STRUCTURAL MODIFICATION OF SYNTHESISED VIBRATION BAR-SYSTEMS AS AN INTRODUCTION TO OPTIMAL CONTROL THESE SYSTEMS

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Abstract: Basing on the design of continuous free systems with definite frequency spectrum, with cascade structure, the dynamical flexibility of such systems has the alternating: pole, zero, pole, zero, etc. Nevertheless, the frequency with the zero value is always a pole. On the other hand, in the case of systems with cascade structure but fixed at one side, the dynamical flexibility has the alternating: zero, pole, zero, pole, etc. respectively. The frequency with the zero value is always a zero. In this paper the method was applied in order to structural modification of the torsionally vibrating mechanical system with cascade structure. This is the method of decomposition of characteristics into continued fraction expansion presented by graphs.

Keywords: dynamical characteristic, hypergraph, cascade structure of mechanical system

1 INTRODUCTION

In the practical application of the results of design of vibrating continuous bar systems with definite frequency spectrum, the same or similar requirements may be fulfilled, both as far as free and fixed systems with cascade or branched structure are concerned. The solution of such task entails modifying the requirements laid down for the dynamical characteristic, which implies that another characteristic should be subjected to synthesis, by first and foremost, alternating frequency $f_0 = 0$, from the pole to zero, in the case of replacing the characteristic of free systems to the fixed ones, or from zero to a pole, in the case of replacing the characteristic of the fixed systems to the free system characteristic. The implications of such modification are further variations, resulting from the fulfillment of the conditions of accomplishing the modified physical characteristic. The variations involve adding or deducting a zero (pole) with different frequency values.

Using the symbols introduced in papers [3-10, 15], a following couple

$$X = \left({}_1X,\ {}_2X\right) \tag{1}$$

is called a *graph,* where: ${}_1X=\{{}_1x_0, {}_1x_1, {}_1x_2,\dots, {}_1x_n\}$ - finite set of vertices, ${}_2X = \{{}_2x_1, {}_2x_2, \dots, {}_2x_m\}$- family of edges, being two-element subsets of vertices, in the form of ${}_2x_k = \left({}_1x_i, {}_1x_j\right)$ (i,j = 0,1, ... ,n) (of. [2]).

The couple

$${}^kX = \left({}_1X,\ {}^k_2X\right) \tag{2}$$

is called a *hypergraph,* where: ${}_1X$ is the set as in (1), and ${}^k_2X = \left({}^k_2X^{(i)}/i\in N\right)$, (k=2,3, ... $\in$N) is a family of subsets of set ${}_1X$; the family ${}^k_2X$ is called a *hypergraph* over ${}_1X$ as well, and ${}^k_2X = \left\{{}^k_2X^{(1)},\ {}^k_2X^{(2)}, \dots, {}^k_2X^{(m)}\right\}$ is a set of edges [2], called *hyperedges* or *blocks,* if

$${}^k_2X \neq \varnothing\ (i\in N), \tag{3}$$

$$\bigcup_{i\in I} {}^k_2X^{(i)} = {}_1X,\ (I\subset N). \tag{4}$$

Using notion of graph and of hypergraph and their connections with structural numbers [1,3,13,14] and system of notation [13,14], methods of modification of mechanical system as task of the synthesis of dynamical characteristic - mobility has been presented.

A characteristic - dynamical flexibility is given in form

$$Y(s) = \frac{\sum\limits_{i=0}^{k} c_i \, th^j \Gamma s}{s\sum\limits_{j=0}^{l} d_j \, th^j \Gamma s}. \tag{5}$$

After transformations [11,12]

$$V(s) = sY(s), \tag{6}$$

$$r = \mathrm{th}\Gamma h, \tag{7}$$

the mobility has been obtained as

$$V(s) = \frac{\sum_{i=0}^{k} c_i r^i}{s \sum_{j=0}^{l} d_j r^j}. \tag{8}$$

where: $c_k, c_{k-1}, ..., c_0, d_l, d_{l-1}, ..., d_0$ are any real numbers, $\Gamma = \sqrt{\frac{\rho}{G}} L = \sqrt{\frac{\rho^{(i)}}{G^{(i)}}} L^{(i)}$, ρ -mass density, G-Kirchhoff's modulus , $L=L^{(i)}$- length of basic element, $s=j\omega$, $j=\sqrt{-1}$, $c_k, c_{k-1}, ..., c_0, d_l, d_{l-1}, ..., d_0$-real numbers, i,j,k,l - natural numbers, k-l=1.

This problem as an introduction to optimal control is discussed in more detail in the paper.

