

**AUTOMATED LASER SCANNER 2D POSITIONING AND ORIENTING BY METHOD OF
TRIANGULATION FOR UNDERGROUND MINE SURVEYING**

*Julian V. Simela, Joshua A. Marshall, and Laeeque K. Daneshmend

The Robert M. Buchan Department of Mining

Goodwin Hall, Queen's University

25 Union Street, Kingston, ON Canada K7L 3N6

*(*Corresponding author: julian.simela@mine.queensu.ca)*

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ABSTRACT

Conventional methods of underground surveying use theodolites/total stations and 3D laser scanners to obtain information about the underground environment. Current methods of geo-referencing these instruments to a mine reference system include triangulation, trilateration and resection. However, despite technological advancements, surveying procedures have remained slow, laborious and relatively unchanged during the last half-century. Recent innovations in the robotics community have shown that automated mapping of underground mining tunnels can be undertaken using 2D/3D laser scanners. These techniques have the potential to, in turn, improve upon current underground surveying and mapping methods. Automating surveying and mapping using current surveying tools without changing the setting up procedures or removing current constraints, or even changing the type of equipment used, does not change the status quo. The problem of moving from a static, tripod based surveying and mapping system to an unconstrained mobile surveying and mapping system is the subject of this paper.

A current challenge for automated mapping is the ability to automatically geo-reference a mobile mapping system to a mine reference system. For a mobile mapping system that uses a horizontally mounted 2D laser scanner to gather data of the underground environment, the challenge of determining its position and orientation within the mine environment is magnified even more. This paper introduces Mobile Automated Scanner Triangulation (MAST), a technique under development at Queen's that is designed to geo-reference a mobile mapping system to a mine reference system for the purposes of underground mine surveying. MAST quickly and automatically determines scanner 2D position and orientation (azimuth) in a mine reference frame by using minimal human input.

KEYWORDS

Underground surveying, mobile robotics, Laser-based positioning

INTRODUCTION

Conventional methods of underground surveying use tripod-based theodolites or total stations and 3D laser scanners to obtain information about the underground environment. On surface, satellite-based Global Positioning Systems (GPS) have become integral tools for positioning, as-built mapping, surveying, and setting out. However, GPS technology is unavailable for use in underground mining environments; hence, positioning, mapping, and setting out tasks are not as easily, quickly and accurately accomplished. Recent innovations by the robotics community (Zlot and Bosse, 2012; Artan et al. (2009); Thrun et al. (2003); Huber and Vandapel (2006); Bakambu et al., 2004, 2000) have shown that automated mapping of underground mining tunnels can be accomplished using 2D and 3D laser scanners. However, surveying was not an intended application. These innovations can potentially improve upon current underground surveying and mapping techniques. Currently, automated volumetric mapping of tunnel environments is accomplished using 3D scanners or combinations of 2D scanners in 3D Simultaneous Localization and mapping (SLAM).

Research into automatic positioning specifically for underground mobile mapping has as a major challenge, which is the inability to automatically geo-reference a mobile mapping system to a mine reference system and obtain position and orientation results within standards used by the mining industry. This paper first gives some background on current positioning techniques, then introduces MAST (Mobile Automated Scanner Triangulation), a new positioning concept being developed to realize instantaneous 2D registration for automated underground surveying systems. MAST uses custom designed wall-mounted

beacons and applies surveying techniques through tools used by the robotics community. The MAST 2D positioning and orienting process, as well as algorithms, are discussed, followed by some simulation results and conclusions.

BACKGROUND

Advances in surveying and mapping instrumentation, technology, and techniques have been more expedient for surface in comparison with underground surveying and mapping operations. On surface, these advances have largely come about through use of robotic total stations, automatic levels, laser scanners, GPS, LiDAR, and even satellite-based mapping systems. Surveying and mapping literature and research has similarly focused on the application of new instrumentation and technologies to improve current positioning techniques for surface rather than underground applications.

Geo-referencing procedures in surveying have remained slow, lengthy, and cumbersome (Zlot and Bosse, 2012; Artan et al., 2011; Shaffer and Stentz, 1993; Goldberg and Ream, 1990), requiring significant human input and have been relatively unchanged in the last half century. Despite technological advancements, recent techniques such as highlighted in Purser et al. (2010); Langdon (2009), to geo-reference a robotic vehicle mounted off-the-shelf laser scanner to mine reference points, have not deviated from the norm (Zlot and Bosse, 2012).

