Analytic Form Solution of the Forward Position Analysis of Three-Legged Parallel Mechanisms Generating SR-PS-RS Structures

R. Di Gregorio

***Abstract*— Spatial parallel mechanisms (SPMs) become parallel structures when the actuators are locked. Parallel structures are constituted by two rigid bodies (platform and base) connected by a number of kinematic chains (legs) with only passive kinematic pairs. A set of SPMs is the one collecting the mechanisms which become SR-PS-RS structures. Such structures have three legs: one leg of type SR, another leg of type PS and the remaining one of type RS (P, R and S stand for prismatic pair, revolute pair and spherical pair respectively). The analytic determination of the assembly modes of the SR-PS-RS structures has not been presented in the literature, yet. This paper presents an algorithm that analytically determines the assembly modes of the SR-PS-RS structures [i.e. that analytically solves the forward position analysis (FPA) of the SPMs that become SR-PS-RS structures when the actuators are locked]. In particular, the closure equation system of a generic SR-PS-RS structure is written in the form of three non-linear equations in three unknowns. The solution of the non-linear system is reduced to the determination of the roots of a twelfth-degree univariate polynomial equation plus a simple back substitution procedure. The proposed solution algorithm is applied to a real case. The result of this study is that the solutions of the FPA of all these SPMs are at most twelve and can be analytically determined through the proposed algorithm.**

***Index Terms*— kinematics, position analysis, parallel mechanisms, parallel structure.**

I. Introduction

SPATIAL parallel mechanisms (SPMs) are traditionally used as testing machines [1] or as simulators [2]. Moreover, some tendon-driven machines have a parallel architecture [see [3] for Refs.]. These applications attracted the interest of factories and research groups that operate in the field of the civil constructions and of the building maintenance [4-9] together with the interest of other engineering fields [see [3] and [10] for Refs.].

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SPMs are constituted by one or more-than-one closed loops and can be described as a number of kinematic chains (legs) that in parallel connect the output link (end effector) to the frame. Such mechanisms require a number of kinematic pairs much greater than their number of degrees of freedom (dof). Therefore, they always contain many non-actuated kinematic pairs (passive joints) together with a number of actuated kinematic pairs (active joints) equal to the mechanism’s dof.

These complex architectures inevitably bring complex kinematic and dynamic behaviors and SPMs’ diffusion is conditioned by the solution of the theoretical problems related to them. For instance, SPMs’ design and control cannot leave aside the identification of their singular configurations (singularities) and the solution of their position analysis [3]. This paper is focused on the position analysis of a family of three-legged SPMs.

The solution of the position analysis involves the solution of two problems: the forward position analysis (FPA) and the inverse position analysis (IPA). The FPA is the determination of the end-effector poses (positions and orientations) compatible with assigned values of the joint variables of the active joints. The IPA is the determination of the values of the joint variables of the active joints compatible with an assigned end-effector pose. Both FPA and IPA are important in the definition of the algorithm that controls the mechanism motion.

A problem widely discussed in the literature has been the FPA of the Stewart platforms [11-15]. Stewart platforms are SPMs with six legs that, for the solution of their FPA, can be modeled as six variable-length segments (Fig. 1). In the Stewart platforms, the end-effector pose is controlled by controlling the lengths of the six segments. The variable length segments can be realized by using different type of hardware, for instance, tightened wires driven by pulleys [6] or kinematic chains of type UPS (U, P and S stand for universal joint, prismatic pair and spherical pair respectively) [2]. Different types of Stewart platforms are obtained by making the legs’ attaching points coincide in the end effector and/or in the frame [11]. The IPA of the Stewart platforms is easy to solve; whereas their FPA involves the solution of non-linear equation