2. FORMULATION OF THE STRUCTURAL MODIFFICATION OF A SYNTHESISED VIBRATION BAR-SYSTEMS

The synthesis consists in investigating the structure of a system with the continuous distribution of parameters and specific requirements set for the realization of the desired mechanical phenomena, because the discrete model is too remote from the real object. Moreover, the problem of synthesizing discrete physical, mechanical and, first of all, electrical and electronic systems is widely in scientific research. However, it is considerably to find examples of methods of synthesizing continuous mechanical systems. Authors of the papers approaching these problems emphasize that the synthesis and the same the modification of systems with the continuous distribution of parameters is only beginning to be developed, so its exact formulation and solution is still to be dealt with. So far, this task has been approached by attempts to determine of a rectangular bar or a sleeve (depending on longitudinal or torsional vibrations) from the frequency equation and only on basis the first natural frequency.

The first attempt at the solution to this problem concerning the frequency spectrum has been made in the Gliwice research centre in [3].

In this paper the continued fraction expansion method was applied in order to modifiication of the torsionally vibrating mechanical system with cascade structure represented by graphs.

2.1. Continued fraction expansion method of the structural modiffication of the dynamical systems represented by graphs

The dynamical flexibility Y(s) of torsionally vibrating mechanical continuous bar system is given in form

$$Y(s) = \frac{c_k \mathrm{th}^k \Gamma s + c_{k-1} \mathrm{th}^{k-1} \Gamma s + \cdots + c_0}{s(c_l \mathrm{th}^l \Gamma s + c_{k-1} \mathrm{th}^{k-1} \Gamma s + \cdots + d_0)} \tag{9}$$

Using the transformation (6) the mobility V(s) is obtained in form

$$V(s) = \frac{c_k \mathrm{th}^k \Gamma s + c_{k\text{-}1} \mathrm{th}^{k-1} \Gamma s + ... + c_0}{d_l \mathrm{th}^l \Gamma s + d_{l\text{-}1} \mathrm{th}^{l-1} \Gamma s + ... + d_0}, \tag{10}$$

After the next transformation (7) called Richards' transformation [3,11,12], the mobility V(r) is given in form

$$V(r) = \frac{c_k r^k + c_{k\text{-}1} r^{k-1} + ... + c_0}{d_l r^l + d_{l\text{-}1} r^{l-1} + ... + d_0}, \tag{11}$$

where: k-l = 1.

The method of the synthesis of transformed mobility function V(r) is presented here, assuming the even number of elements, and when k is an even natural number, then V(r) takes form

$$V(r) = \frac{c_k r^k + c_{k\text{-}1} r^{k-1} + ... + c_0}{d_{k\text{-}1} r^{k-1} + d_{k\text{-}3} r^{k-k} + ... + d_1 r} \tag{12}$$

or

$$V(r) = \frac{L_k(r)}{M_{k-1}(r)}. \tag{13}$$

After dividing in (13) the numerator by denominator - it is a first step of the synthesis - the equation below is obtained

$$V(r) = V_r^{(1)}(r) + \frac{L_{k-2}(r)}{M_{k-1}(r)} = $$
$$= V_r^{(1)}(r) + \frac{1}{\frac{M_{k-1}(r)}{L_{k-2}(r)}} =$$
$$= V_r^{(1)}(r) + \frac{1}{U_2(r)} = \frac{r}{c_r^{(1)}} + \frac{1}{U_2(r)}, \tag{14}$$

where: $c_r^{(1)}$ is value of "i" synthesized discrete elastic element.

Graphical representation of the implementation of equation (14) in a form of a graph is shown in fig. 1.

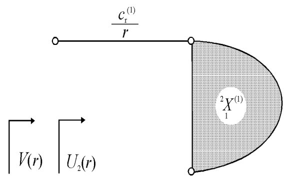

Fig. 1. Graphical illustration of equation (14)

The second step is the realization of the function $U_2(r)$ into (14). When dividing $M_{k-1}(r)$ by $L_{k-2}(r)$, $U_2(r)$ takes form

$$U_2(r) = U_z^{(2)}(r) + \frac{M_{k-3}(r)}{L_{k-2}(r)} = $$
$$= U_z^{(2)}(r) + \frac{1}{\dfrac{L_{k-2}(r)}{M_{k-3}(r)}} = $$
$$= U_z^{(2)}(r) + \frac{1}{V_3(r)} = J_z^{(2)} r + \frac{1}{V_3(r)} \ , \qquad (15)$$

where: $J_z^{(i)}$ -value of "i" synthesized discrete inertial element.

The graph of synthesized mechanical bar system after operation (15) is shown in fig. 2.

