POSITION MODELLING OF PLANAR TWO–LINK MANIPULATOR WITH ELECTROHYDRAULIC CONTROL SYSTEM

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Abstract: The paper deals with the problems of kinematic and dynamic modelling, simulation and experimental test of the planar two-link manipulator with the electrohydraulic control. The kinematic model was constructed on the basis of the kinematic relations and the dynamic model by means of bond graphs. On the test stand the dynamic characterisation and identification of the model mechanism was carried out and the accuracy of hydraulic cylinder positioning was investigated.

Keywords: planar manipulator, position modelling, electrohydraulic control system

1 INTRODUCTION

Multi-link manipulators are used in the construction of machinery, machine tools, elevators and industrial robots. Their main elements are links (arms), joints and power transmission systems. In the endpoint a device or effector performing specified tasks is placed. For the hydraulic drive the cylinder functions also as the mechanism's arm of the length dependent on the piston stroke [2], [4]. For the electrohydraulic control system to be applied the following elements should be selected: power pack, actuators (cylinder or motor), control valve and electronic control and meter system. In order to determine the new position of the endpoint the control valve must be reversed and the manipulator must be positioned. The accuracy of positioning is a effected by state parameters, control parameters, load, dynamic disturbances, rigidity of construction and clearance in kinematic pairs. The analysis of the manipulator is carried out on the basis of its kinematic and dynamic models. The kinematic model is applied in order to determine the trajectory of respective points and in particular to determine the position, velocity and acceleration of the endpoint. The dynamic model enables simulation and dynamic analysis of the manipulator in its transient states. The dynamic model of the manipulator is verified on the research stand. The experiments enable to select such control algorithm which ensures high accuracy of positioning and good dynamic properties of the manipulator's control and power transmission system. Since the conventional algorithms do not always ensure the high precision of position control the methods of prediction control, fuzzy logic or "dead beat" may be more effective. Increasing requirements concerning both the precision of positioning and the rate of manipulators' operation induce simulation and experimental research.

2. KINEMATIC MODEL

The kinematic modelling of a planar two-link manipulator has been carried out under the assumption that the dimensions of manipulator's links are precisely determined and the rigidity of their construction is high. The purpose of modelling and kinematic analysis is to determine the motion trajectory of the manipulator's characteristic points and in particular to establish the manipulator's endpoint. A planar, two-link manipulator with two degrees of freedom and electrohydraulic control has been modelled. Its functional diagram is presented on Fig.1. The characteristic dimensions of the mechanism are shown on kinematic diagram Fig.2. On the basis of the diagram the plane motion of the mechanism in the stationary co-ordinate system Axy has been considered. The link AC rotates around the point A according to the change of τ angle, and the link CD rotates around the point C. In point B the piston rod of hydraulic cylinder is connected with CD link. The length of AB link is changeable and depends on the length of stroke position of the piston rod. Hence, the kinematics of manipulator with CD link driven by the hydraulic cylinder is evaluated. On the kinematic diagram of the manipulator the input parameters are given: τ, l_{AC}, l_{CD}, S_{ABmin}, S_{ABmax}.

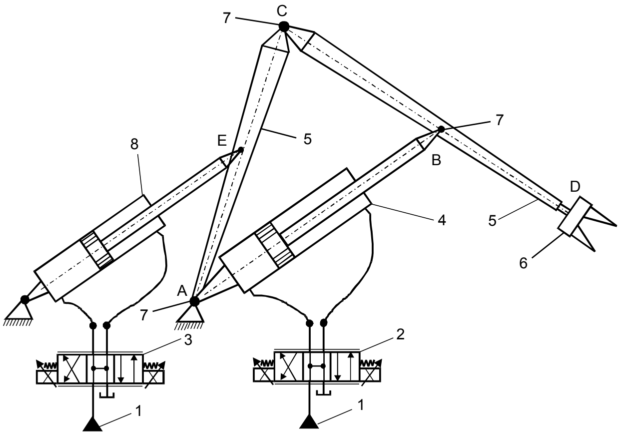


Figure 1. Diagram of planar two-link manipulator with electrohydraulic control: 1 – supply pressure, 2 – servo solenoid valves, 3 – servo solenoid valves, 4 – positioning hydraulic cylinder, 5 – links, 6 – effector, 7 – joints , 8 – positioning hydraulic cylinder

