PHONON MECHANISMS: A KNOW-WHY FOR VIBRATION IN FLEXIBLE-LINK ROBOT ARMS

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Abstract: Utilizing theory of the secondary quantization, a study of vibration behaviors in flexible-link robot arms is possible in microscopic viewpoints. This paper studies the relation of vibration related to phonon coupling interaction by means of Green's function method beginning with energy model. Results confirm analytically the existence of the relationship between the macroscopic study employing a classical beam theory such as the Euler-Bounouli theory and the proposed microscopic method. The approach presented in this paper may be an alternative way to gain some insight into the real vibration and damping mechanism of flexible-link robot arms.

Keywords: flexible arm, vibration theory, phonon mechanism, vibration behavior, flexible-link robot

1 INTRODUCTION

Due to some advantages of flexible-link robot arms over the conventional ones such as lower energy consumption, less overall mass, and faster system responses, various basic researches on flexible-link robot arms in both science and engineering aspects have been intensively carried out [1-5, to name a few]. However, the flexible nature of the link of the arm induces the vibration of the structure. Attempts to construct a model of the arms including the vibration behaviors have been an issue in this area. However, the perfect model has not yet been obtained because the correct damping mechanisms in beam have not perfectly understood yet [6,7].

Although the Kelvin-Voigt damping has a good physical meaning, by which the vibration modes lost there energy to the resistance of the beam material, but this types of damping overdamps the higher modes as reported in [7]. According to [7], the $A^{1/2}$ operator damping in [8] seems to be mathematically fit the experimental data, but the insight of physical meaning is not obviously illustrated. Hence the correct damping mechanisms in beams are still illusive, and are required more studies and further investigation.

In recent years, the knowledge of quantum mechanics has been introduced to control engineering [9-10]. Few papers on quantum control have been published. The area of applying knowledge in quantum mechanics for problem in control system has considered as in its infancy stage. Not only the framework needs to be clarified in the areas, but also the knowledge about using the new thinking tools is also required for control engineers to carry out their work and apply to problem-solving processes fallen into the new paradigm.

This paper attempts to pioneer such an investigation on applying the secondary quantization known and used in physics since the World War II to introduce an alternative way of thinking for control engineering and robotics. At the same time, the equal important objective is paid on the search of understanding the vibration mechanism in the flexible-link robot arms, while relating and depicting the microscopic viewpoints to the macroscopic world. Some insight is expected to gain in this research along with the demonstration of applying tools in quantum mechanics to robotics and automation.

To begin, the energy model of phonons as the vibration quasi-particle acting quantum mechanically and as collective excitations together with their interaction on the whole lattices is given in Section 2. Section 3 shows mathematical methods such as the propagators, approximation and diagrams for many-body systems as relatively new tools in robotics, automation and control engineering. The finding results along with discussion and conclusion are presented in Section 4 and 5 as in fashion.

2. ENERGY MODEL

Phonons are known as "bosons" or particles obeyed Bose-Einstein statistics [11]. Having considered a flexible-link robot arm as a beam, we now think microscopically that phonons are pumping into a flexible-link arm created from the excitation of input energy while the manipulator is in an operation mode. In atomic scale, the proposed Hamiltonian system for phonon mechanism in beam is formulated as

$$H = \sum_k \varepsilon_k b_k^+ b_k + \sum_k \varepsilon_0 - \sum_k V(k)(b_k + b_{-k}^+)(b_{-k} + b_k^+) \quad (1).$$

Here the first two terms represent the unperturbed term or the Hamiltonain for a set of phonons as oscillators extending through the whole lattice, where the second term is the energy at the ground state level of the phonons. The operators b_k , b_k^+ are the annihilation and creation operators obeyed boson commutation rules:.

$$\begin{aligned} [b_j , b_k^+] &\equiv b_j b_k^+ - b_k^+ b_j = \delta_{jk}, \\ [b_j , b_k] &\equiv b_j b_k - b_k b_j = 0, \\ [b_j^+ , b_k^+] &\equiv b_j^+ b_k^+ - b_k^+ b_j^+ = 0. \end{aligned} \quad (2)$$

The third term in equation (1) involves the interaction among phonons with the lattices and themselves, and V(k) is the parametizing microscopic mechanism of interaction acquired a k-dependent frequency.

Since the perturbation term could be equally large as the unperturbed one, the formal perturbation technique cannot be used. Moreover, upon hitting, lattices could absorb the phonon energy effecting probability amplitude of the phonons, making V(k) as a time dependent function. To carry out the analysis, the Green's function propagator method will be employed in the next section.