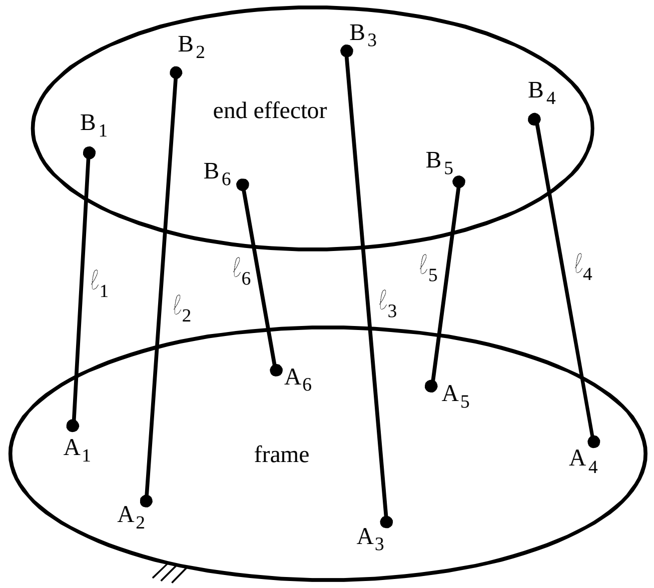

Fig. 1: General Stewart platform

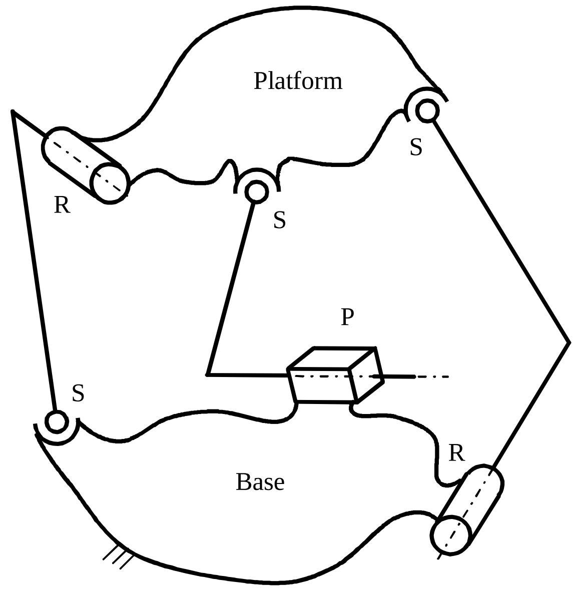

Fig. 2: Structure with topology SR-PS-RS (R, P and S stand for revolute, prismatic and spherical pairs, respectively)

systems and is not an easy problem even though the general case has been recently solved [12-14]. In general, both the IPA and the FPA of an SPM involves the solution of non-linear equation systems and are not easy to solve (see for instance [16]).

When all the active joints are locked (or, which is the same, the values of all the joint variables of the active joints are assigned and kept constant), SPMs become closed structures where two rigid bodies (base and platform) are in parallel connected by a number of kinematic chains (legs) with only passive joints. If the topology of two or more-than-two SPMs differ only for the type and/or the location of their active joints, then they generate closed structures with the same topology when the active joints are locked. As a consequence, some properties of the closed structures are strictly related to properties shared by all the SPMs that generate those closed structures. In particular, the assembly modes, without link permutations, of a closed structure one-to-one correspond to the solutions of the FPA of all the SPMs that generate that structure. Therefore, an algorithm that computes the assembly modes of closed structures with given topology can be applied to solve the FPA of all the SPMs that generate structures with that topology.

A wide set of SPMs is constituted by the mechanisms that become closed structure with topology SR-PS-RS (Fig. 2) where the platform and the base are connected by three legs: one of SR type, another of PS type and the remaining one of RS type (R stands for revolute pair). An algorithm that analytically solves the FPA of all the mechanisms of this set is not presented in the literature yet [17].

This paper presents an algorithm that solves in analytical form the FPA of all the SPMs that generate structure with SR-PS-RS topology when the active joints are locked.

In particular, firstly, the closure equation system of a generic SR-PS-RS structure will be written in the form of three non-linear equations in three unknowns. Then, the solution of the non-linear system will be reduced to the determination of the roots of a univariate-twelfth-degree polynomial equation plus a simple back substitution procedure.

Finally, the proposed solution algorithm will be applied to a real case.

The result of this study is that the solutions of the forward position analysis of all the SPMs that generate structures with SR-PS-RS topology, when the active joints are locked, are at most twelve and can be analytically determined through the proposed algorithm.