Traversing is the primary method for positioning in underground surveying and uses similar procedures to surface surveying. These procedures can be found in various surveying textbooks, such as Anderson and Mikhail (1998); Gracie and Mikhail (1981); Wolf and Ghilani (1997); Schofield and Breach (2007). Specific to mine and tunnel surveying, gyroscopic orientation techniques for control networks are discussed in Lewen (2006). Lam (2010), presents an overview of advances in tunnel engineering surveying operations and briefly discusses methods of transferring surface geodetic networks to underground operations. Optical plummet methods for transferring heights and orientation to underground mining operations are described in Bahuguna (2005). The use of autocollimation and high-precision Inertial Navigation Systems (INS) to transfer orientation into underground workings is outlined in Neuhierl et al. (2006). Legal aspects and regulation of mine surveying is discussed in Cawood et al. (2007) with specific reference to the coal mining industry in South Africa.

Automated mapping is widely discussed in the mobile robotics literature. Online mapping methods relating to Simultaneous Localization and Mapping (SLAM) are discussed in (Madhavan et al., 1998; Bakambu et al., 2004, 2000; Shaffer and Stentz, 1993), as well as many others. Offline SLAM techniques that post-process position and mapping are discussed in Lu and Milius (1997); Artan et al. (2009); Thrun et al. (2003); Huber and Vandapel (2006); Zlot and Bosse (2012). In Zlot and Bosse (2012); Bosse and Zlot (2009), a sweep-matching algorithm for initial 6DOF trajectory estimation is described, which has application to underground mapping.

Surveying is a regulated profession. Current automated robotic mapping has its focus on local rather than global consistency of mapping and is unable to sufficiently articulate accuracy/precision of the mapping process, normalization of the 2D mapping process with current mine mapping standards. That, coupled with the fact that current experimental practice and methodology in mobile robotics is not as well developed and defined as in conventional surveying and mapping (Amigoni et al., 2009), could be significant factors behind difficulties in gaining acceptance by the mining industry.

MAST POSITIONING CONCEPT

Beacon Design and Placement

The MAST process for 2D positioning uses two beacons mounted opposite each other on the walls of a tunnel environment. The beacons are geometric shapes with known design panel lengths p_i , and

known panel deflection angles θ . Typically these would be affixed to the wall using a bolt and levelled to ensure that vertical beacon panels are vertical. The beacons are then surveyed in using a tripod based laser scanner referenced to the mine reference system. Beacon coordinates representing the panel vertical edges would then be extracted from the resulting point cloud data set such that each horizontal and vertical panel edge has a distinct 3D coordinate in the mine reference system. Figure 1 shows a schematic of such a beacon. The three panels P1, P2 and P3 have distinct lengths that will enable distinguishing one panel from the others, for example, by keeping the panel lengths at a constant ratio $p_1 : p_2 : p_3$ of 2:3:1. Similarly, θ_2 , which is the deflection angle between P2 and P3 is always calculated as a function of the depth of the beacon h .

A mobile laser scanner or automated surveying and mapping system placed within visibility and range of the two beacons scans the beacons and the environment in a single horizontal scan. Given known design beacon dimensions and coordinates in the mine reference system and measured distances from the scanner position to the beacons, MAST calculates the scanner centre 2D coordinates and an orientation azimuth indicating the direction the mapping system is facing.

