Cutting and Impedance Model of Soil-Excavator Interaction

Fiorenzo Malaguti

Abstract— **The present work considers a double wedge soil cutting model, suitable to describe the soil-tool interaction in automated digging processes. This one or a similar model will be the base to reach a set of equations by which to fit online typical soil and tool parameters, instead to fit these parameters by time consuming autoregressive methods. The final aim is to reach a predictor of soil-tool interaction to use in the trajectory control of automated digging machine..**

Index Terms— **Automation, digging, soil, online fitting..**

I. Introduction

Many authors have tackled the soil-excavating robot by the numerical identification of a soil cutting model, in practice the numerical methods identify the polynomial coefficients of an equation depending on the cutting depth, *d*, and the cutting angle, *α*, of working tool. The method make simple to build a predictor of the interaction forces, but it's need computing time to converge and some times the convergence isn't certain, moreover the polynomial coefficients don't have a clear physical meaning. On the other hand, the traditional soil mechanics from the agricultural engineering suggested soil cutting models unsuitable for the automated processes, so a double wedge soil cutting model was proposed [1], in order to have a soil model with a physical meaning, that it could be a good base for the numerical identification, but without to require a long computing time to converge.

The necessity to introduce a double wedge is clear, it approximates better the logarithmical spiral shape of soil rupture than a single wedge.

This model, based on the experimental study [2], has allowed to maintain a fixed shape, so to prove the soil cutting frequency depends on the cutting depth, while the single wedge model is unable to prove. However, the double wedge model shown numerical problems and it had to include the 3D extension, this work involves the improved double wedge model to reduce the numerical problem, include the 3D extension.

The author is with the IMAMOTER Institute on Italian National Research Council (CNR), via Canal Bianco 28, 44044 Ferrara,Italy, f.malguti@imamoter.cnr.it .

II. The improved double wedge model

The experimental studies of Makanga, Salokhe and Gee-Clough have shown a soil model where the soil rupture angle *β*, shaped by the free and rupture soil surface, to be in practice constant and, while for a cutting angle $\alpha \le \pi/2 - \phi$ (where ϕ is the internal friction angle of soil) the soil clod could be approximated by a single wedge, for $\alpha > \pi/2 - \phi$ the clod could be approximated by two wedges having a common surface at $\pi/2 - \phi$. The second wedge is contained among the tool surface, the surface of the first wedge at $\pi/2 - \phi$ and the horizontal line connecting the tool edge and the vertex of the first wedge.

Considering fixed the position of this line or, by the discontinuity in the soil rupture shape so created, we introduce numerical and substantial errors, that we weren't able to correct without a long and expensive tests.

McKyes and Ali by small part of their work [3] we have allowed to reconsider the double wedge model, having an apparent validation. As regards previous double wedge model, the base of the wedge on the blade isn't horizontal, but it makes an angle $(\pi/4 + \phi/2 + \delta)$, consequently the angle between the wedge base and horizontal is $\theta = 135° - \delta - \phi/2 - \alpha$. In fact the shape of soil fracture depends on the tool-soil interaction angle *δ*, which has a range for the steel from *1/2* to *7/8* of *ϕ*, so it can be considered as data of the material.

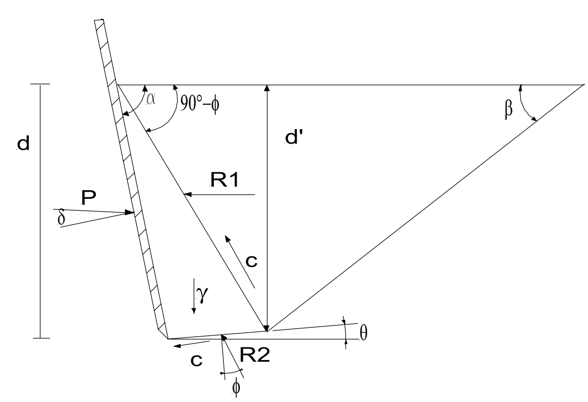

Figure 1 The McKyes's double wedge model

Also the maximum depth of first wedge is variable and equal to

$$d' = d\frac{\cos\phi}{\sin\alpha}\frac{\sin(\pi/4+\delta+\phi/2)}{\sin(-\pi/4+\alpha+\delta+3\phi/2)} \qquad (1)$$

The cutting forces on the blade are carried out, imposing the equilibrium of both wedges, in particular the force R_1 given by the first wedge and working on the second wedge, putted in the following classical form of the general earth-moving equation, where γ and c are respectively the density and the cohesion coefficient of soil, while q is the surcharge,

$$R_1 = \left[\gamma d'^2 N^*_{\gamma H} + cd'N^*_{cH} + qd'N^*_{qH}\right]w \qquad (2)$$

is computed considering the common side of wedges is inclined of $\pi/2-\phi$, the interaction angle is equal ϕ, and the length of wedge is

$$r = d'(\cot(\pi/2-\phi)+\cot\beta)$$

It's possible also to consider the 3D soil cutting model, including the forces produced by two lateral lobes or wedges having circular section, with radius r and a span angle

$$\cos\rho' = d'/r\ \cot(\pi/2-\phi)$$

so the force loading the second wedge is $R = R_1 + 2R_2$.