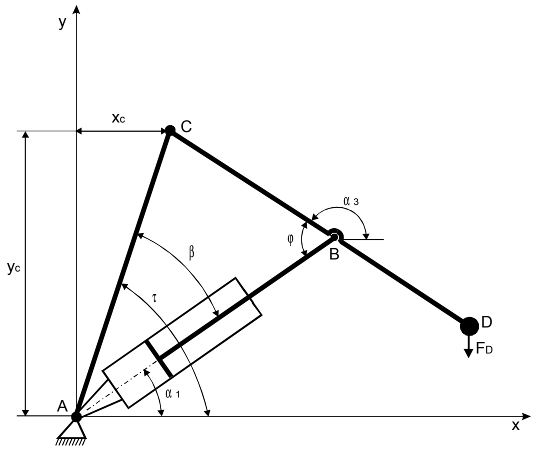


Figure 2. Kinematic diagram of two-link manipulator

From the aforesaid data it follows:

$$\Delta S = S_{AB\max} - S_{AB\min} \tag{1}$$

$$x_c = l_{AC}\cos\tau \tag{2}$$

$$y_c = l_{AC}\sin\tau \tag{3}$$

where: l_{AC} – length of AC link, τ – deflection angle of AC link, S_{ABmin} – minimum stroke of cylinder, S_{ABmax} – maximum stroke of cylinder.

Velocity $\dot{S}_{AB}$ of the cylinder piston is determined from the ratio of flow rate Q_1 or Q_2 to the piston area of cylinder A_1 or A_2 :

$$\begin{cases} \dot{S}_{AB} = \dfrac{Q_1}{A_1} & \textit{for lifting} \\ \dot{S}_{AB} = \dfrac{Q_2}{A_2} & \textit{for lowering} \end{cases} \tag{4}$$

and its acceleration $\ddot{S}_{AB}$ for a given time t of the piston rod advancement is calculated from the formula:

$$\ddot{S}_{AB} = \frac{d^2 S_{AB}}{dt^2} \tag{5}$$

where: A_1 – piston area, A_2 – piston area on rod end,

From the kinematic diagram on Fig.2 the angles α_1, α_3, β, φ have been determined:

$$\alpha_1 = \tau - \beta \tag{6}$$

$$\alpha_3 = \pi - (\varphi - \alpha_1) \tag{7}$$

$$\beta = arc\cos\left(\frac{S_{AB}^2 + l_{AC}^2 - l_{BC}^2}{2\, l_{AC}\, S_{AB}}\right) \tag{8}$$

$$\varphi = arc\cos\left(\frac{S_{AB}^2 + l_{BC}^2 - l_{AC}^2}{2\, l_{BC}\, S_{AB}}\right) \tag{9}$$

where: l_{BC}, l_{CD} – lengths of BC and CD links.

On the basis of the kinematic relations is obtained the equations for point B have been written. They describe its:

— position

$$\begin{aligned} S_{AB}\cos\alpha_1 + l_{BC}\cos\alpha_3 - x_c &= 0 \\ S_{AB}\sin\alpha_1 + l_{BC}\sin\alpha_3 - y_c &= 0 \end{aligned} \tag{10}$$

— velocity

$$\begin{aligned} \dot{S}_{AB}\cos\alpha_1 - S_{AB}\,\omega_1\sin\alpha_1 - l_{BC}\,\omega_3\sin\alpha_3 &= 0 \\ \dot{S}_{AB}\sin\alpha_1 + S_{AB}\,\omega_1\cos\alpha_1 + l_{BC}\,\omega_3\cos\alpha_3 &= 0 \end{aligned} \tag{11}$$

— acceleration

$$\begin{aligned} &\ddot{S}_{AB}\cos\alpha_1 - 2\,\dot{S}_{AB}\,\omega_1\sin\alpha_1 - S_{AB}\,\omega_1^2\cos\alpha_1 + \\ &- S_{AB}\,\varepsilon_1\sin\alpha_1 - l_{BC}\left(\omega_3^2\cos\alpha_3 + \varepsilon_3\sin\alpha_3\right) = 0 \\ &\ddot{S}_{AB}\sin\alpha_1 + 2\,\dot{S}_{AB}\,\omega_1\cos\alpha_1 - S_{AB}\,\omega_1^2\sin\alpha_1 + \\ &+ S_{AB}\,\varepsilon_1\cos\alpha_1 + l_{BC}\left(-\omega_3^2\sin\alpha_3 + \varepsilon_3\cos\alpha_3\right) = 0 \end{aligned} \tag{12}$$

