated substitution into (2.3)-(2.4) yields for $j \le N$:

$$ {}_0\dot{\underline{r}}^j = \sum_{k=0}^{j-1} {}_0A^k \left({}_k\underline{u}^{k+1} + {}_k\underline{\omega}^{k+1} \times {}_k\underline{r}^j \right) \tag{2.7} $$

$$ {}_0\ddot{\underline{r}}^j = \sum_{k=0}^{j-1} {}_0A^k \left({}_k\underline{a}^{k+1} + {}_k\dot{\underline{\omega}}^{k+1} \times {}_k\underline{r}^j + {}_k\underline{\omega}^{k+1} \times \left({}_k\underline{\omega}^{k+1} \times {}_k\underline{r}^j \right) \right) $$

$$ + 2 \sum_{\ell \ge k}^{j-1} \sum_{k=0}^{j-1} {}_0A^k \left({}_k\underline{\omega}^{k+1} \times {}_kA^\ell \left({}_\ell\underline{u}^{\ell+1} + {}_\ell\underline{\omega}^{\ell+1} \times \delta_{k\ell}\, {}_\ell\underline{r}^j \right) \right) \tag{2.8} $$

where $\delta_{k\ell} = 1$ for $k \neq \ell$, $\delta_{k\ell} = 0$ for $k = \ell$, and ${}_0A^0 \equiv I$, the identity matrix.

The following short hand notation for the multiplicative operator ${}_iA^k$ acting on an arbitrary vector ${}_k\underline{\zeta}^j$ will prove to be convenient

$$ {}_i({}_k\underline{\zeta}^j) \triangleq {}_iA^k\, {}_k\underline{\zeta}^j \tag{2.9} $$

Thus by (2.1), (2.8)) the force ${}_0\underline{f}^j$ acting on a mass element m

$$ {}_0\underline{f}^j = \sum_{k=0}^{j} {}_0\underline{f}^{kj} + \sum_{k=0}^{j} {}_0\underline{f}^{kkj} + \sum_{\ell \ge k}^{j} \sum_{k=0}^{j} {}_0\underline{f}^{k\ell j} + {}_0\underline{f}_g \tag{2.10} $$

where

$$ {}_0\underline{f}^{kkj} = m\, {}_0\left({}_k\underline{a}^{k+1} + {}_k\dot{\underline{\omega}}^{k+1} \times {}_k\underline{r}^j \right) \tag{2.11} $$

is the inertial force

$$ {}_0\underline{f}^{kkj} = m\, {}_0\left({}_k\underline{\omega}^{k+1} \times \left({}_k\underline{\omega}^{k+1} \times {}_k\underline{r}^j \right) \right) \tag{2.12} $$

is the centrifugal force,

$$ {}_0\underline{f}^{k\ell j} = 2m\, {}_0\left({}_k\underline{\omega}^{k+1} \times {}_k\left({}_\ell\underline{u}^{\ell+1} + {}_\ell\underline{\omega}^{\ell+1} \times {}_\ell\underline{r}^j \right) \right) \tag{2.13} $$

is the Coriolis force, and

$$ {}_0\underline{f}_g = m\, {}_0\underline{g} \tag{2.14} $$

is the gravity force where ${}_0\underline{g}$ is the gravity acceleration.

Note the force transformation formula:

$$ {}_i\underline{f} = {}_iA^j\, {}_j\underline{f} \tag{2.15} $$

Then ${}_i\underline{f}$ may be found from (2.10) merely by replacing the subscript 0 by i.

By assumption (A4), the total required force at joint i is found by integrating (2.10) over all mass elements from joint i (denoted as J_i) towards the end effector (denoted as E).

Let M^j denote the mass of link j and ${}_k\underline{p}^j$ denote the center of mass of link j with respect to coordinate frame F_k. Then:

$$ {}_i\underline{F}^{(j)} = \sum_{k=0}^{N-1} {}_i\underline{F}^k + \sum_{k=0}^{N-1} {}_i\underline{F}^{kk} + \sum_{\ell \geq k}^{N-1} \sum_{k=0}^{N-1} {}_i\underline{F}^{k\ell} + {}_i\underline{F}_g \tag{2.16} $$

where ${}_i\underline{F}^{(j)}$ is the total force acting on link j with respect to frame F_j.

