Three Dimensional Spatial Metrics for Compactness Assessment of Urban Forms

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Abstract –
Developing measurement tools to assess urban sustainability is one of the new streams in built environment; however, automated methods of urban form assessment are technically difficult. While planar characteristics of urban forms have been studied traditionally using the spatial metrics from remote sensing data such as Landsat images, height information of urban environments is now playing an important role in urban energy exchange (e.g. solar energy gains), urban air circulation (which is affected by interactions between terrain and buildings) and urban microclimate. As incorporating the planar metrics and the height information has been rarely attempted in the sustainable urban form studies, we propose an automatic technique for the quantification of 3-dimensional urban compactness which is one of the main factors influencing sustainability. In this paper, we aim to utilise one of autocorrelation statistics known as Moran’s I for the assessment of urban form compactness, considering both layout and elevation attributes. Additionally, Getis-Ord G statistic is also used for further investigation on concentration of low or high urban features.

Keywords -
Lidar, Sustainable urban form, Urban neighbourhood pattern, GIS

1 Introduction

Accurate and reliable measurement techniques are necessary for urban sustainability assessment as they provide a basis for characterisation of various urban neighbourhoods. Recent metrics such as Leadership in Energy and Environmental Design for Neighbourhood Development (LEED-ND) rating system credits can be used as sustainable urban form metrics. Urban Neighbourhood Pattern and Design (UNPD) is one of the major categories of LEED-ND rating system with a prerequisite of compact development. The main goal of this category is to build a community [1]. A problem of this category is that the spatial dimension of compactness is often ignored. While social dimension is important in urban neighbourhood research, the spatial and structural assessment of urban neighbourhoods is also necessary. Apart from the shortcomings of the existing spatial assessment, height information of buildings has not been utilised appropriately even though building height plays a crucial role in both climatic design and human scale design. Climatic design aims to create convenient and comfort built-up areas by designing 3-dimensional (3D) building shapes and appropriate arrangement of the buildings in an urban environment that requires adaption to the local climate. Therefore, it is important to investigate 3D urban metrics that can quantify spatial dimensions of urban sustainability.

Urban metrics provide measurement methods of comparing the characteristics of different urban districts. There are several metrics for measuring characteristics of urban forms at metropolitan and building block scales. Spatial metrics have been known as effective measurement tools for characterizing urban forms in large study extents such as metropolises [2]. For example, Huang et al. [2] suggested seven metrics to reveal five elements of urban forms, namely, compactness, centrality, complexity, porosity and density to compare cities in developing and developed countries. The metrics characterising building pattern, complexity and compactness also exist; for example, Floor Area Ratio (FAR) and Building Coverage Ratio (BCR). However, metrics quantifying urban neighbourhood characteristics such as compactness remain scarce. Sustainable urban form measurement focuses on decision for either compact or sprawl development at the metropolitan scale. Compactness measurement at the neighbourhood scale is important for sustainable development assessment as the urban morphology and geometry affect wind speed, air quality at ground level [3] and energy exchange [4]. Indeed, traditional spatial metrics cannot characterise 3D urban forms because characterization of 3D urban forms requires both 3D data and adaptive spatial metrics. One of the fundamental barriers for 3D characterisation of urban forms is the large extent of the spatial analysis because 3D data (such as lidar point clouds) over an urban environment may not cover the entire urban area or could not provide a sufficiently high resolution. While remote sensing is reported as a reliable source of information over urban areas [5], the proven advantage of 3D urban remote sensing to analyse 3D
urban patterns and processes has not been fully explored. Urban form patterns have been traditionally derived using spatial urban metrics applied to remote sensing data. This bottom-up approach, deriving information from structure to process [6], can be improved using lidar. The advantage of lidar for urban pattern assessment studies is to provide accurate elevation information of both terrain and objects e.g. buildings and trees. The potential of lidar data for characterising urban forms can be used when we adapt the spatial and urban metrics to 3D space.

This research aims to develop a spatial 3D metric of compactness. We employ Moran's I (MI) as a spatial metric of characterizing 3D compactness for the sustainable urban form assessment purpose. A novelty of this research lies in applying an autocorrelation statistic of MI to elevation information of lidar point clouds to investigate the level of 3D compactness of urban features, including man-made and natural objects.