From the equation (11) angular velocities have been calculated:

$$\omega_1 = \frac{d\alpha_1}{dt} = \frac{\dot{S}_{AB}(\sin\alpha_1\sin\alpha_3 + \cos\alpha_1\cos\alpha_3)}{S_{AB}(\sin\alpha_1\cos\alpha_3 - \cos\alpha_1\sin\alpha_3)} \tag{13}$$

$$\omega_3 = \frac{d\alpha_3}{dt} = \frac{\dot{S}_{AB}}{l_{BC}\left(\cos\alpha_1 \sin\alpha_3 - \sin\alpha_1 \cos\alpha_3\right)} \tag{14}$$

and from the equation (12) angular acceleration has been calculated:

$$\varepsilon_1 = \frac{d^2\alpha_1}{dt^2} = \frac{a\cos\alpha_3 + b\sin\alpha_3}{S_{AB}\left(\sin\alpha_1 \cos\alpha_3 - \cos\alpha_1 \sin\alpha_3\right)} \tag{15}$$

$$\varepsilon_3 = \frac{d^2\alpha_3}{dt^2} = \frac{a\cos\alpha_1 + b\sin\alpha_1}{l_{BC}\left(\cos\alpha_1 \sin\alpha_3 - \sin\alpha_1 \cos\alpha_3\right)} \tag{16}$$

where:

$$a = \ddot{S}_{AB}\cos\alpha_1 - 2\dot{S}_{AB}\,\omega_1 \sin\alpha_1 + \\ - S_{AB}\,\omega_1^2 \cos\alpha_1 - l_{BC}\,\omega_3^2 \sin\alpha_3$$

$$b = \ddot{S}_{AB}\sin\alpha_1 + 2\dot{S}_{AB}\,\omega_1 \cos\alpha_1 + \\ - S_{AB}\,\omega_1^2 \sin\alpha_1 - l_{BC}\,\omega_3^2 \sin\alpha_3$$

Basing on the equations (10)–(16) the equations for the endpoint D have been written. They describe its:

— co-ordinates

$$x_D = x_C + l_{CD}\cos(\pi - \alpha_3) \\ y_D = y_c - l_{CD}\sin(\pi - \alpha_3) \tag{17}$$

— radius

$$r_D = \sqrt{x_D^2 + y_D^2} \tag{18}$$

— velocity

$$\mathrm{v}_{Dx} = l_{CD}\,\omega_3 \sin\alpha_3 \\ \mathrm{v}_{Dy} = -\,l_{CD}\,\omega_3 \cos\alpha_3 \tag{19}$$

$$\mathrm{v}_D = \sqrt{(\mathrm{v}_{Dx})^2 + (\mathrm{v}_{Dy})^2} = l_{CD}\,\omega_3 \tag{20}$$

— acceleration

$$a_{Dx} = l_{CD}\left[\omega_3^2 \cos\alpha_3 + \varepsilon_3 \sin\alpha_3\right] \\ a_{Dy} = -\,l_{CD}\left[-\,\omega_3^2 \sin\alpha_3 + \varepsilon_3 \cos\alpha_3\right] \tag{21}$$

$$a_D = \sqrt{(a_{Dx})^2 + (a_{Dy})^2} \tag{22}$$

On the basis of the kinematic model of the planar, two-link manipulator with the electrohydraulic control the trajectories of the endpoint D have been determined by means of mathematical package MATLAB. The trajectories of $r_{D\,r}$ radius, v_D velocity, a_D acceleration of the endpoint D, obtained for parameters: $l_{AC} = 4$ m, $\tau = 70^o$, $\Delta S = 1{,}2$ m, $l_{CB} = 3$ m, $l_{CD} = 5$ m, $A_1 = 1{,}14\mathrm{x}10^{-3}$ m^2, $Q_1 = 0{,}33\mathrm{x}10^{-3}$ m^3/s, $t = 10$ s are presented on Fig.3.