$$ {}_i\underline{F}^k = \int_{J_i}^{E} \left({}_k\underline{a}^{k+1} + {}_k\dot{\underline{\omega}}^{k+1} \times {}_k\underline{r}^j \right) dm^j = \sum_{j=max\{i,k\}}^{N-1} M^j{}_i\left({}_k\underline{a}^{k+1} + {}_k\dot{\underline{\omega}}^{k+1} \times {}_k\underline{p}^j \right) \tag{2.17} $$

is the total inertial force

$$ {}_i\underline{F}^{kk} = \int_{J_i}^{E} {}_i\left({}_k\underline{\omega}^{k+1} \times \left({}_k\underline{\omega}^{k+1} \times {}_k\underline{r}^j \right) \right) dm^j = \sum_{j=max\{i,k\}}^{N-1} M^j{}_i\left({}_k\underline{\omega}^{k+1} \times \left({}_k\underline{\omega}^{k+1} \times {}_k\underline{p}^j \right) \right) \tag{2.18} $$

is the total centrifugal force

$$ {}_i\underline{F}^{k\ell} = \sum_{j=max\{i,k,\ell\}}^{N-1} 2M^j{}_i\left({}_k\underline{\omega}^{k+1} \times \left({}_\ell\underline{u}^{\ell+1} + {}_\ell\underline{\omega}^{\ell+1} \times \delta_{k\ell}\,{}_\ell\underline{p}^j \right) \right) \tag{2.19} $$

is the total coriolis force, and

$$ {}_i\underline{F}_g = \sum_{j=i}^{N-1} M^j{}_i\underline{g} \tag{2.20} $$

is the total gravity force. The end-effector and payload are taken as part of link N.

The "missing link" in the model (2.16)-(2.20) is the description of the shown positions, velocities and accelerations in terms of the <u>controlled joint positions, velocities and</u>

accelerations. This may not be a trivial task in a truly convention free kinematics, but in principle it can be done.

Let ${}_i\underline{f}$ be the force acting on an arbitrary mass element, and ${}_i\underline{r}$ the position of that mass element, all with respect to coordinate frame F_i. Then the torque acting on this mass element is:

$$ {}_i\underline{n} = {}_i\underline{r} \times {}_i\underline{f} \tag{2.21} $$

${}_i\underline{f}$ is given by (2.10). Again by integrating over all mass elements from link j towards the end effector, the total torque acting on link j can be found:

$$ {}_i\underline{N}^{(j)} = \sum_{k=0}^{N-1} {}_i\underline{N}^{k} + \sum_{k=0}^{N-1} {}_i\underline{N}^{kk} + \sum_{\ell \geq k}^{N-1} \sum_{k=0}^{N-1} {}_i\underline{N}^{k\ell} + {}_i\underline{N}_g \tag{2.22} $$

where

$$ {}_i\underline{N}^{k} = \sum_{j=max\{i,k\}}^{N-1} \int_{link\,j} {}_i\underline{r}^j \times {}_i\left({}_k\underline{a}^{k+1} + {}_k\underline{\dot{\omega}}^{k+1} \times {}_k\underline{r}^j \right) dm^j \tag{2.23} $$

$$ {}_i\underline{N}^{kk} = \sum_{j=max\{i,k\}}^{N-1} \int_{link\,j} {}_i\underline{r}^j \times {}_i\left({}_k\underline{\omega}^{k+1} \times \left({}_k\underline{\omega}^{k+1} \times {}_k\underline{r}^j \right) \right) dm^j \tag{2.24} $$

$$ {}_i\underline{N}^{k\ell} = \sum_{j=max\{i,k,j\}}^{N-1} 2 \int_{link\,j} {}_i\underline{r}^j \times {}_i\left({}_k\underline{\omega}^{k+1} \times {}_k\left({}_\ell\underline{u}^{\ell+1} + {}_\ell\underline{\omega}^{\ell+1} \times \delta_{k\ell\ell}\underline{r}^j \right) \right) dm^j \tag{2.25} $$

$$ {}_i\underline{N}_g = \sum_{j=i}^{N-1} M^j \, {}_i\underline{p}^j \times {}_i\underline{g} \tag{2.26} $$

are the inertial, centrifugal, Coriolis and gravity torques respectively.

Equations (2.23)-(2.25) may further be processed along the lines shown below. As an example the explicit evaluation of the inertial torque is shown. For a full treatment of the other dynamics terms see [4].

$$\int_{link\,j} {}_{i}\underline{r}^{j} \times {}_{i}\left({}_{k}\underline{a}^{k+1}\right) dm^{j} = M^{j}\, {}_{i}\underline{p}^{j} \times {}_{i}\left({}_{k}\underline{a}^{k+1}\right) \tag{2.27}$$

