A BUCKET DISCHARGE CONTROL FOR A BACKHOE EXCAVATOR

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Abstract
The study presents a practical, optimal control system of the discharge process of a backhoe bucket. The process begins when the bucket is filled and ends when the bucket come back to the digging position. In the process, several geometrical constraints are considered, such as, clearance dimension, and obstacles in the excavator working zone. In other words, the operator has to give the path of the discharge process. The problem is considered as a multi body, chain system, driven by hydraulic actuators. The system is decomposed in free body diagrams of separate elements. Starting from the bucket, each element is solved separately, as many times, as many it is needed to get optimal time for given actuator forces.

Key Word: excavator, multi body systems, optimal control, hydraulic actuators, recursive multi body dynamics.

1. INTRODUCTION

A relatively large part of the excavator work time is taken by hauling, dumping and return of the bucket to its digging position. This part of the excavation process is having at least two features, which distinguish them from the earth moving work. The first one, is that the bucket discharging should be considered as a dynamic process. The soil digging, in the contrary, can be regarded as a quasi-static process, in which the acceleration of the excavator attachment members could be neglected.
It is then the aim of the study to present a practical control system of the discharge process of a backhoe bucket. The process begins when the bucket is filled and ends when the bucket came back to the digging position. In the discharge control process several geometrical constrains are taken into consideration. First of all, clearance dimension have to be defined. Secondly, all possible obstacles in the excavator working zone should be considered. In the others words, the operator has to define the trajectory of the bucket motion, assuming its time duration.
The discussed problem consists in optimization of a multi body system dynamics (MSD) with constraints and actuators specific for hydraulic driven machines. It kinematics and dynamics are close to the mechanics of robots. The differences are in environmental and working conditions, hydraulic actuators and randomness of working conditions.
Since last two decades, a large number of publications and monographs are devoted to kinematics and dynamics of systems of rigid bodies and their applications to robotics. Monograph by Duffy [5] and Wttenberg [12] together with edited by Schiehlen handbook[9], are very good sources of the knowledge on MSD. One of the very important branch of MSD is robots dynamics (RD). Also in this field, the number of works is very large, as the problem is of scientific and of important applications.
Only in recent years RD started to be investigated in the field of earth moving machines, mostly in excavators (Budny et al [4], Skibniewski [10].
A profound knowledge in MSD get the possibilities of investigating optimal control of considered systems. A pioneer work on optimal control of manipulators, dealing with analytical and numerical approach to the problem, is due Akulenko et al [1]. A general survey of numerical methods for trajectory optimization, showing a wide range of possible applications, is presented by Betts[3]. Due the complexity of optimal control problems, recently, some attention is paid to detailed, problem oriented methods (Furukawa [7], Hu et al [8] and Eberhardet al [6]. Also the present study is an oriented method which can be applied to bracket kinematic chains.
The paper starts with motion description of the excavator boom, arm and bucket motion. The Lagrange's equations are derived, for this complex system, with four degrees of freedom. It is assumed that the separate hydraulic actuator, with limited power, drives each of the degrees of freedom.
The obtained equations are solved in a recursive way. First, the dynamic of the bucket is considered. Then, the actuator force between the bucket and the arm is applied to the latter. This way, step by step, the dynamics of the boom and excavator body are solved. If some of the actuator forces exceed their limited values, we come back to the bucket increasing the assumed time of discharging. This

causes decreases accelerations and then dynamic forces.

The paper presents a pre-shape input to the discussed control system. It is illustrated with the simplest, possible problem of lifting the excavator bucket in horizontal position.

2. KINEMATICS

Consider an excavator attachment, composed of boom, arm and bucket. Their lengths and angels of rotations are respectively l_1, a_1, l_2, a_2, l_3, a_3.

The planar attachment can rotate, together with the excavator booth, by an angle j? (Fig. 1). The operator of the machine is assuming path $\boldsymbol{x_p}$ of the bucket tip in 3D space, together with a_3, the angle of the bucket. The system is driven by hydraulic actuators. It is the aim of the study to design a control system, allowing to travel of the end effector, from initial point $\boldsymbol{x_{po}}$ to a final one $\boldsymbol{x_{pf}}$ in the shortest time.