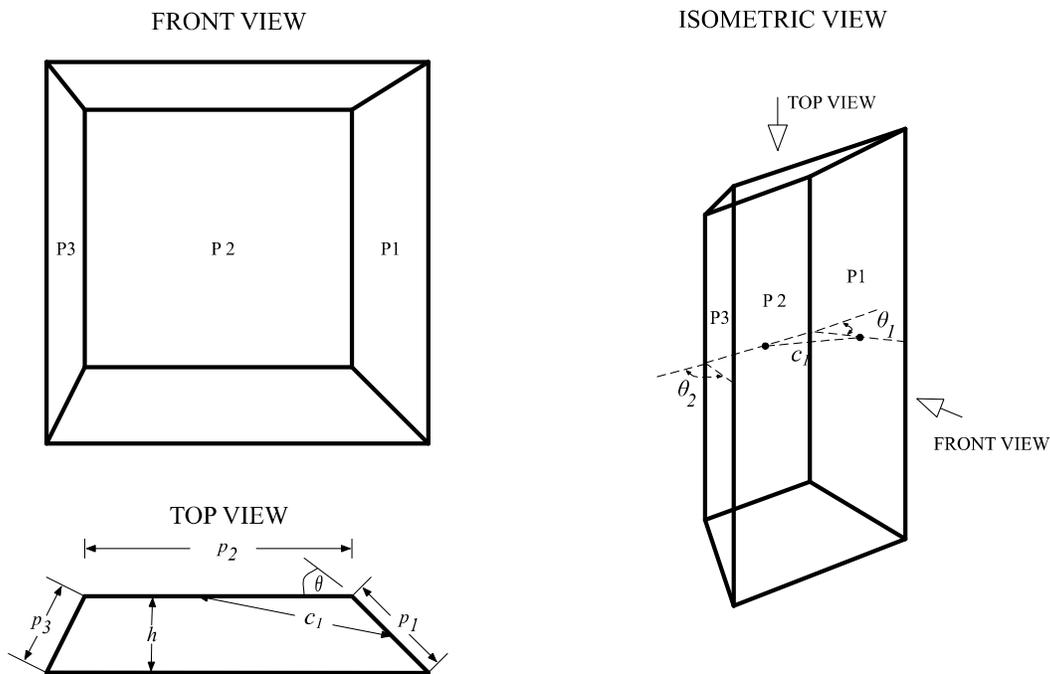


Figure 1: Beacon Schematic – Front, Top and Isometric Views

MAST Algorithm

For each scan, the MAST algorithm goes through four steps; segmentation, template matching, alignment of scanned beacon data from an arbitrary scanner reference frame to a mine reference frame and finally determining of horizontal position, azimuth orientation and positioning statistics. In this paper the derivation of positional statistics is not a focus and has been left out for brevity. However, the reader is referred to Gracie and Mikhail (1981), Anderson and Mikhail (1998), Wolf and Ghilani (1997) for standard methods of determining positioning statistics in surveying.

Data Segmentation

Figure 2 illustrates the segmentation flow chart. In data segmentation, principal component analysis is used to fit a linear model to a file of sequentially incremented point coordinate data in the arbitrary scanner reference frame. Various line segment parameters such as length, orientation, normal vector and deflection angle are collected. It is assumed that consecutively extracted lines share a common node point.

For any two consecutive panels (P1 and P2 or P2 and P3), thresholds governing panel lengths, p_1 , p_2 , centroid distances, c_j and panel deflection angle θ_j , are used to maximize the possibility of positively identifying each beacon panel. In reality, design dimensions and line segment parameters computed from point cloud data are unlikely to correspond due to the randomness of scanner position and orientation relative to beacon positions. There is an extremely low probability that a scan point will fall exactly at the edge or intersection of any two panels resulting in the measured segment lengths shorter than design lengths.

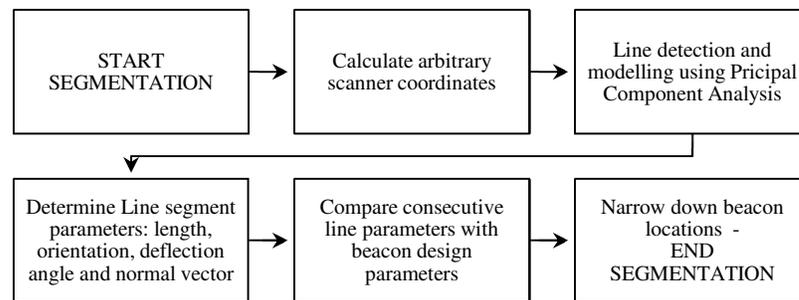


Figure 2: Segmentation Process

To positively identify a single or two consecutive panels, calculated values should fall within given thresholds such that any of the desired combinations of p_1 , p_2 , θ_j , c_j in Table 1 are obtained. Thresholds on panel lengths, deflection angles and centroid distances are set to maximize the possibility of correctly identifying one or two beacon panels. If threshold constraints are too tight, each line segment will have unique point identifiers with no common point, with the result that segmentation fails. Similarly, if threshold constraints are too relaxed, the deflection angle criterion θ_j that is crucial to correct panel segmentation is never achieved and segmentation fails.