By (2.2):

$$\int_{link\,j} {}_{i}\underline{r}^{j} \times {}_{i}\left({}_{k}\underline{\dot{\omega}}^{k+1} \times {}_{k}\underline{r}^{j}\right) dm^{j} = M^{j}\, {}_{i}\underline{d}^{j} \times {}_{i}\left({}_{k}\underline{\dot{\omega}}^{k+1} \times {}_{k}\underline{d}^{j}\right) + M^{j}\, {}_{i}\underline{d}^{j} \times {}_{i}\left({}_{k}\underline{\dot{\omega}}^{k+1} \times {}_{k}A^{j}\, {}_{j}\underline{p}^{j}\right)$$

$$+ M^{j}\, {}_{i}A^{j}\, {}_{j}\underline{p}^{j} \times {}_{i}\left({}_{k}\underline{\dot{\omega}}^{k+1} \times {}_{k}\underline{d}^{j}\right) + {}_{i}A^{j} I^{j} \left({}_{i}A^{j}\right)^{T} \left({}_{i}A^{k}\, {}_{k}\underline{\dot{\omega}}^{k+1}\right) \tag{2.28}$$

where

$$I^{j} = \int_{link\,j} \left[tr({}_{j}\underline{r}\, {}_{j}\underline{r}^{T}) I - ({}_{j}\underline{r}\, {}_{j}\underline{r}^{T}) \right] dm^{j} \tag{2.29}$$

is the inertia matrix of link j including the actuators intertia as observed from F_i. I is the identity matrix.

The first term in (2.28) is due to a version of the parallel axis theorem. The second and third terms in (2.28) become zero if the origin of F_i is placed at the center of mass of link j.

3. Coupling Effects Between the Manipulator and the Moving Base

The dynamic model of the previous section becomes invalid if the robot base accelerates, however the modeling approach remains the same. One must start with an inertial system denoted as the "room frame" F_R. Thus the new modeling starting point that replaces equation (2.1) is:

$${}_{R}\underline{f} = m\, {}_{R}\underline{\ddot{r}}^{j} \tag{3.1}$$

Refer to Figure 3.1. Let F_B be a coordinate frame attached to the robot base. It is assumed that the robot is rigidly attached to its base. Thus frame F_o of the robot and frame F_B are related through a constant homogeneous transformation matrix ${}_BT^o$.

$$F_B = {}_BT^o F_o \tag{3.2}$$

In particular F_o and F_B may be selected in such a way that ${}_BT^o$ becomes a pure translation.

The entire modeling of the previous chapter may now be repeated starting with F_R, going through F_B, then F_o etc. In other words the two-degree of freedom moving base in the case of horizontal base motion is viewed as link B together with joint B of the robot. This modeling approach however does not allow much insight into the vehicle dynamic features.

In this section the coupling forces and torques between the moving base and the moving robot are studied. From the practical point of view it indeed makes sense to "mentally" separate the manipulator from the base (each may even have its own controller). In the case of a non-actuated wheeled base, the manipulator motion may cause the base to move, but the base motion in return affects the robot motion. It will be shown that this coupling can be resolved in a very simple algebraic manipulation.

Consider a particle dm on one of the manipulator links. Its total acceleration a_t with respect to the room frame is the sum of the base acceleration a_B and a_o, its acceleration under the assumption of a stationary base.

The total force $\underline{F}_t$ that acts on the base is:

$$\underline{F}_t = \int_{J_o}^{E} \underline{a}_t \, dm = \int_{J_o}^{E} \underline{a}_o \, dm + \int_{J_o}^{E} \underline{a}_B \, dm \tag{3.3}$$