In this paper, we review the current conceptual approaches in urban pattern modelling and the theoretical background of autocorrelation statistics of MI and Getis-Ord G (GOG). To fill the gap of 3D compactness measurements at the urban neighbourhood scale, we propose novel 3D metrics that will be tested in the simulation and implementation step. Before applying the statistics, the lidar data will be classified to ground and non-ground points. The statistics then will be applied to Digital Surface Model (DSM) and Normalised DSM (NDSM) of 6 urban districts grouped in two urban land uses including diverse object shapes, elevation and urban fabric.

2 Urban Pattern Conceptual Approaches

At the metropolitan scale of urban pattern recognition, two main conceptual approaches can be recognized; namely, traditional and modern perspectives. As Herold et al. [6] discussed, in the traditional top-down view (i.e. from process to structure), processes produce structures, while in the modern bottom-up view (i.e. from structure to process), structures can be a representative of processes. Although these conceptual approaches are reported for spatio-temporal analysis of urban forms, the main idea works for a snapshot of urban forms. That is, the modern view employs remote sensing data and spatial metrics for analysis of urban patterns whereas the traditional perspective believes that the processes are the major drivers of urban form and structure.

While the modern approach is acceptable in urban growth modelling at the metropolitan scale, it can be enhanced using remotely sensed 3D data at the neighbourhood scale. However, there are two main problems for such improvement. One problem that could be emerged when using 3D data is that the height information will be ignored spontaneously if the study extent is an entire metropolis or a big city. To address this problem, the scale of analysis has to be modified to an urban neighbourhood. Therefore, 3D data can be used for characterising the 3D pattern of urban neighbourhoods.

Another problem is lack of appropriate spatial metrics adapted to 3D space. The conventional spatial metrics are capable of characterizing planar urban forms in a large extent. However, we need metrics that are capable of characterizing urban forms at the neighborhood scale.

In general, urban fabric is categorised to ‘fine’, ‘coarse’ and ‘mixed’ where fine fabric includes mostly small size objects and coarse fabric mostly contains large objects. In many cases change of fabric from coarse to fine, or reverse, can be a sign of land use change. For example, industry sites are mostly recognized as coarse fabric because these areas include large size buildings. On the other hand, fine fabrics are mostly residential areas. In 3D space, these categories can be expanded based on the amount of high, medium or low buildings. Table 1 shows possible 3D urban fabrics. In 3D space, a central business district can be either mixed-high or coarse-high fabric. Downtowns can be examples of coarse high fabric.

Table 1. Urban fabric categories in 3D space

<table>
<thead>
<tr>
<th>Planar fabric</th>
<th>Height attribute of urban fabric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fine</td>
<td>High</td>
</tr>
<tr>
<td>Coarse</td>
<td>Coarse</td>
</tr>
<tr>
<td>Mixed</td>
<td>Mixed</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
</tr>
<tr>
<td></td>
<td>Low</td>
</tr>
</tbody>
</table>

Figure 1 contains our proposed bottom-up approach for neighbourhood pattern analysis in 3D space. As can be seen, the overall goal is to start from Step 1: Structure, to achieve Step 4: Pattern. In this proposed bottom-up approach, remotely sensed 3D data would be employed using developed 3D urban metrics to derive, measure and compare urban neighbourhood patterns.

3 Moran’s I for Measuring 3D Compactness

The relationship between pair variables is known as correlation and the degree of correlation is measured through statistical coefficients [7]. Sign and the numerical value of correlation coefficient are important; for example a positive or direct relationship
is determined by a plus sign and a value close to 1 (|a| \rightarrow 1) shows higher strength of a relationship than a value close to 0 (|a| \rightarrow 0) [7]. Autocorrelation refers to the correlation between pairs of observations in a single variable [7]. This means that, in autocorrelation, the relationship between the values of a variable is investigated. Autocorrelation can be calculated for a variable changes over time, for linear spatial series and for two-dimensional spatial series. The phenomenon of spatial autocorrelation can be defined as the relationship among the values of an attribute in distributed areal units on a planar surface [7].

The null hypothesis for spatial autocorrelation is Complete Spatial Randomness (CSR) between observation values and p-value is calculated to test the hypothesis: if p is small (less than 0.05), the null hypothesis is rejected and it indicates that clusters in observations exist.