Options 1 and 2 are considered optimal to correctly detect two consecutive panels of a beacon. Options 3 through 6 are sub-optimal and work to identify single beacon panels. The end result of the segmentation process for each beacon is a list of point number ranges corresponding to point cloud data falling on each identified panel. Specifically, the start and end point numbers for each panel will be used to analyse correspondence between arbitrary and mine beacon coordinate data.

Table 1: Segmentation Options for Positive Panel Identification

Option	p_1	p_2	θ_j	c_j
1	*	*	*	*
2	*	*	*	
3		*	*	*
4	*		*	*
5	*		*	
6		*	*	

Template Matching

Each beacon vertical panel edge has a distinct x - y coordinate in the mine reference system. In the horizontal plane, each beacon can be identified using four unique x - y coordinate pairs and a unique coordinate pair identifier. Given known panel dimensions and inter-panel relationships from the design, the panel edge and intersection coordinates in the scanner reference frame, and mine beacon coordinate data, one can determine whether correspondence exists between each segmentation point range and two unique beacon coordinate pair identifiers.

Alignment of Arbitrary Scanner Data to Mine Reference Frame

Alignment of arbitrary scanner data to the mine reference frame is a two-step process. The first step uses a four-parameter transformation, after Gracie and Mikhail (1981), Anderson and Mikhail (1998), Wolf and Ghilani (1997), to rotate, translate and scale arbitrary data to proximity of its correct position in the mine reference system.

Let P represent a $2 \times n$ beacon coordinate matrix in the arbitrary scanner coordinate system (S_{Arb}) and N , a similarly sized beacon coordinate matrix in the mine reference system (S_{Mine}). The transformation of coordinates in (S_{Arb}) to (S_{Mine}) is as follows: $N = uR_\alpha P + t$ where u is a 2×2 uniform scale change matrix and R_α , a 2×2 rotation matrix with respect to angle α t is a $2 \times n$ translation matrix formed by concatenating a 2×1 translation vector $[t_x \ t_y]^T$. The transformation parameters $\hat{x} = [u \ \alpha \ t_x \ t_y]^T$ are found from $\hat{x} = (A^T A)^{-1} A^T f$. Each identified beacon point in P contributes to a 2×4 matrix $\begin{bmatrix} X_{iP} & -Y_{iP} & 1 & 0 \\ Y_{iP} & X_{iP} & 0 & 1 \end{bmatrix}$ concatenated to form a $2n \times 4$ matrix A . Similarly, f is a $2n \times 1$ matrix formed by concatenating each beacon coordinate $[X_{iN} \ Y_{iN}]^T$ in N .

The second step uses a modified version of the Iterative Closest Point (ICP) algorithm in Low (2004) to align or fit 2D point sets to given lines. The goal with each ICP iteration is to optimize M_{Opt} where: $M_{Opt} = \operatorname{argmin}_M \sum_i ((M \cdot s_i - d_i) \cdot n_i)^2$. M is composed of a rotation matrix R_β and a translation matrix $T_{(t_x, t_y)}$ such that $M = T_{(t_x, t_y)} \cdot R_\beta$. The source data matrix $s_i = [s_{ix} \ s_{iy} \ 1]^T$, corresponds to the scan data point set. The destination data matrix $d_i = [d_{ix} \ d_{iy} \ 1]^T$, corresponds to the projection of s_i onto the line segment joining the corresponding reference points and $n_i = [n_{ix} \ n_{iy} \ 0]^T$ is the unit normal vector corresponding to each line segment.

When β is very small, the small angle theorem is applied. This leads M to being approximated by $\hat{M} = T_{(t_x, t_y)} \cdot \hat{R}(\beta)$. As a result, \hat{M}_{Opt} can then be written as: $\hat{M}_{Opt} = \operatorname{argmin}_{\hat{M}} \sum_i ((\hat{M} \cdot s_i - d_i) \cdot n_i)^2$ with each point in the point set written as a linear expression in the unknown parameters β , t_x and t_y . The reader is referred to Low (2004) for any intermediate steps, left out for brevity's sake. The vector of parameter estimates $\hat{x} = [\hat{\beta} \ \hat{t}_x \ \hat{t}_y]^T$ and residuals, v , are obtained from $\hat{x} = (A^T A)^{-1} A^T b$ and $v = b - A\hat{x}$ respectively. The refined mine coordinates B_{MineR} for the current iteration are obtained from the results of the previous iteration by: $B_{MineR} = [G_x \ G_y]^T = R_\beta B_{Mine} + T_{(t_x, t_y)}$.