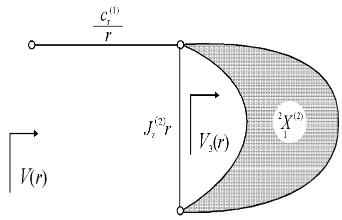

Fig. 2. Graphical illustration of equations (14) and (15)

The synthesized mobility function after operations (14-15) is given in the following form

$$V(r) = V_r^{(1)} + \cfrac{1}{U_z^{(2)}(r) + \cfrac{1}{V_3(r)}} . \qquad (16)$$

The third step is the realization of the mobility function $V_3(r)$ into (16) as

$$V_3(r) = V_r^{(3)}(r) + \frac{L_{k-4}(r)}{M_{k-3}(r)} = $$
$$= V_r^{(3)}(r) + \frac{1}{\dfrac{M_{k-3}(r)}{L_{k-4}(r)}} = $$
$$= V_r^{(3)}(r) + \frac{1}{U_4(r)} = \frac{1}{c_r^{(3)}} + \frac{1}{U_4(r)} . \qquad (17)$$

After execution of the operation (17) the mobility function V(r) takes form

$$V(r) = V_r^{(1)} + \cfrac{1}{U_z^{(2)}(r) + \cfrac{1}{V_r^{(3)}(r) + \cfrac{1}{U_4(r)}}} . \qquad (18)$$

The form (18) corresponds with mobility function (12) of a graph (see fig. 3). The mobility determined at the point indicated by the arrow is identical with (12). This graph is a model of discrete system but after transformation it is a continuous system (see [3]).

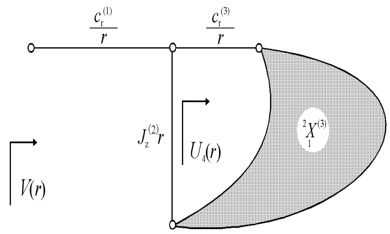

Fig. 3. Graphical illustration of the implementation of equation (18)

The process of the synthesis after steps (14-18) is to be continued until the function $U_k(r)$ will take form

$$U_k(r) = U_z^{(k)}(r) = J_z^{(k)} r . \qquad (19)$$

Finally the mobility (12) as a continued fraction is obtained in form

$$V(r) = V_r^{(1)} + \cfrac{1}{U_z^{(2)}(r) + \cfrac{1}{V_r^{(3)}(r) + \cfrac{1}{U_z^{(4)}(r) + \begin{matrix} \cdot \\ \cdot \\ \cdot \end{matrix} + \cfrac{1}{V_r^{(k-1)}(r) + \cfrac{1}{U_z^{(k)}(r)}}}}} =$$

$$=\frac{r}{c_r^{(1)}}+\cfrac{1}{J_z^{(2)}(r)+\cfrac{1}{\cfrac{r}{c_r^{(3)}}+\cfrac{1}{J_z^{(4)}r+\cdots+\cfrac{1}{\cfrac{r}{c_r^{(k-1)}}+\cfrac{1}{J_z^{(k)}}}}}} \cdot \quad (20)$$

The form (20) corresponds with mobility function (12) of a polar graph $\underset{00}{X}$ (see fig. 4). The mobility determined at the point indicated by the arrow is identical with (12). This graph is a model of discrete system but after transformation it is a continuous system (comp. [3]).

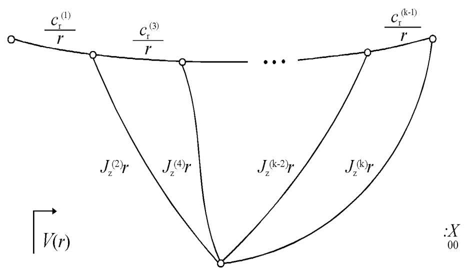

Fig. 4. Graphical illustration of equation (20)

Structural modification of dynamical characteristic of synthesized mechanical system is possible when k is an odd natural number and then V(r) takes form

$$V(r)=\frac{c_k r^k + c_{k-2} r^{k-2} + \ldots + c_1 r}{d_{k-1} r^{k-1} + d_{k-3} r^{k-3} + \ldots + d_0} \quad . \quad (21)$$

The process of the synthesis of the mobility function (21), after steps consistent with (14-18), is to be continued until the mobility V(r) will take form

$$V_k(r)=V_r^{(k)}(r)=\frac{r}{c_r^{(k)}} \,. \quad (22)$$

Finally the mobility function (21) as a continued fraction is obtained in form

$$V(r)=V_r^{(1)}(r)+\cfrac{1}{U_z^{(2)}(r)+\cfrac{1}{V_r^{(3)}(r)+\cfrac{1}{U_z^{(4)}(r)+\cdots+\cfrac{1}{U_z^{(k-1)}(r)+\cfrac{1}{V_r^{(k)}(r)}}}}}=$$

$$=\frac{r}{c_r^{(1)}}+\cfrac{1}{J_z^{(2)}+\cfrac{1}{\cfrac{r}{c_r^{(3)}}+\cfrac{1}{J_z^{(4)}r+\cdots+\cfrac{1}{J_z^{(l-1)}r+\cfrac{1}{\cfrac{r}{c_r^{(l)}}}}}}} \,. \quad (23)$$

The form (23) corresponds with mobility function (21) of a polar graph $\underset{00}{X}$ (see fig. 5). The mobility determined at the point indicated by the arrow is identical with (21).