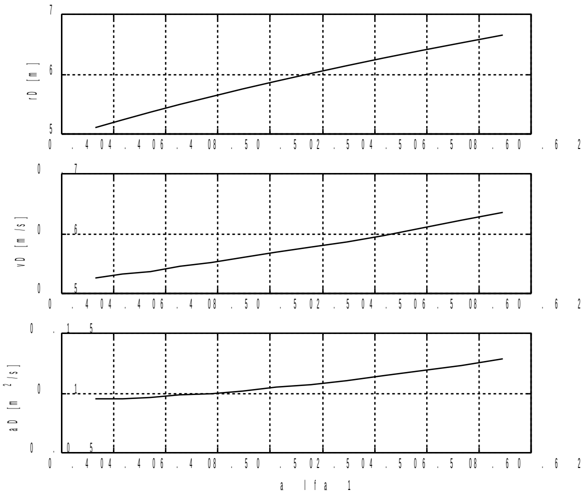


Figure 3. Trajectories of the endpoint D in the function of α_1 angle

3. DYNAMIC MODEL

Modelling of the dynamics of the planar two-link manipulator with the electrohydraulic control system is much simpler if the method of bond graphs. is used [3]. It should be stressed that the structure of bond graphs is the same as the dynamic structure of mechanical, hydraulic and electrical systems for which the energy flux is considered [2]. On the basis of the kinematic diagram of the analysed manipulator its dynamic model has been presented on Fig.4. The model has been worked out on the basis of the bond graph method which comprises the following typical elements: *C, I, R, TF, MTF, O, 1, SF, SE*. By means of bond graphs the following parameters for the electrohydraulic control manipulator have been determined: $SE_{p.}$, SE_o, SE_g, SE_q, R_1, R_2, R_3, R_4, R_{p1}, R_{p2}, R_t, R_s, R_b, C_1, C_2, $I_{s\ r}$, I_o, v_D, F_D. It should be stressed that the dynamic model of the proportional valve was presented on bond graphs in theform of a circle. This is other graphic representation of the hydraulic bridge circuit showing the distribution of flow rate in this valve effectively. From bond graphs on Fig.4 the following equations of the dynamic model for the particular elements of the manipulator can be written:

— for the proportional valve

$$Q_i = \frac{1}{\sqrt{R_i}}\sqrt{|p - p_i|}\,sign(p - p_i) \tag{23}$$

where hydraulic resistance R_i depends on μ, ρ and passage area *f(x)* in the function *x*:

$$R_i = \frac{\rho}{2\left[\mu\ f(x)\right]^2} \tag{24}$$

(in the proportional valve the travel displacement x of the spool is proportional to the control voltage u and the relation is presented on the flow characteristic),

— for pipe between valve and cylinder

$$p_1 = R_{p1} Q_1 \tag{25}$$

$$p_2 = R_{p2} Q_2 \tag{26}$$

— for cylinder

$$p_{s1} = p_{s1}(0) + \frac{1}{C_1}\int_{t_o}^{t}\left(Q_{s1} - \frac{(p_{s1} - p_{s2})}{R_t} - A_1 \dot{S}_{AB}\right) dt \tag{27}$$

$$p_{s2} = p_{s2}(0) + \frac{1}{C_2}\int_{t_o}^{t}\left(- Q_{s2} + \frac{(p_{s1} - p_{s2})}{R_t} + A_2 \dot{S}_{AB}\right) dt \tag{28}$$

$$\dot{S}_{AB} = \dot{S}_{AB}(0) + $$

$$+ \frac{1}{I_s}\int_{t_o}^{t}\left(A_1 p_{s1} - A_2 p_{s2} - R_s \dot{S}_{AB} - SE_q \, sign\, \dot{S}_{AB}\right) dt \tag{29}$$