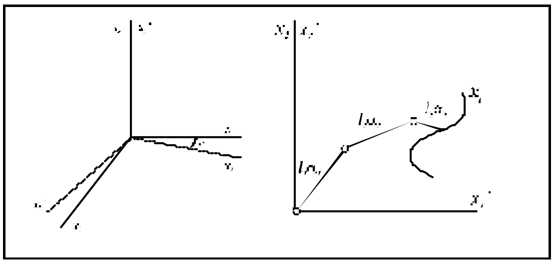

Fig. 1. Coordinate system of excavator's attachment

Denoting by $\boldsymbol{x_p}$ position of the effecter tip, the kinematics of considered mechanism is represented by the vector relation:

$$\boldsymbol{x}_p = \boldsymbol{A}\boldsymbol{l}; \quad \boldsymbol{A} = \begin{bmatrix} c_1c_4 & c_2c_4 & c_3c_4 \\ s_1 & s_2 & s_3 \\ c_1s_4 & c_1s_4 & c_1s_4 \end{bmatrix}; \tag{1}$$

$$\boldsymbol{l}^T = \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix}$$

where c_j and s_j denote $\cos a_j$ and $\sin a_j$ respectively. In further considerations the sub index p is omitted as the position of the only of points of the assumed path are considered.

Velocity of the point $\boldsymbol{P}$ $\mathbf{v} = [v_1, v_2, v_3]^T = [\dot{x}_1, \dot{x}_2, \dot{x}_3]^T$ is obtained by taking time derivative of (1):

$$\dot{\boldsymbol{x}} = \dot{\boldsymbol{A}} \cdot \boldsymbol{l} \tag{2}$$

where:

$$\dot{\boldsymbol{A}} = \dot{\boldsymbol{A}}_1 \cdot \dot{a} + \dot{\boldsymbol{A}}_2 \cdot \dot{j};$$

$$\dot{\boldsymbol{A}}_1 = \begin{bmatrix} -s_1c_4l_1 & -s_2c_4l_2 & -s_3c_4l_3 \\ c_1l_1 & c_2l_2 & c_3l_3 \\ -s_1s_4l_1 & -s_2s_4l_2 & -s_3s_4l_3 \end{bmatrix}; \tag{3}$$

$$\dot{\boldsymbol{A}}_2 = \begin{bmatrix} -c_1s_4l_1 & -c_2s_4l_2 & -c_3s_4l_3 \\ 0 & 0 & 0 \\ c_1c_4l_1 & c_2c_4l_2 & c_3c_4l_3 \end{bmatrix}$$

Taking inverse of the $\dot{\boldsymbol{A}}$ matrix, we find the relation of angular velocities $\dot{a}_1, \dot{a}_2, \dot{j}$ of mechanism elements with the tip displacement vector. These angular velocities, in turn, are dependant on the elongation velocities of hydraulic cylinders. This dependence has to be determined from geometrical relations between, cylinders lengths, constant parameters of attachment and a_j. Detailed derivations of these relations are given in [4].

In further considerations, the problem is discretized. The path between $\boldsymbol{x_0}$ and $\boldsymbol{x_f}$ is divided in k_0 elements. Unknowns are the time intervals T_k needed for the bucket tip to travel along k-th element of the path.

3. DYNAMICS

Consider a free body diagram of the third element (the bucket) of the system (Fig.2). Gravity forces of the bucket and soil are known. Unknown are reaction forces at the point O'_3, being the hinge, between second and third elements. Unknown is also the control moment $\boldsymbol{M_{C3}}$ of the third hydraulic actuator. It is limited ($-\boldsymbol{M}_{C30} \le \boldsymbol{M}_{C3} \le \boldsymbol{M}_{C30}$) by the pressure in the actuator cylinder. Taking moment equation with respect to the hinge O'_3, we come to a vector equation with only one unknown vector and unknown velocities and accelerations $\dot{a}_3, \ddot{a}_3, \dot{j}, \ddot{j}$. The unknowns are found from the solution of optimum control discussed in the next chapter.