II. Closure Equations

Figure 3 shows a generic SR-PS-RS structure and the notations that will be used. With reference to Fig. 3, A is the center of the spherical pair of the base. A_0 is the foot of the perpendicular through A to the axis of platform's revolute pair which is parallel to the unit vector **m**. **m** and **n** are two unit vectors fixed in the platform and mutually orthogonal. θ is the joint variable of platform's revolute pair (θ is equal to zero when the vector ($\mathbf{A}$–$\mathbf{A}_0$) is parallel to **n** and has the same direction as **n**). a is the length of the segment AA_0. B is the center of the spherical pair that joins the platform to the RS leg. B_0 is the foot of the perpendicular through B to the axis of base's revolute pair which is parallel to the unit vector **u**. **u** and **v** are two unit vectors fixed in the base and mutually orthogonal. φ is the joint variable of base's revolute pair (φ is equal to zero when the vector ($\mathbf{B}$–$\mathbf{B}_0$) is parallel to **v** and has the same direction as **v**). b is the length of the segment BB_0. C is the center of the spherical pair that joins the platform to the PS leg. **s** is a unit vector parallel to the sliding direction of the

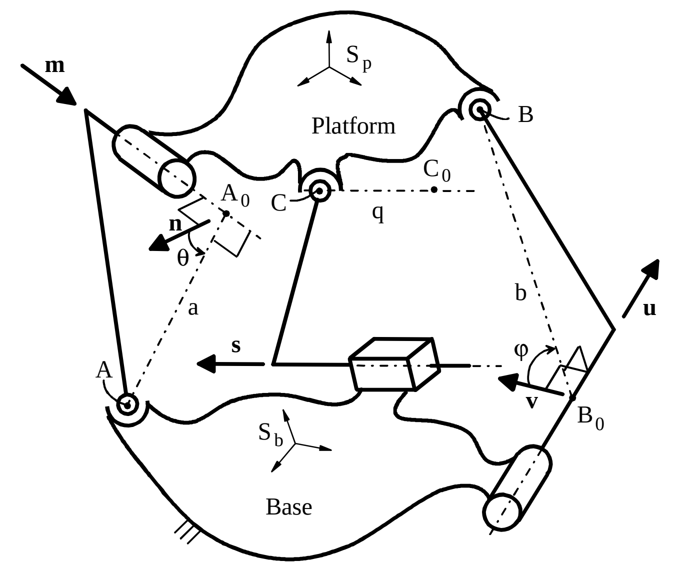

Fig. 3: Notations

prismatic pair. C_0 is a base's point lying on the line through C that is parallel to **s**. q is the joint variable of the prismatic pair and is the signed distance from C_0 to C (q is positive when the vector ($\mathbf{C}-\mathbf{C}_0$) has the same direction as **s**). S_b and S_p are two Cartesian reference systems embedded in the base and in the platform respectively. Hereafter, the vectors with a superscript b (p) on the left are projected on S_b (S_p) and, if they are position vectors, they are defined in S_b (S_p) and projected on S_b (S_p). Finally, d will be the length of the segment BC.

With these notations, the closure equations of a SR-PS-RS structure can be written in the following way:

$$({}^b\mathbf{B}-{}^b\mathbf{C})\cdot({}^b\mathbf{B}-{}^b\mathbf{C})=d^2 \tag{1a}$$
$$({}^b\mathbf{B}-{}^b\mathbf{A})\cdot({}^b\mathbf{B}-{}^b\mathbf{A})=({}^p\mathbf{B}-{}^p\mathbf{A})\cdot({}^p\mathbf{B}-{}^p\mathbf{A}) \tag{1b}$$
$$({}^b\mathbf{C}-{}^b\mathbf{A})\cdot({}^b\mathbf{C}-{}^b\mathbf{A})=({}^p\mathbf{C}-{}^p\mathbf{A})\cdot({}^p\mathbf{C}-{}^p\mathbf{A}) \tag{1c}$$

where

$${}^p\mathbf{A}={}^p\mathbf{A}_0+a\,({}^p\mathbf{n}\cos\theta+{}^p\mathbf{m}\times{}^p\mathbf{n}\sin\theta) \tag{2a}$$
$${}^b\mathbf{B}={}^b\mathbf{B}_0+b\,({}^b\mathbf{v}\cos\varphi+{}^b\mathbf{u}\times{}^b\mathbf{v}\sin\varphi) \tag{2b}$$
$${}^b\mathbf{C}={}^b\mathbf{C}_0+q\,{}^b\mathbf{s} \tag{2c}$$