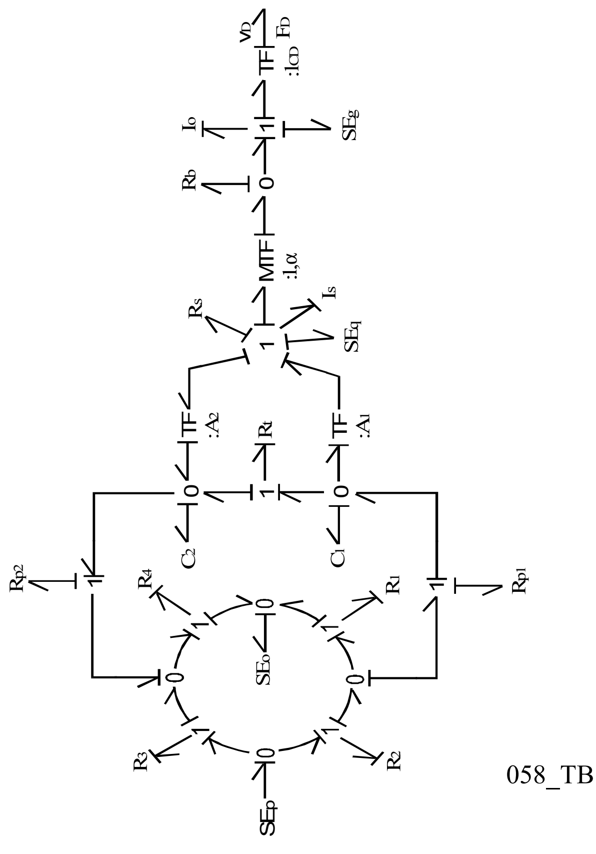
— for CD link

$$\omega_3 = \omega_3(0) + $$

$$+ \frac{1}{I_0}\int_{t_o}^{t}\left[-\left(A_1 p_{s1} - A_2 p_{s2}\right) l_{BC} - R_b(\omega_1 - \omega_3) + SE_g\right] dt \tag{30}$$

— for endpoint D

$$\mathrm{v}_D = \omega_3 \, l_{CD} \quad (31$$



$$x_D = x_D(0) + \int_{t_o}^{t} \mathrm{v}_D \, dt \tag{32}$$

where: C_1, C_2 – hydraulic capacity in the cylinder chambers; I_o – inflated moment of inertia of manipulator's link; I_{sr} – inflated mass on the cylinder piston; p_{s1}, p_{s2} – pressure in the hydraulic cylinder chambers, Q_1, Q_2 – flow rate, R_1, R_2, R_3, R_4 = hydraulic resistance connected with flow through gaps of the proportional valve; R_b – resistance characterising the losses of viscous friction in kinematics pairs; R_{p1}, R_{p2} – hydraulic resistance in the pipes between the hydraulic valve and cylinder; R_s – resistance characterising viscous friction in the cylinder; R_t – resistance characterising leaks losses in the cylinder, p – supply pressure; SE_g – moment resulting from terrestial gravity force and external load, SE_q – moment of Coulomb's friction; v_D – velocity in the endpoint D, x – spool displacement, x_c, y_c – coordinates of point C position, μ – discharge coefficient of valve, ρ – mass density of oil.

Fig.4. Bond graph of manipulator with electrohydraulic control

Basing upon the dynamic model presented on Fig.4 by means of bond graphs the calculations were carried out with the use of simulation package 20-sim [1]. The purpose of the simulation was to determine the dynamic characteristics of the manipulator after the proportional valve has been recontrolled for the voltage supplied $u(t)$. The following time characteristics were obtained: pressure $p_1(t)$ i $p_2(t)$ in the hydraulic cylinder, velocity of the cylinder piston $\mathrm{v}_S(t)$ and velocity $\mathrm{v}_D(t)$ and placement $x_D(t)$ of the endpoint D. On the basis of dynamic characteristics the transient states observed at recontrolling of the manipulator positioning were analysed. The changes of parameters of valve, input signal and disturbances were taken into account. The examples of dynamic characteristics of the analysed two-link manipulator obtained for the following values of parameters:
$p = 10\text{x}10^6$ N/m², $I_s = 150$ kg, $I_o = 6000$ Nsm²,
$SEq = 920$ N, $A_1 = 1{,}14\text{x}10^{-3}$ m², $A_2 = 0{,}63\text{x}10^{-3}$ m²,
$C_1 = 2{,}9\text{x}10^{-12}$ m⁵/N, $C_2 = C_1$; $R_s = 1000$ Ns/m,
$R_t = 1{,}2\text{x}10^{14}$ Ns/m⁵; $R_b = 100$ Nsm,
$R_1 = 7.27\text{x}10^{11}$ Ns²/m⁸, $R_2 = 2{,}5\text{x}10^{12}$ Ns²/m⁸, $R_1 = R_4$, $R_2 = R_3$, $SEg = 1500$ Nm
are shown on Fig 5 and Fig.6.