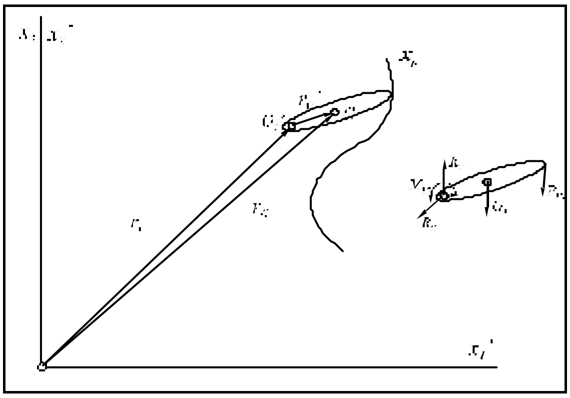

Fig. 2. Free body diagram.

The discussed moment equation can be found from known dynamic relation

$$\frac{d\boldsymbol{J}}{dt} = \dot{\boldsymbol{J}} = \underbrace{\boldsymbol{M}_{3ext}}_{known} + \underbrace{\boldsymbol{M}_{3C}}_{unknown} \qquad (4)$$

where $\boldsymbol{J}$ is the angular momentum of the considered body with respect to O_3', given by the following dynamic relation:

$$\boldsymbol{J}_{O_3'} = \left(\boldsymbol{r}_{3C} \times M_3 \dot{\boldsymbol{r}}_{30}\right) + \dot{\boldsymbol{r}}_{30} \times M_3 \left(w_3 \times r_{3C}'\right) + I_{O_3'} w_3 \qquad (5)$$

where (Fig. 2):

$\boldsymbol{r}_{30} = \boldsymbol{x} - \boldsymbol{l}_3$ - vector between the center of mass *C* and O_3'

M_3 is the mass of the bucket,

w - vector of the angular velocity equal to $\dot{\boldsymbol{a}}_3 + \dot{j}$, and

$I_{O_3'}$ - moment of inertia of the body with respect to O_3' .

After discretization of (4) we get:

$$\boldsymbol{J}_{k+1} - \boldsymbol{J}_k - T_k \boldsymbol{M}_{3ext} - T_k \boldsymbol{M}_{3C} = 0 \qquad (6)$$

with constraints imposed on control torque $-\boldsymbol{M}_{C30} \leq \boldsymbol{M}_{C\,3} \leq \boldsymbol{M}_{C\,30}$.

4. OPTIMUM CONTROL PROBLEM (MINIMUM TIME OF TRAVELING FROM $\boldsymbol{x}_0$ TO $\boldsymbol{x}_f$.)

Consider a minimum time problem by assuming cost function as a sum of all T_k . There are two kinds of constraints – equality and inequality constraints. The first one are moment equations. The latter one, limitations imposed on control moment given with (6). With the following assumptions and notation, our optimum control problem, for the third body, can be stated as follows.

Find:

$$\sum_{k=1}^{k_0} T_k \rightarrow min \qquad (7)$$

under constraints expressed by (6).

The Lagrangian of the optimum problem is given by:

$$L = -\sum_{k=1}^{k_0} \left[T_k + ?_{1k}\left(\boldsymbol{J}_{k+1} - \boldsymbol{J}_k - \boldsymbol{T}_k \boldsymbol{M}_{3ext} - \boldsymbol{T}_k \boldsymbol{M}_{3Ck}\right) + \right.$$
$$\left. + ?_{2k}\left(\boldsymbol{M}_{3Ck} - \boldsymbol{M}^0_{3Ck}\right) + ?_{3k}\left(\boldsymbol{M}_{3Ck} + \boldsymbol{M}^0_{3Ck}\right)\right] \qquad (8)$$

Optimality conditions, in the form of derivatives of *L* with respect to design variables T_k and M_{3Ck} for $k = 1,2,\ldots, k_0$ and $i = 1,2,3$ are:

$$\frac{dL}{dT_k} = -1 - ?_{1k}\left(\boldsymbol{M}_{3ext} + \boldsymbol{M}_{3Ck}\right) = 0$$
$$\frac{dL}{dM_{3iCk}} = -l_{1k} T_k + l_{2ik} + l_{3ik} = 0 \qquad (9)$$
$$l_{2k}\left(M_{3ext} - M^0_{3Ck}\right) = 0$$
$$l_{3k}\left(M_{3ext} + M^0_{3Ck}\right) = 0$$