The introduction of (2) into (1) transforms system (1) as follows

$$e_1\cos\varphi+f_1\sin\varphi=g_1 \tag{3a}$$
$$e_2\cos\varphi+f_2\sin\varphi=g_2+j_1\cos\theta+k_1\sin\theta \tag{3b}$$
$$j_2\cos\theta+k_2\sin\theta=g_3 \tag{3c}$$

where

$$e_1=e_{01}+q\,e_{11}; \tag{4a}$$
$$e_{01}=2\,b\,({}^b\mathbf{B}_0-{}^b\mathbf{C}_0)\cdot{}^b\mathbf{v}; \tag{4b}$$
$$e_{11}=-2\,b\,{}^b\mathbf{v}\cdot{}^b\mathbf{s}; \tag{4c}$$
$$f_1=f_{01}+q\,f_{11}; \tag{4d}$$
$$f_{01}=2\,b\,({}^b\mathbf{B}_0-{}^b\mathbf{C}_0)\cdot({}^b\mathbf{u}\times{}^b\mathbf{v}); \tag{4e}$$
$$f_{11}=-2\,b\,({}^b\mathbf{u}\times{}^b\mathbf{v})\cdot{}^b\mathbf{s}; \tag{4f}$$
$$g_1=-q^2+g_{11}\,q+g_{01}; \tag{4g}$$
$$g_{01}=d^2-b^2-({}^b\mathbf{B}_0-{}^b\mathbf{C}_0)\cdot({}^b\mathbf{B}_0-{}^b\mathbf{C}_0); \tag{4h}$$
$$g_{11}=2\,({}^b\mathbf{B}_0-{}^b\mathbf{C}_0)\cdot{}^b\mathbf{s}; \tag{4i}$$
$$e_2=2\,b\,({}^b\mathbf{B}_0-{}^b\mathbf{A})\cdot{}^b\mathbf{v}; \tag{4j}$$
$$f_2=2\,b\,({}^b\mathbf{B}_0-{}^b\mathbf{A})\cdot({}^b\mathbf{u}\times{}^b\mathbf{v}); \tag{4k}$$
$$j_1=-2\,a\,({}^p\mathbf{B}-{}^p\mathbf{A}_0)\cdot{}^p\mathbf{n}; \tag{4l}$$
$$k_1=-2\,a\,({}^p\mathbf{B}-{}^p\mathbf{A}_0)\cdot({}^p\mathbf{m}\times{}^p\mathbf{n}); \tag{4m}$$
$$g_2=a^2-b^2+({}^p\mathbf{B}-{}^p\mathbf{A}_0)\cdot({}^p\mathbf{B}-{}^p\mathbf{A}_0)-({}^b\mathbf{B}_0-{}^b\mathbf{A})\cdot({}^b\mathbf{B}_0-{}^b\mathbf{A}); \tag{4n}$$
$$j_2=2\,a\,({}^p\mathbf{C}-{}^p\mathbf{A}_0)\cdot{}^p\mathbf{n}; \tag{4o}$$
$$k_2=2\,a\,({}^p\mathbf{C}-{}^p\mathbf{A}_0)\cdot({}^p\mathbf{m}\times{}^p\mathbf{n}); \tag{4p}$$
$$g_3=-q^2+g_{13}\,q+g_{03}; \tag{4q}$$
$$g_{13}=-2\,({}^b\mathbf{C}_0-{}^b\mathbf{A})\cdot{}^b\mathbf{s}; \tag{4r}$$
$$g_{03}=a^2+({}^p\mathbf{C}-{}^p\mathbf{A}_0)\cdot({}^p\mathbf{C}-{}^p\mathbf{A}_0)-({}^b\mathbf{C}_0-{}^b\mathbf{A})\cdot({}^b\mathbf{C}_0-{}^b\mathbf{A}). \tag{4s}$$

Equations (3) form a system of three non-linear equations in three unknowns: φ, θ and q.