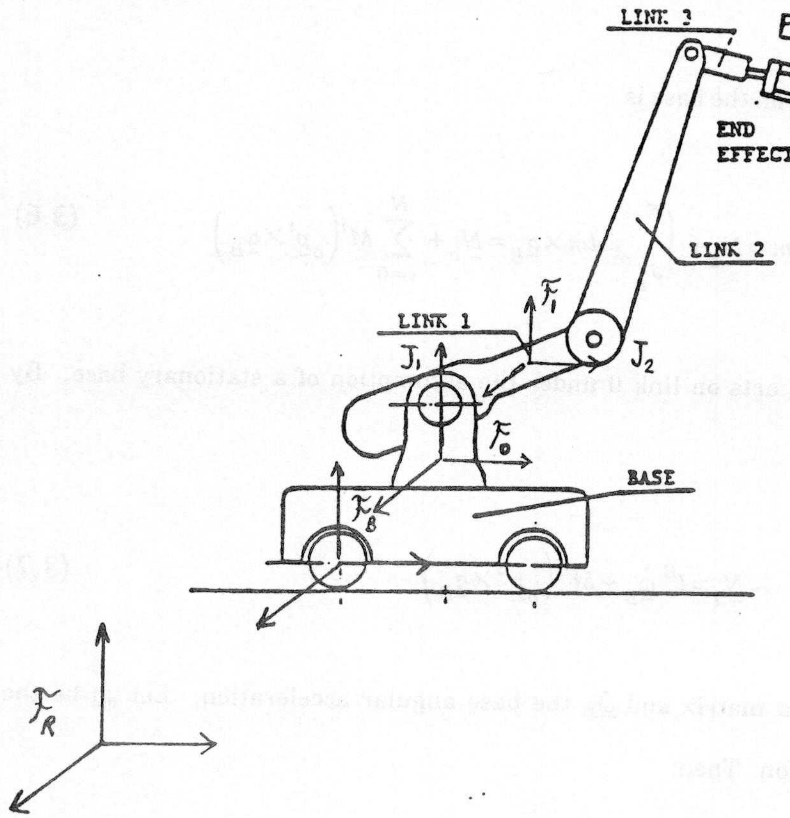

Figure 3.1. Robot Manipulator with Wheeled Base.

Let M_B be the base mass. The force that accelerates the base if it is not actuated is the reaction force $-\underline{F}_t$. Thus:

$$M_B \underline{a}_B = -\int_{J_o}^{E} \underline{a}_o \, dm - \int_{J_o}^{E} dm \, \underline{a}_B \tag{3.4}$$

Hence

$$M \underline{a}_B = -\underline{F}_o \tag{3.5}$$

where M is the total vehicle mass and $\underline{F}_o$ is the force acting on link 0 when the base is stationary!

The total torque acting on the base is

$$\underline{N}_t = \int_{J_o}^{E} {}_o\underline{r} \times \underline{a}_t \, dm = \underline{N}_o + \int_{J_o}^{E} {}_o\underline{r} \, dm \times \underline{a}_B = \underline{N}_o + \sum_{i=0}^{N} M^i \left({}_o\underline{p}^i \times \underline{a}_B \right) \tag{3.6}$$

where $\underline{N}_o$ is the torque that acts on link 0 under the assumption of a stationary base. By Euler's equation:

$$-\underline{N}_t = I^B \dot{\underline{\omega}}_B + M^B \left({}_o\underline{p}^B \times \underline{a}_B \right) \tag{3.7}$$

where I^B is the base inertia matrix and $\dot{\underline{\omega}}_B$ the base angular acceleration. Let ${}_o\underline{h}$ be the vehicle center of mass position. Then:

$$-\underline{N}_o = I^B \dot{\underline{\omega}}_B + M \, {}_o\underline{h} \times \underline{a}_B \tag{3.8}$$

${}_o\underline{h}$ is of course time varying.

Equations (3.5) and (3.8) illustrate the coupling effects between the base and the manipulator. These equations can be readily used for the analysis of the induced base motion as will be shown in the subsequent sections. The reader is cautioned though that $\underline{F}_0$ and $\underline{N}_0$ represent not the manipulator true motion but the idealized motion under the assumption of nonmoving base. Therefore $\underline{F}_0$ and $\underline{N}_0$ should be viewed as planned force and torque, and cannot be calculated on-line from measured manipulator joints, positions, velocities and accelerations.

Equations (3.5), (3.8), although vividly illustrating the coupling effects, cannot be used directly to determine a_B and ω_B as no friction effects have yet been taken into account. $\underline{F}_0$, $\underline{N}_0$ will appear as driving inputs in the base equations of motion. The resulting $\underline{a}_B$, $\dot{\underline{\omega}}_B$ may then be used to evaluate the precise manipulator motion.

IV. An Example: Wheeled Base with Two Fixed and One Free Spinning Wheels

From here on a particular though generic example will be analyzed. Consider a non-actuated wheeled robot base that moves on a horizontal plane. A particular wheel configuration is selected for this example. However, similar analysis can be carried for other configurations. Refer to Figure 4.1. It is assumed that the base back wheels numbered as wheels 2 and 3 are fixed, rigid and have the same axis of motion. There is a single front wheel (wheel 1) which is free spinning.

Let μ_r, μ_s be the dynamic rolling and sliding respectively friction coefficients between the wheels and the ground. It is assumed that

$$\mu_r < \mu_s \tag{4.1}$$

Similarly, let $\mu_r^{(s)}$, $\mu_s^{(s)}$ denote the static friction coefficients.

$$\mu_r < \mu_r^{(s)} \qquad \mu_s < \mu_s^{(s)} \tag{4.2}$$

Fixed wheels are restricted to roll in one direction but may slide in any direction.