Inspecting (9), we come to the conclusion, that control moment should have one of its extreme values: M^0_{3Ck} or $-M^0_{3Ck}$. Bearing in mind Pontriagin’s general results, we assume one control switching, changing $-M^0_{3Ck}$ to M^0_{3Ck}. It means that we have two sets of equations (4) and (6) with positive and negative value of M^0_{3Ck}

It means, they have to be solved with different initial-boundary conditions. They are:

$$\begin{aligned} & t = t_0; \quad a_3 = a_{30}; \quad \dot{a}_3 = 0; \\ & j = j_{30}; \quad \dot{j}_{30} = 0; \quad \boldsymbol{x} = \boldsymbol{x}_0; \\ & t = t_f; \quad a_3 = a_{3f}; \quad \dot{a}_3 = 0; \\ & j = j_{3f}; \quad \dot{j}_{3f} = 0; \quad \boldsymbol{x} = \boldsymbol{x}_f; \end{aligned} \qquad (10)$$

Once the time t_f of control system is known, we can solve all dynamic equations of the body, finding hinge reactions. Now, we can proceed to the dynamics of the second element (arm). The considerations are exactly the same as with the bucket. We find angular momentum and then moment equations with respect to O_2', hinge joining first (boom) and second (arm) elements. From these equations, we find control moment $\boldsymbol{M_{2Ck}}$, needed to move the second element, within the time found solving control problem of the first element. Next, we proceed, in the same way, to the first element finding $\boldsymbol{M_{1Ck}}$. Finally, we write the moment equation for the booth, again finding its control moment $\boldsymbol{M_{bCk}}$. From all four elements we are getting control moments. If moments for the booth, the first and second elements are smaller than assumed limit values, then the problem is solved. If one the mentioned moments is larger than its limit value, we have to go back to the first element, decreasing its $\boldsymbol{M^0_{3Ck}}$, verifying again, through dynamic equations, if the remaining moments are within assumed limit values.

5. EXAMPLE

In order to illustrate the method, in a relatively simple way, we discuss below a planar motion of an excavator bucket along a vertical line (Fig. 3). Additionally, the bucket during all its way is remaining in a horizontal position.

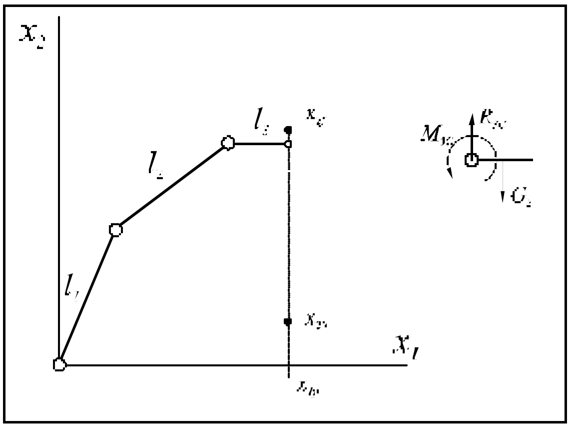

Fig. 3. Bucket motion along vertical line.

We have to find t_f, minimum traveling time from x_{20} to x_{2f}. To make the presentation more clear, all the attachment elements are replaced by straight beams of a constant thickness. We start writing moment equation for the bucket. Due the fact that the motion is planar and that the bucket remains horizontal, its angular velocity is equal to zero. Substituting in (4) and (5) for :

$$\boldsymbol{r}_0 = \left(x_{1p} - l_3\right)\boldsymbol{i}_1 + x_{2p}\boldsymbol{i}_2; \quad \boldsymbol{r}'_C = \frac{l_3}{2}\boldsymbol{i}_1;$$

$$\dot{\boldsymbol{r}}_0 = x_{2p}\boldsymbol{i}_2; \quad \boldsymbol{r}_C = \left(x_{1p} - \frac{l_3}{2}\right)\boldsymbol{i}_1 + x_{2p}\boldsymbol{i}_2;$$

we get, after transformations, the following moment equation of the third element with respect to O'_3:

$$\ddot{x}_{2p}\left(x_{1p} - \frac{l_3}{2}\right) = -G_3\frac{l_3}{2} + M_{3C};$$

$$-M^0_{3C} \le M_{3C} \le M^0_{3C}$$

or in a dis cretized form

$$x_{2p,k+1} - 2x_{2p,k} + x_{2p,k-1} + \overline{G}_3 - u_{3k} = 0$$

where

$$\overline{G}_3 = G_3\frac{l_3}{\left(2x_{1p} - l_3\right)}; \qquad u_{3k} = \frac{2M_{3Ck}}{2x_{1p} - l_3}$$