An one-to-one correspondence exists between the triplets of (φ, θ, q) values that satisfy system (3) and the assembly modes of the SR-PS-RS structure.

In the next section, an equation in the only variable φ will be obtained by the successive elimination of θ and q from Eqs. (3), then an analytic solution procedure for system (3) will be proposed.

III. Analytic Solution

Equation (3b) and (3c) are linear in cosθ and sinθ and give the explicit expressions

$$\cos\theta=\frac{t\,k_2-k_1g_3}{j_1k_2-k_1j_2},\quad \sin\theta=\frac{j_1g_3-t\,j_2}{j_1k_2-k_1j_2}, \tag{5}$$

where $t = e_2\cos\varphi + f_2\sin\varphi - g_2$. Expressions (5) depend only on q and φ.

The introduction of expressions (5) into the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ yields

$$(t\,k_2-k_1g_3)^2+(j_1g_3-t\,j_2)^2=(j_1k_2-k_1j_2)^2 \tag{6}$$

Equation (6) contains only the two unknowns φ and q. The couples of (φ, q) values that solve Eq. (6) are also solutions of Eqs. (3b) and (3c) where cosθ and sinθ are computed through expressions (5).

By expanding the expressions appearing in (6) and taking into account (4), Eq. (6) becomes

$$h_4\,(q^2)^2+h_3\,(q^2)\,q+h_2\,(q^2)+h_1\,q+h_0=0 \tag{7}$$

where

$h_4 = k_1^2 + j_1^2$; $h_3 = -2\, g_{13}\, h_4$; (8a)

$h_2 = (g_{13}^2 - 2\, g_{03})\, h_4 + 2\, t\, (k_1k_2 + j_1j_2)$; (8b)

$h_1 = 2\, g_{13}\, g_{03}\, h_4 - 2\, t\, g_{13}\, (k_1k_2 + j_1j_2)$; (8c)

$h_0 = g_{03}^2\, h_4 - 2t\, g_{03}\, (k_1k_2+j_1j_2)+t^2\, (k_2^2+j_2^2)-(j_1k_2-k_1j_2)^2$. (8d)

Moreover, Eq. (3a) can be rearranged as follows

$$q^2 = w_1\, q + w_0 \qquad (9)$$

where

$w_1 = -e_{11} \cos\varphi - f_{11} \sin\varphi + g_{11}$; (10a)

$w_0 = -e_{01} \cos\varphi - f_{01} \sin\varphi + g_{01}$; (10b)

Equations (7) and (9) are two polynomial equations in q whose coefficients depend only on φ. Such equations constitute a system of two equations in the two unknowns φ and q whose solutions coincide with the solutions of system (3) where cosθ and sinθ are computed through expressions (5).

By substituting the right-hand side of Eq. (9) for q^2 into Eq. (7), expanding the resulting equation and, finally, substituting again the right-hand side of Eq. (9) for q^2 into the resulting equation, Eq. (7) becomes

$$r_1\, q + r_0 = 0, \qquad (11)$$

where

$r_1 = (w_0 + w_1^2)(w_1\, h_4 + h_3) + w_1\, (w_0\, h_4 + h_2) + h_1$, (12a)

$r_0 = w_0\, [w_1\, (w_1\, h_4 + h_3) + w_0\, h_4 + h_2] + h_0$. (12b)

Equation (11) yields

$$q = -\frac{r_0}{r_1} \qquad (13)$$

that, when introduced into (9), gives the following compatibility equation

$$r_0^2 + w_1\, r_0\, r_1 - w_0\, r_1^2 = 0 \qquad (14)$$

Equation (14) contains only the unknown φ. The analysis of (14) shows that the values of φ which make r_1 equal to zero are solution of (14) only if they also make r_0 equal to zero, and, in addition, the values of φ, that make r_0 and r_1 contemporarily equal to zero, are roots of Eq. (14) with even multiplicity.