3. GREEN'S FUNCTION AS PHONON PROPAGATOR

The propagator method treated by using Green's functions and diagrammatic techniques as shown in Section 4 have successful applied to extract "know-why" in many macroscopically physical phenomena such as low temperature superconductivity, phase transition of matter, ferromagnetic behaviors, and other problems in nuclear physics and quantum electronics. In the Green's function propagator method the poles of the propagator provides the energies of the excited states. Using the standard treatment of the Green's function and taking the Fourier transforms, we have the propagator in the forms of

$$G(k,\omega) = \int_{-\infty}^{+\infty} dt e^{i\omega t} G(k,t) \quad (3),$$

where

$$G(k,t) = -i < 0 | T\{b_k(t) b_k^+(0)\} | 0 > \quad (4).$$

The ket vector | 0 > represents the ground state for the system of interacting phonons, where the T operator is the time operator for bosons having property:

$$\begin{aligned} T\{\alpha(t_1)\beta(t_2)\} &= \alpha(t_1)\beta(t_2) \ if \ t_1 > t_2 \\ &= \beta(t_2)\alpha(t_1) \ if \ t_1 < t_2 \end{aligned} \quad (5).$$

The physical meaning of the Green's function propagator in (4) can be directly read off as the the probability amplitude that a phonon in the state k is pumped into the interacting ground state at time t=0 and propagates in the system, which the phonon in state k at time t = t and t>0 can be observed. For the non-interacting case for which a phonon propagates freely, we assume G (k,t) in the form of

$$G_0(k,t) = -iA_t e^{-i\omega_0 t} \quad (6),$$

and the backward in time propagator for the free propagation of a phonon can be obtained by changing the +t in equation (6) to –t.

Taking the Fourier's transform of (6) as defined in (3), and using the residue theorem, we have

$$G_0(k,\omega) = \frac{1}{\omega - \omega_0 + i\delta} \quad (7).$$

Here δ represents the positive infinitesimal used to remove the oscillating terms at time t approaching to infinity, which may be related to the damping feature.

Similarly, for the backward phonon propagation in time as aforementioned, we have

$$G_0^{back}(k,\omega) = \frac{1}{\omega + \omega_0 - i\delta}$$

(8).

4. ANALYTICAL RESULTS AND DISCUSSION

Using the Heisenberg picture and the Hamiltonian given in equation (1)and then taking the Fourier transform defined in (3), the equation of motion for each Green's function propagator can be obtained as shown by the Dyson's equation:

$$G(k,\omega) = G_0(k,w) + G_0(k,w)\Sigma\, G(k,\omega) \quad (9),$$

where Σ is the self-energy defined as

$$\Sigma = V(k) + V(k)G_0^{back} + V^2(k)[G_0^{back}]^2 + \ldots \quad (10).$$

Equation (8) can be found by summing all possibility sequences of interaction between each phonon and lattices or among phonon hitting other phonons. With the aid of Feynman diagrams, Figure 1 can be obtained.

G(k,t) =

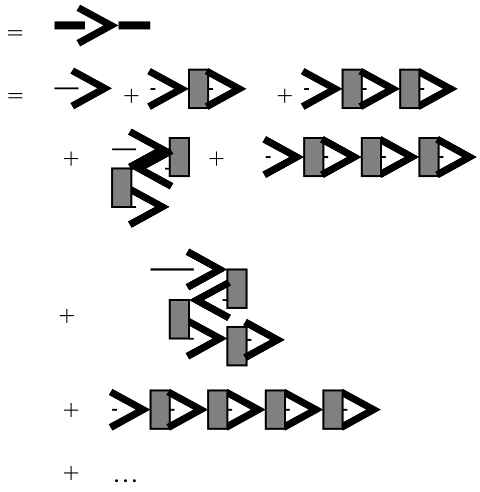
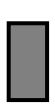


Figure 1. Expansion of phonon propagators.

The symbol  represents an interaction at time t. Assume also that each interaction with the lattice the phonon lost its energy to the lattice resulting a vibration effect. The summing of all phonons propagators via statistical average would effect the lattice to vibrate upon hitting. The more hitting rate of phonons to the whole lattice, the higher the lattice vibration and the more energy lost from the phonons. This physical phenomena associates damping mechanism would guarantee the convergent of the series in Figure 1.

Translating Figure 1 into an equation, we then get:

$$G = G_0 + G_0VG_0 + G_0VG_0^{back} + G_0VG_0VG_0 + G_0VG_0VG_0^{back} + \ldots \quad (11).$$

We sum up the series in (11) by using the geometric series to get equation (9). Alternatively, the series can be summed graphically as shown in Figure 2.

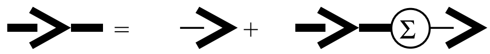


Figure 2. Dyson's equation as shown in equation (9).

With the damping mechanism, thus the G(k, ω) form holds only for ω satisfying the convergence condition that

$$\left|G_0(k,w)\Sigma\right| < 1 \quad (12).$$

From (10) with the aid of the geometric series, we have

$$\Sigma = \frac{V(k)}{1 - V(k)G_0^{back}(k,\omega)} \quad (13),$$

and

$$\left|V(k)G_0^{back}(k,w)\right| < 1 \quad (14).$$

We now reach the condition for ω range in which the phonon propagators to be valid as shown by the inequalities (11) and (14). Again with damping mechanism associated with phonon-lattice interaction of both forward and backward propagators, prove for convergence of equations (11) and (14) to hold can be omitted and further assumptions are not required.