Use of this two-step process ensures quick convergence of the ICP algorithm, usually within five iterations. A cumulative sum of the rotations β_{Cum}° , is then used to determine the azimuth of the laser scanner centre from each finally adjusted point position.

Scanner Horizontal Position and Azimuth Orientation

The azimuth of the laser scanner centre from an adjusted point is determined from $Az_{ScanCentre} = h_a^\circ + 180^\circ + \beta_{Cum}^\circ$ where h_a° represents the range of scanner angular measurements coinciding with the segmented data. Since the laser scanner centre position is calculated based on the segmented point cloud, a least squares approach is adopted. This will also enable quantifying accuracy of positioning. Each laser

scanner measurement $h = [h_a^\circ \quad h_d]^\top$ has an associated $\sigma = \text{diag}[\sigma_{h_a^\circ} \quad \sigma_{h_d}]$ where h_d, σ are the distance measurement and standard deviations of the scanner measurements. The scanner centre coordinates can be expressed in the form: $X_p = G_x + h_d \sin(Az_{ScanCentre})$ and $Y_p = G_y + h_d \cos(Az_{ScanCentre})$ and written in the least squares technique of adjustment of indirect observations (Anderson and Mikhail, 1998; Gracie and Mikhail, 1981) matrix form $v + B\Delta = f$.

The parameter estimates $\Delta_{(X_p, Y_p)}$ are obtained from sequentially applying: $N = B^T W B$, $t = B^T W f$ and $\Delta_{(X_p, Y_p)} = n^{-1} t$, where W is a weight matrix indicating confidence placed on field measurements, B and f are a coefficient matrix of the parameters and a vector of constant terms of the condition equations respectively. N is a coefficient matrix of the normal equations, t , a vector of constants of the normal equations. The residuals are calculated from: $v = f - B\Delta$. The precision of positioning is determined from the covariance matrix $\Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta} = \sigma_0^2 (B^T W B)^{-1}$, where σ_0^2 is a reference variance, $Q_{\Delta\Delta}$, a cofactor matrix of the parameter estimates and this is used in post-adjustment statistical analysis. The azimuth or direction faced by the scanner corresponds to the mid h_a° value adjusted for β_{Cum}° . The reader is further referred to Gracie and Mikhail (1981); Anderson and Mikhail (1998); Wolf and Ghilani (1997) for standard post adjustment statistical analysis methods as applied to surveying.

SIMULATION RESULTS AND DISCUSSION

Simulation Results

In this research, scan data of the environment was modelled or generated based on known scanner and beacon positions and orientations. As with normal surveying practice, reference point locations and coordinates are known quantities. The challenge was then to recover through simulation the original scanner position and orientation using only the scan distance data and the beacon coordinates. This modelling and simulation experiment was undertaken using MATLAB® Version 7.10.0.499 (R2010a). In this simulation, the tunnel width was kept constant. Beacon 2 was kept at a constant 0° azimuth orientation and Beacon 1 orientation was varied over a $\pm 10^\circ$ range to the Beacon 2 mid-panel normal. Beacons were placed in either divergent, parallel, or convergent positions. For scanner positions closest to the walls, the platform was assumed parallel to the closest wall, otherwise in a direction that is the mean of the two wall directions. Beacon parameters (size and panel inclination) and the angular resolution were also varied.

Seven scanner locations forming a triangular shape were investigated in determining position and orientation accuracy. Positions 1 nearest Beacon 2, is at the apex of this triangle. Position 7 is directly opposite position 1 and midway the base of this triangle. Positions 5 and 6 form the other two corners of the triangle and these three positions are also within proximity to alternate locations of Beacon 1. Midway between the beacon positions are the other three scanner locations.

Figure 3 on the next page shows examples of partial and full beacon segmentation. The top four sub-figures illustrate segmentation in the arbitrary scanner frame and the bottom four, orientation of the segmented data to the mine reference frame after the application of ICP. Table 2 shows a subsection of results obtained during this research. These show simulated results based on laser scanner angular resolutions of 0.25° and 0.50° respectively.