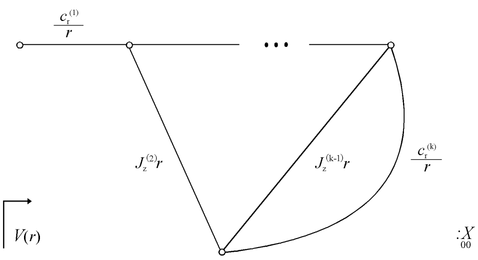

Fig. 5. Polar graph as an illustration of the implementation of the equation (23)

When $k-1=-1$, the method of the synthesis of transformed inverse function $U(r)=\frac{1}{V(r)}$, is presented here, as well, assuming the even number of elements. Then U(r), as a third case of synthesis of the function of mechanical bar system, is given in the following form

$$U(r)=\frac{d_l r^l + d_{l-2} r^{l-2} + \ldots + d_0}{c_{l-1} r^{l-1} + c_{l-3} r^{l-3} + \ldots + c_1 r} \,. \quad (24)$$

or

$$U(r)=\frac{L_l(r)}{M_{l-1}(r)}\,. \tag{25}$$

The equation (24) or (25) as a continued fraction is obtained in form

$$U(r)=U_z^{(1)}(r)+\cfrac{1}{V_r^{(2)}(r)+\cfrac{1}{U_z^{(3)}(r)+\cfrac{1}{V_r^{(4)}(r)+\begin{array}{c}.\\.\\.\end{array}+\cfrac{1}{U_z^{(l-1)}(r)+\cfrac{1}{V_r^{(l)}(r)}}}}}=$$

$$=J_z^{(1)}r+\cfrac{1}{\cfrac{r}{c_r^{(2)}}+\cfrac{1}{J_z^{(3)}r+\cfrac{1}{\cfrac{r}{c_r^{(4)}}+\begin{array}{c}.\\.\\.\end{array}+\cfrac{1}{J_z^{(l-1)}r+\cfrac{1}{\cfrac{r}{c_r^{(l)}}}}}}}. \tag{26}$$

The Eq. (26) represents the inversion of mobility function of dynamical structure in form of polar graph $\underset{00}{X}$ (fig. 6).

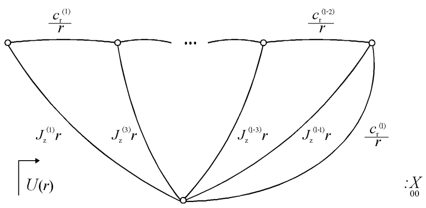

Fig. 6. Graphical illustration of equation (26)

The structural modification of dynamical characteristic – U(r) of synthesized mechanical system is possible when k is an odd natural number and $k-l=-1$ and then U(r) of the transformed function of bar system - takes form

$$U(r)=U_z^{(1)}(r)+\cfrac{1}{V_r^{(2)}(r)+\cfrac{1}{U_z^{(3)}(r)+\cfrac{1}{V_r^{(4)}(r)+\begin{array}{c}.\\.\\.\end{array}+\cfrac{1}{V_r^{(l-1)}(r)+\cfrac{1}{U_z^{(l)}(r)}}}}}=$$

$$=J_z^{(1)}r+\cfrac{1}{\cfrac{r}{c_r^{(2)}}+\cfrac{1}{J_z^{(3)}r+\cfrac{1}{\cfrac{r}{c_r^{(4)}}+\begin{array}{c}.\\.\\.\end{array}+\cfrac{1}{\cfrac{r}{c_r^{(l-1)}}+\cfrac{1}{J_z^{(l)}r}}}}}. \tag{27}$$

The Eq. (27) represents the transformed inversion of the transformed mobility function of dynamical structure in form of polar graph $\underset{00}{X}$ (Fig. 7).

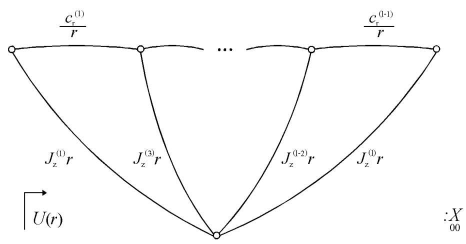

Fig. 7. Polar graph as an illustration of the implementation of the equation (27)

The above formulas and sentences in [3] are a base for the computer aided synthesis and structural modification of longitudinally or torsionally vibrating mechanical systems by the continued fraction expansion method.

3. CONCLUSION

Presented problems are introduction to optimal control of set of models of discrete and continuous mechanical systems.

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