It is easy to see that we can get renormalized phonon frequencies from the poles of G(k,ω) by using equations (9) and (10) by direct substitution . For example, if the interaction term is assumed to be in the form of

$$V(k) = \frac{1}{2}\omega_0 \cos(ka) \qquad (15)$$

, which is the form obtained from considering the atoms of the flexible material vibrated harmonically with coupling constant $\frac{1}{2}m\omega_0^2$ and interatomic distance a, we can obtain

$$G(k,\omega) = \frac{\omega + \frac{1}{2}\omega_0(2 - \cos(ka))}{\omega^2 + \omega_0^2(1 - \cos(ka))} \qquad (16)$$

In equation (16), we drop the damping associated term δ out for the sake of convenience. We then have the renormalized phonon frequencies from the poles of G:

$$\omega = \omega_0(1 - \cos(ka))^{\frac{1}{2}} \qquad (17)$$

As we can see, equation (7) is nothing but the phonon dispersion law. This implies that the phonon mechanism rules out the vibration, microscopically. The vibration modes also involve with this mechanism and the series truncation using only the dominant modes in consideration can be done by employing the inequalities (12) and (14). The question of how many modes exist or should be included in modeling and control of the flexible robot arms can be now partly answered, analytically and intuitively.

Referring to [3], Figure 3 is plotted based on Timoshenko beam theory. The curves imply some insight into the effects of microscopic world by phonon mechanism resulting macroscopically via statistical average of the collective excitation phenomena. Some evidence relating to the damping effects and vibration modes due to the decreasing of the probability amplitudes of phonons upon hitting lattice and other interaction together with the macroscopic effects corresponding to phonon frequencies are shown.

Concerning a question of the control spill over effects in which the inappropriate control signal excites the neglected higher order vibration modes, there is no difficulty to answer this question intuitively in the light of inequalities (12) and (14). Let us recall the meaning of the 'self-energy.' This term here can be interpreted as the case that the ordinary phonon interacts with many-body system. Consequently, the interaction creates the collective excitation effects viewed as 'the cloud' and the cloud in turn reacts back on the phonon. The phonon motion is then disturbed. Hence, the other way of looking at this phenomenon is that the phonon changes its own energy by interacting with itself via the system.

In normal case, the amplitudes of the higher order vibration modes are less than the lower order ones, which holds microscopically for probability amplitudes of the phonons. The control signal is generated in order to suppress all vibration modes. If this cannot be completely done, at least, the dominant lower order modes should be inherently preserved. In contrast to the good control law, the inappropriate control signal pumps more phonons into the system. The pumping phonons then exchange energy with lattice and among themselves. With the collective excitation effect, either low frequency phonons or the environment around themselves could transfer their energy back to the high frequency phonons. The control spill over effect is now described microscopically. It is possible to show the explanation mathematically and should be a topic to be covered for the future work.

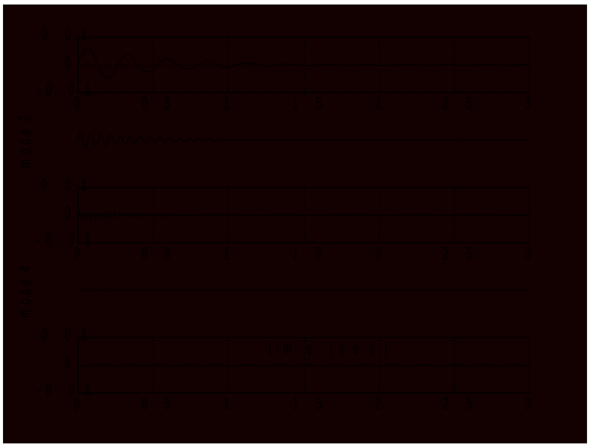


Figure 3. The first five vibration modes of a pined-pined model flexible-link robot arm [3].

5. CONCLUSION

In summary, we have used a theory of the secondary quantization in quantum mechanics to study vibration behaviors of flexible-link robot arms. The relation of vibration related to phonon coupling interaction is studied by means of Green's function method beginning with energy model.

The major result is analytically confirmed the high possibility to study the topic in microscopic viewpoints by employing the methods shown in this paper. Intuitively, the existence of the relationship among the macroscopic studies employing classical theories and the proposed microscopic ones should be agree in sense of looking at the macroscopic effects as a statistical average of the collectively microscopic behaviors.

Based upon the best of the literature review of the author, this work is perhaps the first attempt to study the vibration of flexible-link robot arms in microscopic point of views using quantum field theory. The approach may be an alternative way to gain some insight into the real vibration mechanism and to reveal the correct damping mechanism of flexible-link robot arms, which would benefit for the future work and further investigation.

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