Recalling the above discussion about optimality criteria, we come to two distinct equations of motion. The first one for the positive u^0_3 and the second for its negative value:

$$\ddot{x}^{(1)}_{2p} = -\overline{G}_3 + u^0_3; \qquad \ddot{x}^{(2)}_{2p} = -\overline{G}_3 - u^0_3;$$

With boundary conditions (10), velocities for both extreme values of control moments are:

$$\dot{x}^{(1)}_{2p} = \left(-\overline{G}_3 + u^0_3\right)t; \quad \dot{x}^{(1)}_{2p} = \left(\overline{G}_3 t + u^0_3\right)\left(t_f - t\right)$$

The intersection of these two lines gives the control switch time t_s with respect to t_f :

$$t_s = \frac{\overline{G}_3 + u^0_3}{2u^0_3}t_f$$

From the condition that the distance $x_{2f} - x_{20}$ is a sum of distances with $+u^0_3$ and $-u^0_3$ active, we find finally minimum time.

$$t_f = 2\sqrt{\frac{x_{2pf} - x_{2p0}}{B}}$$

where

$$B = \frac{\left(u + \bar{G}_3\right)}{2}\cdot\left[\frac{\bar{G}^2_3 - u^2}{4u^2} + \left(1 - \frac{u + \bar{G}_3}{2u}\right)^2\right]$$

Knowing t_f we can find time dependent motion of the bucket. Its motion equations with respect to the center of mass, recalling that $\dot{a}_3 = 0$ (Fig. 3) are:

$$\frac{G_3}{g}\ddot{x}_{2p} = R_{32} - G_3; \qquad R_{32}\frac{l_3}{2} = M_{3C}$$

Now we can proceed, in the same way, to the second member (arm) shown in Fig.4, and then to the first one (boom). Finally, we verify obtained values of M_{2C} and M_{1C} if they do not exceed their assumed limits M_{2C}^{0} and M_{1C}^{0}. If this is the case, the problem of minimum time is solved. If one, or both, of the moments are larger than their limits, we go back to the bucket, decreasing value of M_{3C}^{0}. The percent by which we decrease it, is equal to the largest percent of violation of M_{2C}^{0} or M_{1C}^{0}.

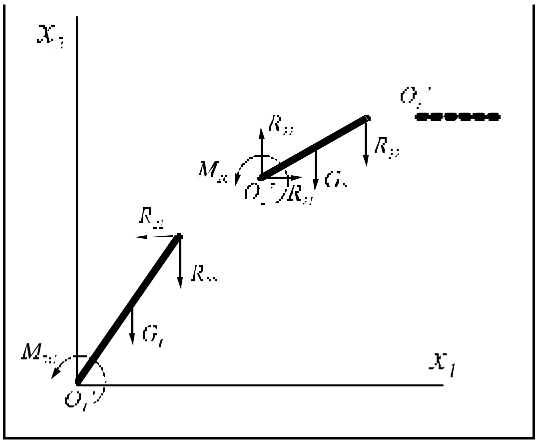

Fig. 4. External forces loading attachment elements.

6. CONCLUSIONS

Minimum time, 3D problems of multibody dynamic system control, with several degrees of freedom and arbitrary constraints, are very complex. Nonlinear state equations, together with constraints, and optimality conditions, constitute a system of equations and inequalities of a very limited value for applications.

In the present study, we propose an approach allowing to decompose the controlled system in separate elements, considering them in a recursive way. This can be applied to bracket kinematic chains. From presented consideration we find minimum time needed to travel of the end effector of the system, along a prescribed path. Beside that, it shown that control moments are taking always their extreme values – positive or negative. The presented relations allow to find the switch time in this ban-bang problem.

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