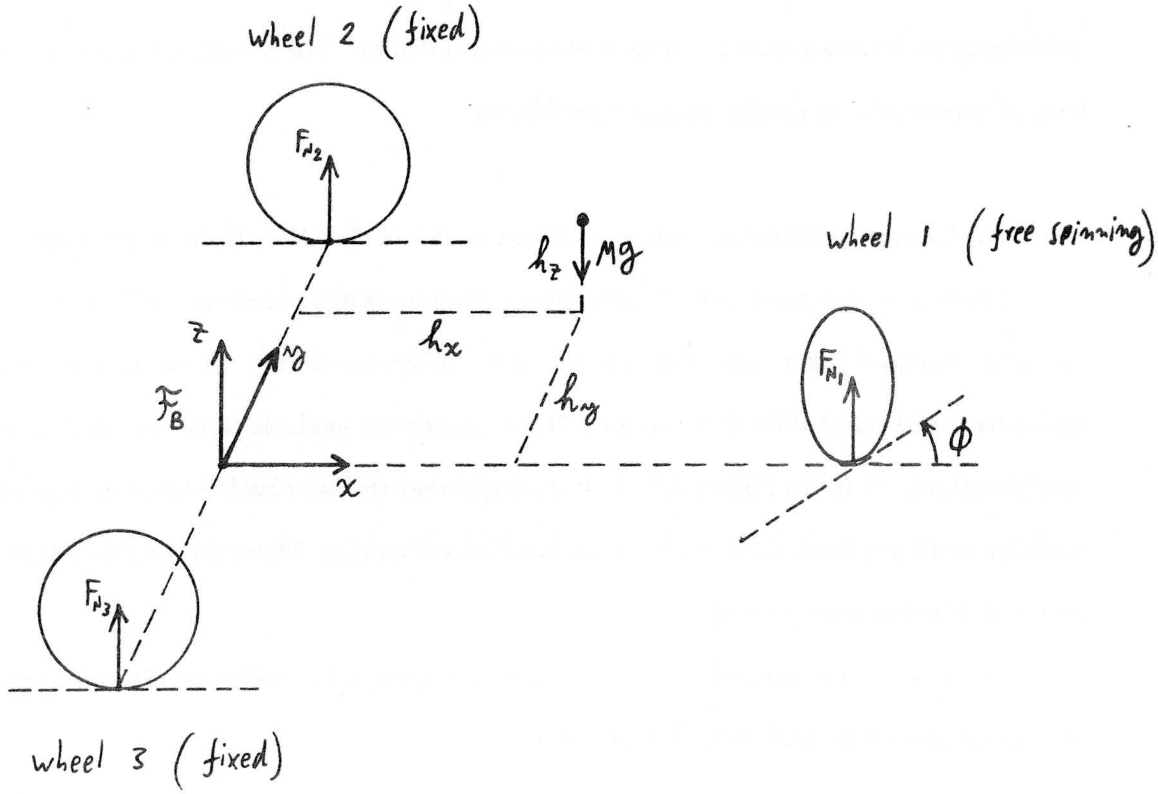

Figure 4.1. Geometry of the Wheeled Base.

Let v and a be the linear velocity and acceleration respectively of the robot base and let β be the angular velocity of the wheels. Let F be the applied force on the wheel and F_N be the ground reaction force. Then the equation of motion of the fixed wheels along the direction of rolling is:

$$v=a=0 \quad if \quad F<\mu_r^{(s)} F_N \tag{4.3}$$

$$v=R\omega \quad and \quad Ma=F-\mu_r F_N \quad if \quad \mu_r^{(s)} F_N \leq F<\mu_s^{(s)} F_N \tag{4.4}$$

$$Ma=F-\mu_s F_N \quad if \quad \mu_s^{(s)} F_N \leq F \tag{4.5}$$

where M is the vehicle mass and R is the radius of the wheel.

The equations of motion of the fixed wheels in the direction perpendicular to the direction of rolling are:

$$v_\perp=a_\perp=0 \quad if \quad F_\perp<\mu_s^{(s)} F \tag{4.6}$$

$$Ma_\perp=F_\perp-\mu_s F_N \quad if \quad F_\perp \geq \mu_s^{(s)} F_N \tag{4.7}$$

where $F_\perp$ is the applied force in the direction perpendicular to the rolling direction.

A free spinning wheel aligns itself at steady state with the direction of the total applied force. Thus:

$$\lim_{t\to\infty} v_\perp = \lim_{t\to\infty} a_\perp=0 \tag{4.8}$$

The equations of motion of a free spinning wheel along the direction of the applied force are similar to (4.3)-(4.5).

A convenient choice of a right-hand system F_B, the base coordinate frame, is now being taken. Its origin is placed on the ground as shown in Figure 4.1.