Discussion

It is apparent from the results obtained that positional accuracy is achievable in the sub-centimetre range and is not dependent on the location of the scanner relative to the position and orientation of the beacons. However, orientation accuracy seems to be affected by scanner angular resolution. Considering that for scanner angular resolutions of 0.25° and 0.50° respectively, the orientation errors are consistently less than 0.1667° (10 arc seconds) and 0.25° (15 arc seconds) respectively, whilst maintaining sub-centimetre positional accuracy indicates the potential in this procedure.

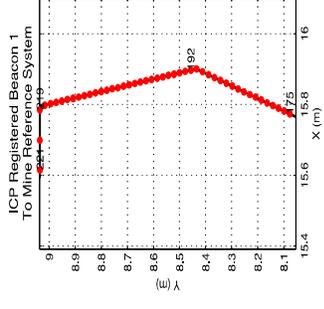
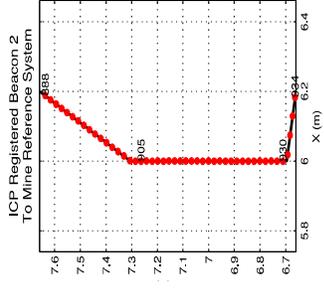
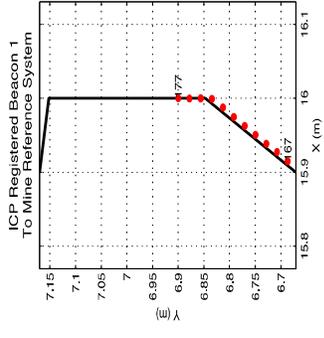
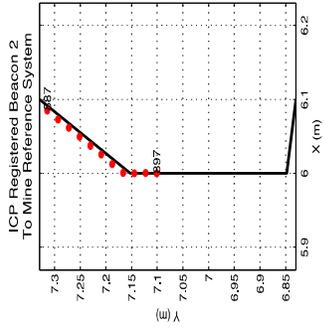
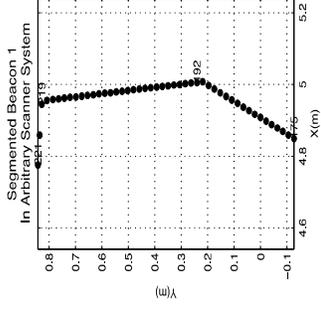
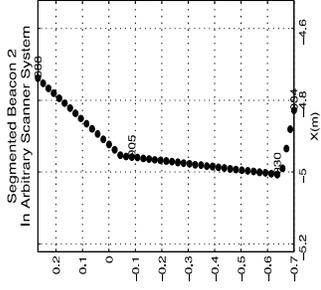
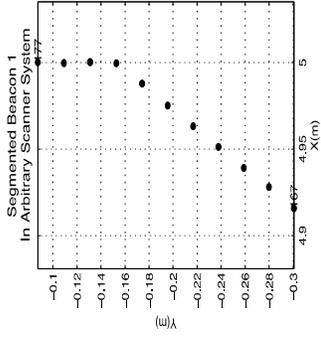
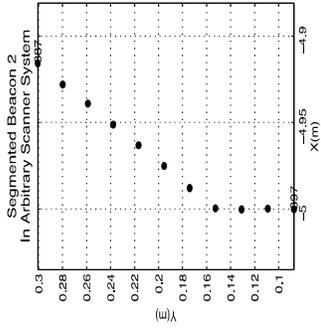


Table 2: Simulated Position and Orientation Results

Pos	True Postn.		Calctd. Postn		95%CI [+/- (m)]		Position Error		True Azimuth		Calc. Azimuth		95%CI +/- deg		Orient. Err.		Bcn	
	X(m)	Y(m)	X(m)	Y(m)	X(m)	Y(m)	X(m)	Y(m)	deg	min	sec	deg	min	sec	deg	min		sec
4	11.000	7.000	11.000	7.000	0.0000	0.0000	0.000	0.000	360	0	0.0	0	9	7.1	0	9	7.1	12
3	10.932	7.781	10.932	7.781	0.0000	0.0000	-0.000	-0.000	355	0	0.0	355	0	24.9	0	9	53.8	13
1	7.000	7.000	6.999	7.002	0.0000	0.0000	-0.001	0.002	360	0	0.0	0	3	59.3	0	12	19.3	6
3	10.932	7.781	10.932	7.782	0.0000	0.0000	0.000	0.000	355	0	0.0	355	3	46.5	0	12	21.5	6
6	14.863	8.563	14.864	8.561	0.0000	0.0000	0.001	-0.002	350	0	0.0	350	1	54.0	0	10	36.2	6
4	11.000	7.000	11.000	7.000	0.0000	0.0000	-0.000	-0.000	360	0	0.0	0	3	57.0	0	13	37.1	6
7	15.000	7.000	15.001	6.999	0.0000	0.0000	0.001	-0.001	360	0	0.0	0	6	1.6	0	19	2.2	6
2	10.932	6.219	10.931	6.217	0.0000	0.0000	-0.001	-0.001	5	0	0.0	5	3	17.1	0	14	11.6	6
5	14.863	5.437	14.864	5.435	0.0000	0.0000	0.000	-0.003	10	0	0.0	10	0	30.0	0	13	19.9	6