The dynamic model of the manipulator was experimentally verified. Verification consisted in identification of the dynamic model through analysis of the accuracy of approximation of the real physical phenomena described by this model. [4].

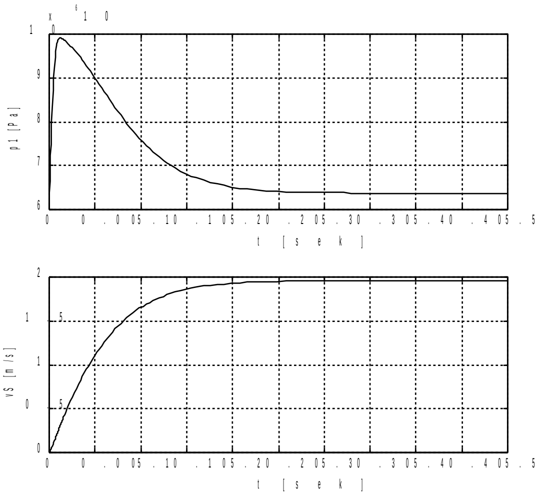


Figure 5. Pressure p_1 and velocity v_s responses of the cylinder

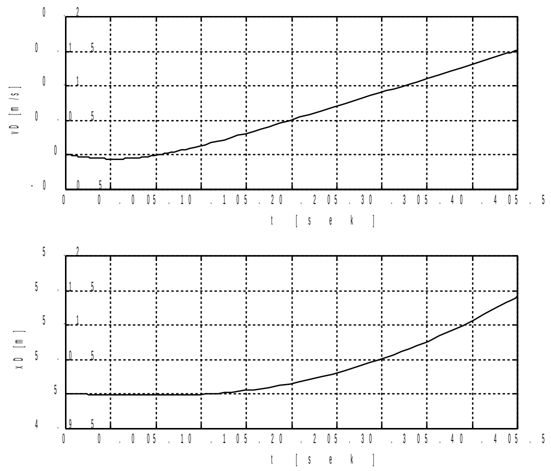


Figure 6. Velocity v_D and placement x_D responses of the endpoint of manipulator

4. CONTROL OF POSITIONING

The basic task of the position control is reaching the set position of the end point with the minimum deviation. It is however, necessary for the control time to be as short as possible, the position set not to be overshot and outer disturbances and the metering noise to be effectively damped. Electrohydraulic position systems usually show strong non-linearities resulting from non-linear flow through the control valve and the zone of insensitivity in the electrohydraulic amplifier. On Fig.7 the diagram of the research stand used to check the accuracy of the system positioning with electrohydraulic control is presented.

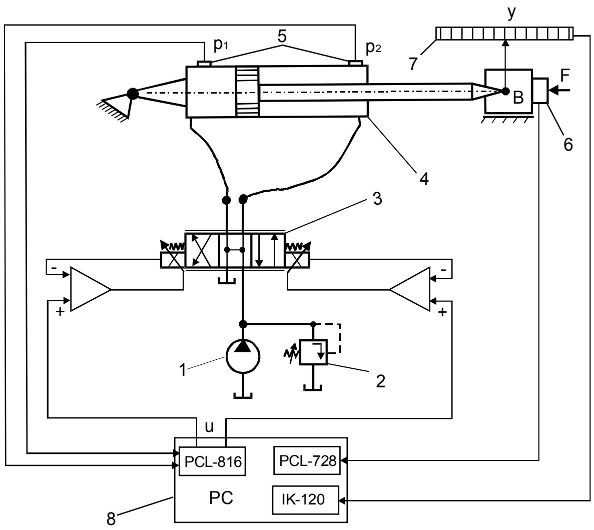


Figure 7. Diagram of the research stand for examining the accuracy of positioning of the hydraulic cylinder: 1 – pomp, 2- pressure relief valve, 3 – servo solenoid valve, 4- positioning cylinder, 5 – pressure converters, 6 – force converter, 7 – photo-optical converter of positioning, 8 - PC measurement cards