Let $\underline{F}$ and $\underline{N}$ be the applied force and torque with respect to F_B. $\underline{F}$ and $\underline{N}$ are found through force transformation of $-\underline{F}_0$ and $-\underline{N}_0$ (acting on F_0) that have been discussed in the previous section.

Let F_{N_1}, F_{N_2} and F_{N_3} be the normal reaction forces of the ground on each wheel.

Let ${}_B\underline{h} = (h_x, h_y, h_z)^T$ be the position of the system center of mass with respect to F_B. ${}_B\underline{h}$ varies as the manipulator moves.

The lengths s, d are as shown in Figure 6.1.

V. Stability Conditions

Necessary and sufficient condtions for the stability of the moving vehicle are:

$$F_z - Mg + F_{N_1} + F_{N_2} + F_{N_3} = 0 \tag{5.1}$$

$$N_x - Mgh_y + F_{N_2}d - F_{N_3}d = 0 \tag{5.2}$$

$$N_y + Mgh_x - F_{N_1}s = 0 \tag{5.3}$$

where

$$F_{N_1} \geq 0,\ F_{N_2} \geq 0,\ F_{N_3} \geq 0 \tag{5.4}$$

Combining (5.1)-(5.4), the following stability conditions are obtained:

$$h_x \leq -\frac{N_y}{Mg} \tag{5.5}$$

$$F_z + Mg\left(\frac{s-h_x}{s} - \frac{h_y}{d}\right) + \left(-\frac{N_y}{s} + \frac{N_x}{d}\right) \geq 0 \tag{5.6}$$

$$F_z + Mg\left(\frac{s-h_x}{s} + \frac{h_y}{d}\right) - \left(\frac{N_y}{s} + \frac{N_x}{d}\right) \geq 0 \tag{5.7}$$

Conditions (5.5)-(5.7) need to be evaluated for every planned manipulator motion.

VI. Kinematic Relationships and Dynamic Constraints on the Base Motion

Assuming that the manipulator motion is planned to never violate the vehicle stability requirements (5.5)-(5.7), another assumption is now being made regarding the type of base motion. In this paper only a pure roll motion will be discussed. Combination of sliding and rolling motion requires a considerably more complicated solution and is thus being avoided.

Let ϕ denote the angle by which the front wheel is steered from the rolling forward direction. If all wheels are precisely aligned, the vehicle at any given time traverses along a circular path (the circle parameters are of course time varying), with center O as shown in Figure 6.1.

Let ω_B, $\dot{\omega}_B$ denote the angular velocity and acceleration respectively of the vehicle about a normal to the surface axis of rotation that passes through O. Let $\beta_1, \beta_2, \beta_3$ be the angular speeds of each of the wheels respectively. Then for $\phi \neq 0$:

$$R_1\beta_1 = s\omega_B \csc\phi \tag{6.1}$$

$$R\beta_2 = (s\cot\phi - d)\,\omega_B \tag{6.2}$$

$$R\beta_3 = (s\cot\phi + d)\,\omega_B \tag{6.3}$$

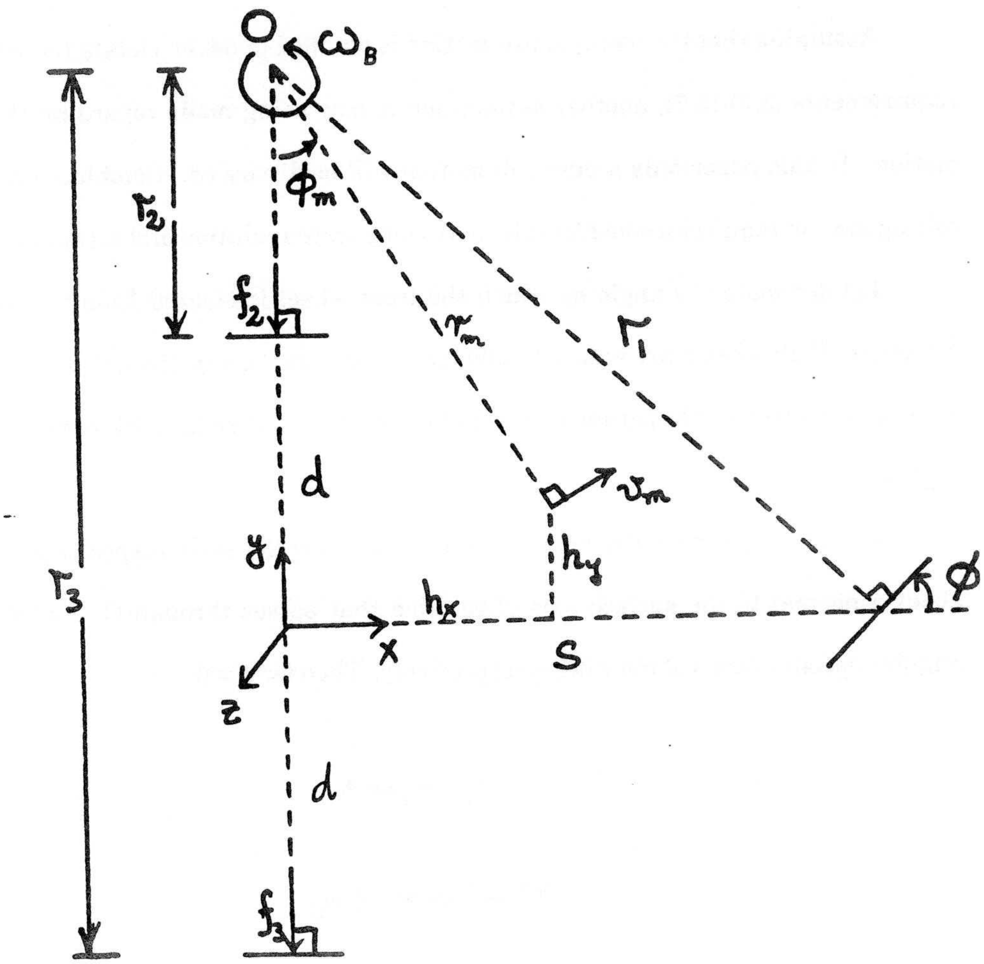

Figure 6.1. Kinematics of a Rolling Base.

where R_1 is the radius of the front wheel and R is the radius of each of the back wheels. Obviously, by (6.2)-(6.3) the back wheels must not be on the same shaft as any nonzero angular velocity ω_B requires that $\beta_2 \neq \beta_3$. The value of ϕ determines the ratio between β_2 and β_3. When $\phi=0$:

$$\omega_B=0 \qquad \beta_2=\beta_3 \qquad \beta_1=\beta_2\frac{R}{R_1} \tag{6.4}$$

Let r_m, ϕ_m be as shown in Figure 6.1:

$$r_m=\left(h_x^2+(s\cot\phi-h_y)^2\right)^{1/2} \tag{6.5}$$

$$\phi_m=\cot^{-1}\left((s\cot\phi-h_y)/h_x\right) \tag{6.6}$$

To find the kinematic constraint on ω_B note that ϕ, h_x and h_y are all time-varying. Thus

$$\omega_B=\dot{\phi}_m=\frac{sh_x\dot{\phi}+(s\cot\phi-h_y)\sin^2\phi\dot{h}_x+h_x\sin^2\phi\dot{h}_y}{r_m^2\sin^2\phi} \tag{6.7}$$

Let v_x, v_y be the components of the linear velocity of the base:

$$v_x=\omega_B r_m\cos\phi_m \qquad v_y=\omega_B r_m\sin\phi_m \tag{6.8}$$

Then the components of the linear acceleration of the base are:

$$a_x=\dot{v}_x=\dot{r}_m\cos\phi_m\omega_B-r_m\sin\phi_m\omega_B^2+r_m\cos\phi_m\dot{\omega}_B \tag{6.9}$$

$$a_y=\dot{v}_y=\dot{r}_m\sin\phi_m\omega_B+r_m\cos\phi_m\omega_B^2+r_m\sin\phi_m\dot{\omega}_B \tag{6.10}$$

Finally, the assumption of pure roll motion involves a dynamic constraint with respect to the friction forces. It is the friction force in the direction perpendicular to the roll direction that keeps the two back wheels from slipping. Let $f2_{\perp}$ and $f3_{\perp}$ be these perpendicular friction forces. Thus:

$$f_{2_\perp} \leq \mu_s^{(s)} F_{N_2} \qquad\qquad f_{3_\perp} \leq \mu_s^{(s)} F_{N_3} \tag{6.11}$$

For simplicity, is is assumed that the front wheel is weightless. Therefore it will react to external forces instantaneously. Thus the perpendicular friction force on the free spinning wheel is assumed to be identically zero.