Pos	True Postn.		Calctd. Postn		95%CI [+/- (m)]		Position Error		True Azimuth		Calc. Azimuth		95%CI +/- deg		Orient. Err.		Bcn	
	X(m)	Y(m)	X(m)	Y(m)	X(m)	Y(m)	X(m)	Y(m)	deg	min	sec	deg	min	sec	deg	min		sec
3	10.932	7.781	10.932	7.782	0.0000	0.0000	0.001	0.000	355	0	0.0	355	10	2.8	0	42	36.3	9
4	11.000	7.000	11.000	7.000	0.0000	0.0000	-0.000	-0.000	360	0	0.0	0	14	43.6	0	52	59.6	6
4	11.000	7.000	11.000	7.000	0.0000	0.0000	-0.000	-0.000	360	0	0.0	0	11	30.1	0	47	50.9	9
1	7.000	7.000	7.000	7.003	0.0000	0.0000	-0.000	0.003	360	0	0.0	359	59	38.5	0	29	5.2	6
3	10.932	7.781	10.933	7.780	0.0000	0.0000	0.001	-0.001	355	0	0.0	355	11	56.4	0	45	13.1	6
6	14.863	8.563	14.864	8.561	0.0000	0.0000	0.001	-0.002	350	0	0.0	350	3	32.0	0	27	35.7	6
4	11.000	7.000	11.000	7.000	0.0000	0.0000	-0.000	-0.000	360	0	0.0	0	14	43.6	0	52	59.6	6
7	15.000	7.000	15.000	6.999	0.0000	0.0000	0.000	-0.000	360	0	0.0	0	2	27.3	0	29	34.7	6
2	10.932	6.219	10.932	6.218	0.0000	0.0000	-0.000	-0.000	5	0	0.0	5	3	33.8	0	35	48.0	6
5	14.863	5.437	14.864	5.437	0.0000	0.0000	0.001	-0.000	10	0	0.0	10	11	42.6	1	1	59.7	6

The top two results in the top table of Table 2 represent the segmentation and orientation shown in Figure 3. The first result in the top table and the top three results in the second table all represent sub-optimal segmentation. Sub-optimal segmentation will correctly identify only one of the required two beacon panels. This is shown in the first two diagrams in the bottom row of Figure 3. As can be observed from these results, partial segmentation is not an impediment to accurate positioning and orientation. With positions 1 and 5 through 7, the scanner was positioned close to one beacon therefore the point density on one beacon was significantly greater than on the other. It can be concluded from this that scan density and angular resolution are not major factors to positioning and orientation accuracy.

Beacons are similar in design however with differing panel inclination and beacon sizes. Beacons tested range in length from 0.499 m to 1.996 m and with depths ranging 0.100 m to 0.400 m respectively. From these results there is no indication that beacon size within this range is a factor to positioning and orientation accuracy. From these results there is no indication that beacon size within this range is a factor to positioning and orientation accuracy.

CONCLUSIONS

Typically positioning in surveying practice is done in 3D and is a function of how well a user can manipulate an instrument and how fine the instrument can be read. Automating and removing the human element from the measurement equation will improve upon the speed and precision of measurement. It has been shown that the MAST positioning concept is a viable process for automatic 2D positioning and has the potential to produce fast and acceptable positioning and orientation results with minimal human input. Beacons are a critical feature to MAST positioning and play a critical role in determining scanner position and orientation. Extending this concept to 3D positioning and orientation is the subject of future work.

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