Well the total cutting force P has the following form, referable anyway to general earth-moving equation

$$P = R_0 + P_\gamma + P_c \qquad (3)$$

where R_0 considers the horizontal and vertical forces due to the first wedge.

In this case with horizontal free surface

$$R_0 = \frac{\cos(\theta+\phi)}{\sin(\alpha+\delta+\phi+\theta)}R$$

$$P_\gamma = \frac{\sin(\theta+\phi)}{\sin(\alpha+\delta+\phi+\theta)}W_2 \qquad (4)$$

$$P_c = \frac{cd\left[\left(1+\frac{d'}{d}\right)\frac{\cos\phi}{\sin\theta} - \frac{d'}{d}\frac{\sin\theta}{\cos\phi}\right]}{\sin(\alpha+\delta+\phi+\theta)}w$$

III. COMPARISON

The double wedge model, carried out from the McKyes's proposal [3], has been compared with the single wedge model, where the β angle is equal to the value, minimizing its *Nγ* coefficient. This comparison is displayed in the fig. 2, where the cutting forces depend on the cutting angle α.

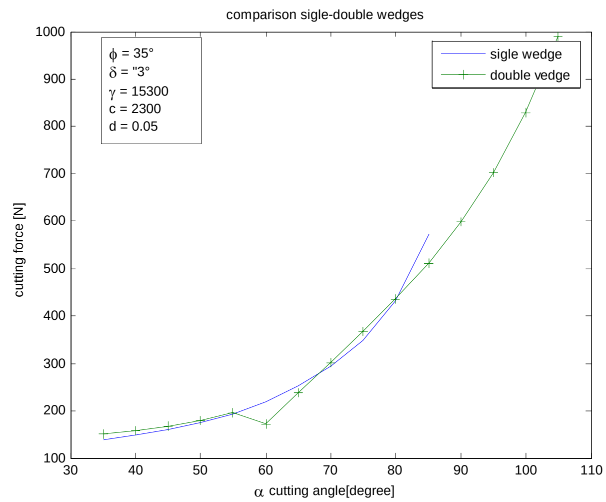

Figure 2 Comparison of cutting forces for single-double wedge models

We can note immediately the discontinuity in the double wedge graph, this problem is well known from the previous work [1], where the previous double wedge model was arranged also for values of α smaller then 90°-ϕ. In this case we wanted to verify the McKyes's proposal, in spite of this discontinuity, the McKyes's model has a better behaviour for angles of α higher then 80° in accordance to McKyes's results, more close probably to the logarithmic spiral shape of the soil failure.

For the same reasons the McKyes's model has a better behaviour than our double wedge model, in fact we supposed the our model in some case can include a more higher volume of soil.

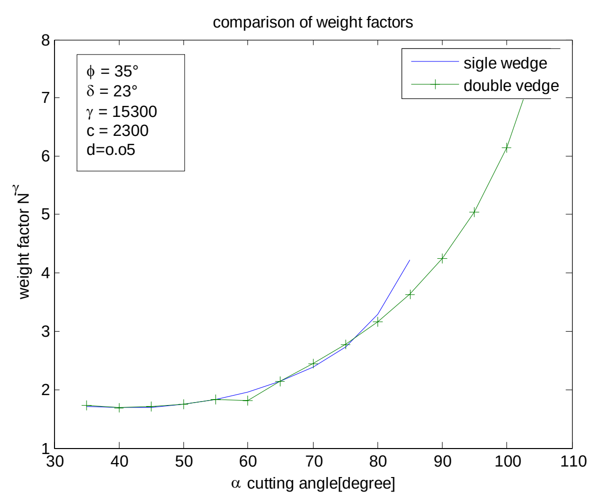

Figure 3 Comparison Nγ factors for single-double wedge models

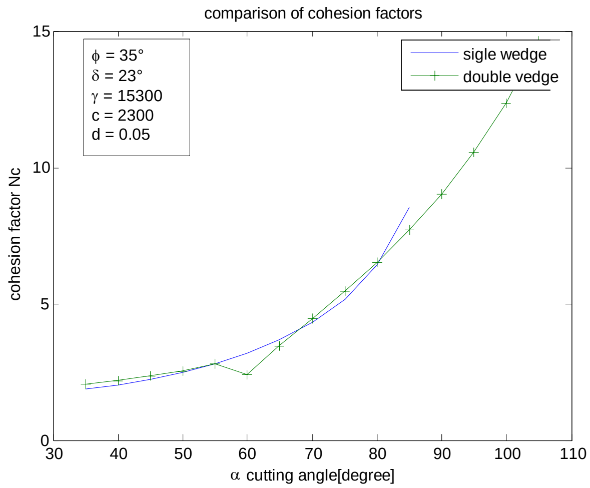

Figura 4 Comparison *Nc* factors for single-double wedge models

IV. IDENTIFICATION

As we can observe, the equations of soil cutting process are very complex, not linear and this nonlinearity is intensified by the large use of trigonometric functions, more over they depend on a lot of parameters.