By using expressions (12) (see the Appendix), it can be demonstrated that, when r_0 and r_1 are contemporarily equal to zero, the fourth-degree polynomial equation (7) can be factorized as the product of Eq. (9) by a polynomial quadratic in q [i.e. Eq. (9) is contained in Eq. (7)]. Therefore, two values of q are associated to each value of φ, that makes r_0 and r_1 contemporarily equal to zero, in the solution of the system constituted by Eqs. (7) and (9). These two values of q are the two solutions of the quadratic Eq. (9) where the value of φ, that makes r_0 and r_1 contemporarily equal to zero, has been introduced in the expressions of w_0 and w_1 [see (10)].

By introducing the expressions of r_0, r_1, w_0 and w_1 [see (8), (10) and (12)] into Eq. (14) and expanding the resulting equation with the help of an algebraic manipulator [18], Eq. (14) can be written in the following form:

$$\sum_{\substack{\alpha,\beta=0,7\\ \alpha+\beta\le 7}} i_{\alpha\beta} \cos^{\alpha}\varphi \sin^{\beta}\varphi = 0 \qquad (15)$$

where the coefficients $i_{\alpha\beta}$ are geometric constants.

Equation (15) can be transformed into a univariate polynomial equation in x=tan(φ/2) by introducing the following trigonometric identities into (15)

$$\sin\varphi = \frac{2x}{1+x^2}, \quad \cos\varphi = \frac{1-x^2}{1+x^2} \qquad (16)$$

and rationalizing the resulting equation. So doing, with the help of an algebraic manipulator [18], Eq. (15) becomes

$$\sum_{\gamma=0,12} p_\gamma x^\gamma = 0 \qquad (17)$$

where the coefficients p_γ are geometric constants containing only the coefficients $i_{\alpha\beta}$.

Equation (17) is a univariate twelfth-degree polynomial equation in x. Therefore, the values of φ that solve Eq. (14) are at most twelve and can be found by solving Eq. (17) with a standard software package, that gives all the roots of a polynomial equation in the complex field, and by using relationships (16). Equation (17) is the eliminant of system (3).

Finally, it is worth noting that, if $(j_1k_2 - k_1j_2)$ is equal to zero for a given couple of (q, φ) values that solve Eqs. (7) and (9), the two Eqs. (3b) and (3c) are not independent and expressions (5) cannot be used to compute θ. In this case, two values of θ that contemporarily solve Eqs. (3b) and (3c) exist. Both these solutions can be computed by solving the quadratic equation in tan(θ/2) that is obtained from either Eq. (3b) or Eq. (3c) after the introduction of the following trigonometric identities:

$$\sin\theta = \frac{2\tan(\theta/2)}{1+\tan^2(\theta/2)}, \quad \cos\theta = \frac{1-\tan^2(\theta/2)}{1+\tan^2(\theta/2)}. \qquad (18)$$

The obtained results suggest the implementation of the following algorithm for solving the closure equations of a SR-PS-RS structure [ATAN2(y, z) is a standard function which gives the angle between $-\pi$ and π radians whose sine and cosine are $y/(z^2+y^2)^{1/2}$ and $z/(z^2+y^2)^{1/2}$ respectively]:

(a) all the values of x that solve Eq. (17) are computed by using a standard software package that computes all the roots of a polynomial equation in the complex field;

(b) for each real value of x, computed in the previous step, the corresponding values of φ are computed by using relationships (16) and the ATAN2 function;

(c) for each real value of φ, computed in the previous step, the values of r_0 and r_1 are computed by using relationships (12); if r_0 and r_1 are not contemporarily equal to zero, then the corresponding value of q is computed through relationship (13), else the corresponding two values of q are computed by solving Eq. (9);

(d) for each couple of (φ, q) values, calculated in the previous step, if $(j_1k_2 - k_1j_2)$ is not equal to zero, then one value of θ is computed by using relationships (5) and the ATAN2 function; else two values of θ are computed by solving one out of Eqs. (3b) and (3c) after the introduction of relationships (18).

Since the real solutions of Eq. (17) are at most twelve, the assembly modes of any SR-PS-RS structure are at most twelve and the solutions of the forward position analysis of any spatial parallel mechanism, which generates an SR-PS-RS structure when the active joints are locked, are as many.