The stand was equipped in electronic amplifier, photo-optical converter of position, converters of pressure and force and PC measuring cards. On the stand the following quantities were measured: pressure in the cylinder chambers and in the feeding line and the position of point B. The accuracy of the position measurement was 0,5 μm. The displacement velocity of this point was determined by means of numerical integration. The research on the stand enabled the identification of the manipulator's dynamic model. The identification experiments were performed on a laboratory model of electrohydraulic control system. The identification of the electrohydraulic control system was performed by introducing to the input of the amplifier of proportional valve a step signal, combination of harmonic signals and random binary signals (for constant amplitude and variable amplitude). On Fig.8 the runs of simulation and real positioning are compared. The tests conducted on the stand concerned mostly the accuracy of positioning of hydraulic cylinder with electrohydraulic control with regard to the effect of parameters of the proportional valve, outer load, input signal and digital controllers. The sensitivity of the electrohydraulic control system to the change of operating conditions was also investigated. As the result of the research the control algorithms ensuring high precision of positioning control resistant to measuring disturbances and considerable changes of force and loading mass were worked out. It was observed that the minimum values of the offset occurred when the conventional controllers of PI or PID types were used. However, for considerable change of parameters a good quality of control was achieved by means of adaptive control. In addition to classical methods of control other controllers like state controller, fuzzy controller and "dead beat" controller were used. Fuzzy controllers ensure simple control of the system but high accuracy of positioning is difficult to obtain. It was observed that the "dead beat" controller was less sensitive to the change of parameters than the classical controller PI [4]. Fig.9 presents the transient of positioning of B point of the co-ordinate *y(t)* and control error *Δu(t)* after have been applied the sequence controller.

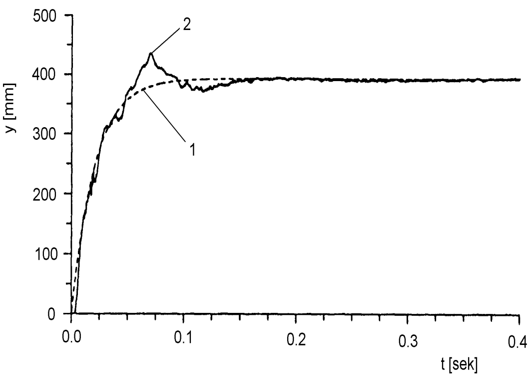


Figure 9. Comparison of simulation (1) and real position transient (2) [4]

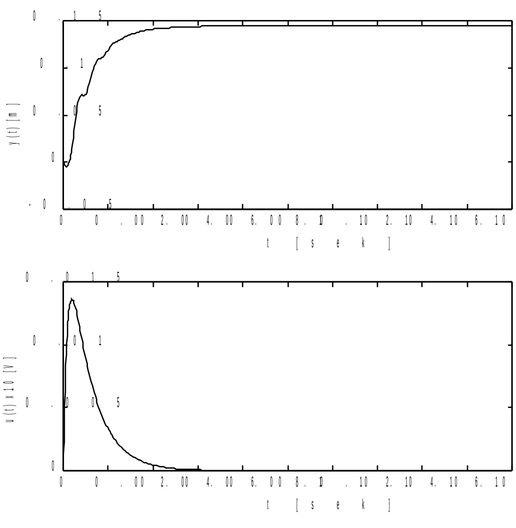


Figure 10. Position *y(t)* and control error *Δu*(t) transient with the use of sequence controller [4]

ACKNOWLEDGMENTS

The paper deals with kinematic and dynamic modelling, digital simulation and experimental test of the planar two-link manipulator with electrohydraulic control. The analysed manipulator is one of the kinematic chains of multi-link mechanisms where the electrohydraulic control is applied. The kinematic diagram which enables determining the trajectories of manipulator's characteristic points and in particular its endpoint D is presented. The manipulator's dynamic diagram is presented on the basis of its kinematic diagram by means of bond graphs. The model has been used in digital simulation and in dynamic analysis dealing with transient processes taking place after re-controlling of proportional valve for different control signals. The trajectories of motion were determined by means of mathematical package MATLAB and the dynamic characteristics by means of simulation package 20-sim. The dynamic characteristics obtained enable to evaluate the effect of parameters of the proportional valve, outer load and control signals on stability and accuracy of positioning. The identifications of the dynamic model and experiments were carried out on the research stand. Their purpose was to determine the effect of different disturbances and control parameters on accuracy of manipulator's positioning. The control algorithms which enable the selection of control parameters for given operating conditions of the manipulator with electrohydraulic control have been worked out. The results of simulation and experimental research have been used in design and construction of numerous manipulators with electrohydraulic control [4].

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