VII. Base Equations of Motion

The base equations of motion under the assumption of pure roll motion are as follows:

$$M a_x = F_x - \mu_r F_{N_2} - \mu_r F_{N_3} - \mu_r F_{N_1} \cos\phi \tag{7.1}$$

$$M a_y = F_y - f_{2_\perp} - f_{3_\perp} - \mu_r F_{N_1} \sin\phi \tag{7.2}$$

Let Izz denote the moment of inertia of the system about the z-axis of F_B. Neglecting the cross inertia terms, the torque equation about the normal axis at 0 is :

$$M a_x (s \cot\phi - h_y) + M a_y h_x + (I_{zz} + M s^2 \cot^2\phi)\, \dot{\omega}_B \simeq N_z + F_x s \cot\phi$$

$$- \mu_r F_{N_2} (s \cot\phi - d) - \mu_r F_{N_3} (s \cot\phi + d) - \mu_r F_{N_1} s \csc\phi \tag{7.3}$$

Substituting F_{N_1}, F_{N_2}, F_{N_3} from (5.1)-(5.3) and a_x, a_y from (6.9) into (7.1) and (7.3) results in a system of two nonlinear coupled first order differential equations for the variables $\phi(t)$ and

$\omega_B(t)$. The inputs to the equations are the forces F_x, F_y, F_z and the torques N_x, N_y, N_z that are determined by the planned motion of the manipulator.

In addition the equations are parametrized by h_x, h_y and I_{zz} that depend on the actual manipulator changing configuration. Needless to say that only numerical solution of the equations is possible. The motion planning problem: "How should the manipulator be moved such that its wheeled base follows a prescribed planar trajectory" remains open.

The base will perform a spinning motion if at least one of the dynamic conditions for pure roll motion (6.11) is violated. In that case:

$$f_{2_\perp}\delta_2 \cos\Psi_2 + f_{3_\perp}\delta_3 \cos\Psi_3 + \mu_r F_{N_2}\delta_2 \sin\Psi_2 - \mu_r F_{N_3}\delta_3 \sin\Psi_3$$

$$-\mu_r F_{N_1}\delta_1 \cos\Psi_1 + (I_{zz} + M(h_x^2 + h_y^2))\dot{\omega}_s = F_x h_y - F_y h_x + N_z \tag{7.4}$$

where

$$\delta_1 \triangleq \left((s-h_x)^2 + h_y^2 \right)^{1/2} \qquad \Psi_1 \triangleq \Phi - tan^{-1} \frac{h_y}{s-h_x}$$

$$\delta_2 \triangleq \left(h_x^2 + (d-h_y)^2 \right)^{1/2} \qquad \Psi_2 \triangleq tan^{-1} \left(\frac{d-h_y}{h_x} \right) \tag{7.5}$$

$$\delta_3 \triangleq \left(h_x^2 + (d+h_y)^2 \right)^{1/2} \qquad \Psi_3 \triangleq tan^{-1} \left(\frac{d+h_y}{h_x} \right)$$

where $\dot{\omega}_s$ is the spinning acceleration. If no spinning occurs, then by substituting $\dot{\omega}_s = 0$ into (7.4), a constraint equation on $f_{2_\perp}$ and $f_{3_\perp}$ is introduced.

For an arbitrary manipulator motion the following is an algorithm for an on-line solution of the base motion given the system mass and the friction coefficients.

Step 1. Measurement phase Measure the manipulator joint positions, velocities and accelerations, the base angular velocity and the angle of the front wheel.

Step 2: Compute the location of the system center of mass (h_x, h_y) and the moment of inertia with respect to the base z-axis I_{zz}.

Step 3: Compute the force and torque acting on the robot 0 link. Transform the forces to F_B with reversal of the sign.

Step 4: Compute the reaction forces on the wheels. Check stability and verify rolling conditions.

Step 5: From equations (6.1),(6.3) predict new values of ϕ and ω_B. These may be used for feedback and feedforward control. Then update the value of the vehicle linear acceleration (a_x, a_y).

Step 6: Solve $f_{2_\perp}$ and $f_{3_\perp}$ from (7.2) and (7.4).

Step 7: Go to Step 1.

VIII. Summary

The motion planning of a moving-base robot manipulator is a complex issue due to the coupling effects between the moving base and the moving manipulator, the changing configuration of the vehicle and the variable friction effects. A framework for analysis has been proposed in this paper.

The importance of taking into account force and torque components not necessarily along the manipulator principal axes of motion has been demonstrated. This necessitated a slight generalization to the familiar closed-form Newton-Euler dynamic model for an open-kinematic chain manipulator.

Many open research problems exist, including the control of an actuated moving base for precise horizontal path tracking, the control of the manipulator motion in the presence of a moving base, sliding base motion and vehicle motion along non-planar surfaces.

Most of the paper evolved around a particular example. By no means should it be implied that the authors advocate this particular wheel configuration. The purpose of the example was merely to illustrate an approach to the dynamic modeling of mobile robots. Optimal vehicle design is a separate and exciting issue.

IX. References

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