IV. CASE STUDY

With reference to the introduced notations (see Fig. 3), the proposed procedure has been applied to an SR-PS-RS structure defined by the following data (the lengths are measured in an arbitrary length unit): ${}^b\mathbf{B}_0$ = {30, 0, 10}^{T}, ${}^b\mathbf{C}_0$ = {20, 10, 50}^{T}, ${}^b\mathbf{A}$ = {10, 0, 10}^{T}, ${}^b\mathbf{s}$ = {– 1, 0, 0}^{T}, ${}^b\mathbf{u}$ = {0, 1, 0}^{T}, ${}^b\mathbf{v}$ = {– 1, 0, 0}^{T}, ${}^p\mathbf{A}_0$ = {0, 0, 30}^{T}, ${}^p\mathbf{C}$ = {– 10, 10, 30}^{T}, ${}^p\mathbf{B}$ = {20, 0, 30}^{T}, ${}^p\mathbf{m}$ = {0, –1, 0}^{T}, ${}^p\mathbf{n}$ = {1, 0, 0}^{T}, a = 35 and b = 40.

By using the above-reported procedure, the four real assembly modes listed in Table I and the eight complex assembly modes listed in Table II have been determined.

In Tables I and II, each row corresponds to one assembly mode (real or complex); columns 2, 3 and 4 report the values of the joint variables φ, q and θ; whereas, columns 5, 6 and 7 report the components of the position vectors ${}^b\mathbf{B}$, ${}^b\mathbf{C}$ and ${}^p\mathbf{A}$ that, together with the data, allow closure equations (1) to be easily checked.

All the twelve solutions verify system (1) which proves that the elimination process used to deduce Eq. (17) does not introduce extraneous roots.

V. CONCLUSION

The analytic-form solution of the forward position analysis of all the spatial parallel mechanisms that generate SR-PS-RS

TABLE I: RESULTS OF THE NUMERICAL EXAMPLE: REAL ASSEMBLY MODES OF THE SR-PS-RS STRUCTURE (THE LENGTHS ARE MEASURED IN AN ARBITRARY LENGTH UNIT, WHEREAS φ AND θ ARE MEASURED IN DEGREES).

No.	φ	q	θ	${}^b\mathbf{B}$	${}^b\mathbf{C}$	${}^p\mathbf{A}$
1	14.51	27.6635	–33.01	{–8.72498, 0, 20.01878}^{T}	{–7.66351, 10, 50}^{T}	{29.34998, 0, 10.93226}^{T}
2	14.51	27.6635	33.01	{–8.72498, 0, 20.01878}^{T}	{–7.66351, 10, 50}^{T}	{29.34998, 0, 49.06774}^{T}
3	36.82	–3.33861	–49.68	{–2.02092, 0, 33.97208}^{T}	{23.33861, 10, 50}^{T}	{22.64592, 0, 3.31363}^{T}
4	36.82	–3.33861	49.68	{–2.02092, 0, 33.97208}^{T}	{23.33861, 10, 50}^{T}	{22.64592, 0, 56.68637}^{T}

TABLE II: RESULTS OF THE NUMERICAL EXAMPLE: COMPLEX ASSEMBLY MODES OF THE SR-PS-RS STRUCTURE ($j=\sqrt{-1}$, THE LENGTHS ARE MEASURED IN AN ARBITRARY LENGTH UNIT, WHEREAS φ AND θ ARE MEASURED IN DEGREES).