The fundamental earth-moving equation form seems previously puts in explicit evidence some soil parameters, the density γ, the cohesion coefficient c, and a working parameter the cutting depth d. while the function coefficients N_γ, N_c, N_q, depend on the internal soil friction angle ϕ, the soil-tool friction angle δ, depending itself on ϕ, and the cutting angle α of tool, a working parameter. Moreover we don't consider the surcharge q a parameter, but rather a state of our system, depending on c, ϕ, δ, the tool design and trajectory.

Also the cutting depth d and the cutting angle α aren't easily measurable, because the soil isn't transparent, however it's easy understand, that having accurate values of the soil parameters, it's possible predict the variations of soil cutting force by variations of d and α, or vice versa it could obtain information on the profile of terrain to digging, using the classical Taylor Series expansion

$$P_{k+1} = P_k + \left[\frac{\partial P}{\partial d} \Delta d + \frac{\partial P}{\partial \alpha} \Delta \alpha \right]_k$$

Really, the soil parameters have a large influence on the soil cutting forces and they can change quickly, so we need to identify also this parameters on line. The number of parameters to identify depend on computing time and resources, so we need to choice a small but significant number of them.

We can choice the basic parameters of soil mechanics, γ, ϕ, c, here considered as apparent parameters or "working condition" parameters without to lose of generality, in fact many parameters like humidity or water content, temperature, void index and other depend on the apparent parameter in working condition and only exceptional cases like planetary exploration can justify additional costs to measure other parameters. Moreover , the soil-tool friction angle and cohesion coefficient are related to ϕ and c.

From the cutting force equation it's possible carried out a system of equations

$$\begin{bmatrix} f_1(x_1, x_2, \ldots, x_n, d_1, \alpha_1) \\ f_2(x_1, x_2, \ldots, x_n, d_2, \alpha_2) \\ \vdots \\ f_n(x_1, x_2, \ldots, x_n, d_n, \alpha_n) \end{bmatrix} = 0$$

Applying the cited Taylor series expansion to all the parameters, a matrix representation for the increment Δx_n can be derived

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} = J(x)^{-1} \begin{bmatrix} -f_1(x_1, x_2, \ldots, x_n, d_1, \alpha_1) \\ -f_2(x_1, x_2, \ldots, x_n, d_2, \alpha_2) \\ \vdots \\ -f_n(x_1, x_2, \ldots, x_n, d_n, \alpha_n) \end{bmatrix}$$

where J is the Jacobian matrix of previous function.
So the vector of the parameters can be computed

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{(k+1)} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_k - \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix}$$

Now an estimator based on Newton-Raphson Method can be used for the real-time estimation of the soil parameters.

The N_γ and N_c factors depend fundamentally on the cutting angle α and the internal friction angle of soil, but they are complex trigonometric function so difficult to derive.

The both factors seem to be fitted by a cubic polynomial, depending on α, in figure 5 and example of the fitted $N\gamma$ factor is shown.

This approach could take to a more handy equations of these factor, including a polynomial dependency on ϕ.

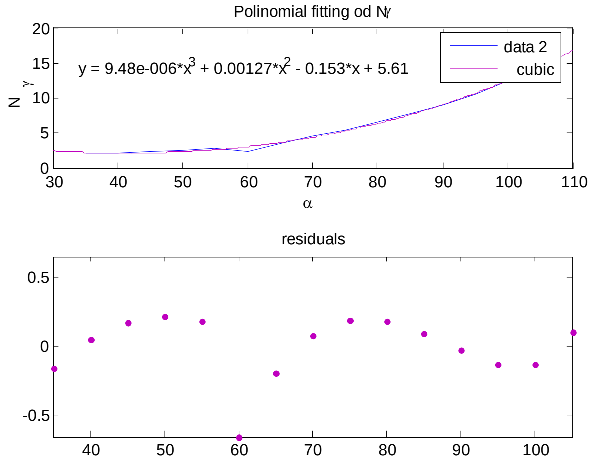

Figura 5 Fitting by cubic polynomial of *Nγ*

V. Conclusion

This work involves about the rightful verification of Mckyes's double wedge model to be suitable for a real-time estimation techniques of the working and soil parameters in automated digging processes, moreover to be an improvement of the double wedge model of soil cutting considered in [1]. In fact Mckyes's double wedge model doesn't need modification of the term, $1/\sin(\alpha+\delta+\phi+\theta)$, moreover the 3D effect can be considered easily.

This verification is certainly positive, in spite of the discontinuity in the model, that we think to solve considering a permanent double wedge model as in [1].

In [1] we have considered an attractive method to measure the cutting depth and the ratio between the weight and cohesive factors by the saw toothed contour of soil cutting forces, but perhaps difficult to use in practice by the strong conditions of soil cutting processes. So here we have introduced a method to estimate soil parameters based on the solution of the system of non linear equation of soil cutting model, more suitable for real-time processes than least square fitting of a long list of data.

References

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