No.	φ	q	θ	${}^b\mathbf{B}$	${}^b\mathbf{C}$	${}^p\mathbf{A}$
1	–114.87–j 97.80	–12.1327+j 43.7052	–119.45–j107.26	{77.88950 +j 96.73143, 0, –93.31503 + j 44.83781}^{T}	{32.13270 –j 43.70522, 10, 50}^{T}	{–57.26450–j 96.73143, 0, –71.41912+j 54.61768}^{T}
2	–114.87–j 97.80	–12.1327+j 43.7052	119.45 +j 107.26	{77.88950 +j 96.73143, 0, –93.31503 + j 44.83781}^{T}	{32.13270 –j 43.70522, 10, 50}^{T}	{–57.26450–j 96.73143, 0, 131.4191–j 54.61768}^{T}
3	–114.87+j 97.80	–12.1327–j 43.7052	–119.45 +j 107.26	{77.88950 –j 96.73143, 0, –93.31503 –j 44.83781}^{T}	{32.13270 +j 43.70522, 10, 50}^{T}	{–57.26450+j 96.73143, 0, –71.41912–j 54.61768}^{T}
4	–114.87+j 97.80	–12.1327–j 43.7052	119.45 –j 107.26	{77.88950 –j 96.73143, 0, –93.31503 –j 44.83781}^{T}	{32.13270 +j 43.70522, 10, 50}^{T}	{–57.26450+j 96.73143, 0, 131.4191+j 54.61768}^{T}
5	89.21–j 31.81	19.9702+j 23.4323	–101.90+j 36.55	{29.35845 –j 23.36262, 0, 56.31960–j 0.323585}^{T}	{0.0297556 –j 23.43234, 10, 50}^{T}	{–8.73345+j 23.36262, 0, –11.45793–j 4.921525}^{T}
6	89.21–j 31.81	19.9702+j 23.4323	101.90 –j 36.55	{29.35845 –j 23.36262, 0, 56.31960–j 0.323585}^{T}	{0.0297556 –j 23.43234, 10, 50}^{T}	{–8.73345+j 23.36262, 0, 71.45793+j 4.921525}^{T}
7	89.21+j 31.81	19.9702–j 23.4323	–101.90–j 36.55	{29.35845 +j 23.36262, 0, 56.31960+j 0.323585}^{T}	{0.0297556 +j 23.43234, 10, 50}^{T}	{–8.73345–j 23.36262, 0, –11.45793+j 4.921525}^{T}
8	89.21+j 31.81	19.9702–j 23.4323	101.90 +j 36.55	{29.35845 +j 23.36262, 0, 56.31960+j 0.323585}^{T}	{0.0297556 +j 23.43234, 10, 50}^{T}	{–8.73345–j 23.36262, 0, 71.45793–j 4.921525}^{T}

structures when the active joints are locked has been presented.

In particular, since the determination of the assembly modes of the SR-PS-RS structures one-to-one correspond to the FPA solutions of the mechanisms that generate those structures, the closure equation system of a generic SR-PS-RS structure has been written in the form of three non-linear equations in three unknowns. The solution of the non-linear system has been reduced to the determination of the roots of a univariate twelfth-degree polynomial equation with real coefficients plus a simple back substitution procedure.

The proposed solution algorithm is applied to a real case.

The result of this study is that the solutions of the forward position analysis of all the spatial parallel mechanisms that generate SR-PS-RS structures when the active joints are locked are at most twelve and can be analytically determined through the proposed algorithm.

APPENDIX

If r_0 and r_1 contemporarily vanish, then expressions (12) yield:

$$h_1 = -(w_0 + w_1^2)(w_1 h_4 + h_3) - w_1 (w_0 h_4 + h_2), \tag{A.1}$$
$$h_0 = -w_0 [w_1 (w_1 h_4 + h_3) + w_0 h_4 + h_2]. \tag{A.2}$$

The substitution of expressions (A.1) and (A.2) for h_1 and h_0 respectively into Eq. (7) yields

$$h_4 (q^2)^2 + h_3 (q^2) q + h_2 (q^2) - q [(w_0 + w_1^2)(w_1 h_4 + h_3) + w_1 (w_0 h_4 + h_2)] - w_0 [w_1 (w_1 h_4 + h_3) + w_0 h_4 + h_2] = 0 \tag{A.3}$$

Equation (A.3) can be factorized as follows [note that, by expanding (A.3) and (A.4), the resulting equations coincide]:

$$[h_4 q^2 + (w_1 h_4 + h_3) q + w_1 (w_1 h_4 + h_3) + w_0 h_4 + h_2] (q^2 - w_1 q - w_0) = 0 \tag{A.4}$$

If the second factor of the expression at the left-hand side of Eq. (A.4) is equated to zero, an equation that coincide with Eq. (9) is obtained, which proves that, if r_0 and r_1 contemporarily vanish, Eq. (9) is contained in Eq. (7).

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