MI and GOG are known as autocorrelation statistics measuring compactness and clusters of low or high values, respectively. In a defined study area and for an observation i and neighbour observations of j in a distance of d, MI for attribute of x is defined as [8]:

\[
I(d) = \frac{n \sum w_{ij} (x_i - \bar{x})(x_j - \bar{x})}{\sum W(x_i - \bar{x})} 
\]  
(1)

and GOG is defined as [8]:

\[
G(d) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} x_i x_j 
\]  
(2)

where \(w_{ij}\) is spatial weight and defined as the inverse distance between i and j, and \(W\) is the sum of all weights.

Getis and Ord [8] compared MI and GOG. They stated that “G statistic measures overall concentration or lack of concentration of all pairs of \((x_i, x_j)\) such that i and j are within d of each other. MI on the other hand, is often used to measure the correlation of each \(x_i\) with all \(x_j\)'s within d of i and, therefore, is based on the degree of covariance within d of all \(x_j\)” [8, p.196].

If we assume \(K_1\) and \(K_2\) as constants of the MI and GOG equations, as below, MI can be calculated based on GOG and then it helps for better comparison.

\[
K_1 = \frac{1}{\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} x_i x_j} 
\]  
(3)

\[
K_2 = \frac{n}{\sum W(x_i - \bar{x})} 
\]  
(4)

\[
G(d) = K_1 \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} x_i x_j 
\]  
(5)

\[
I(d) = K_2 \sum \sum w_{ij} (x_i - \bar{x})(x_j - \bar{x}) 
\]  
(6)

Therefore,

\[
I(d) = \left(\frac{K_2}{K_1}\right) G(d) - K_2 \bar{x} \sum (w_{ij} - w_{ji}) x_i + K_2 \bar{x}^2 W 
\]  
(7)

MI in Equation (1) and G in Equation (2) are unequal when the weighted sums \(\sum w_{ij}\) and \(\sum w_{ji}\) are not equal to \(W \bar{x}\) and therefore, the weight patterns are different [8].

In characterizing the correlation among the same set of weighted observations, the difference between MI and GOG is that MI quantifies the spatial autocorrelation of a feature attribute considering both feature locations and feature values, concurrently but GOG measures the compression of high or low values of an attribute [9].

These two statistics can be compared using their corresponding z-scores. A z-score for MI statistic is defined as [9]:

\[
z_I = \left\{I(d) - E[I(d)]\right\} / \sqrt{Var[I(d)]} 
\]  
(8)

where E and V represent the expected mean and variation, respectively.

In addition, a z-score for G statistic is defined as [8]:

\[
z_G = \left\{G(d) - E[G(d)]\right\} / \sqrt{Var[G(d)]} 
\]  
(9)

Measuring compactness through MI only considers the object’s location on a planar surface, hence, they are characterising only planar compactness but if we apply the autocorrelation statistics of MI and GOG on elevation attribute of urban objects we would achieve 3D compactness considering both layout and elevation of objects. G statistic can be used as a complementary statistic to derive the information for determining concentration of high or low values.

Two spatial statistics of MI and GOG have been used so far for measuring centrality/compactness [10-12] and concentration of high or low values [9]. MI has been recognized as an effective measurement tool of compactness of socio-economic data at the metropolitan scale [10]. However, in literature, its potential to characterize 3D compactness of urban fabric has been rarely explored.

4 Methodology

This research involves in two steps including simulation and application to a case study. The patterns in the simulation step are obtained from the well-known morphological urban patterns.

4.1 Simulation

We obtained 30 and more schematic buildings in a constant simulation extent since at least 30 square input features are required for calculation of MI and GOG [9]. Three different patterns are constructed for constant features by changing the elevation attribute of buildings which make three types of neighborhood pattern in 3D space. These are mono-centric, polycentric and decentralized (see Figures 2a to 2c).

(a) (b) (c)

Figure 2. Monocentric, polycentric and decentralised neighbourhood 3D patterns
The study extent is a coarse-high fabric surrounded by fine fabric. The aerial image over all the study extent is shown in Figure 3a. While the aerial photo can demonstrate whether a fabric is fine or coarse, 3D data is required for 3D fabric determination. We used lidar point clouds with 20 cm horizontal and 12 cm vertical accuracy. Figure 3b shows the triangulated lidar point clouds and the districts which are considered for applying autocorrelation statistics. Lidar data in Figure 3b shows the DSM. To exclude the objects’ height, NDSM is constructed from extracted non-ground points (see Figure 4a). A DSM contains elevation of both terrain and attached objects whereas a NDSM includes only absolute height information of objects such as buildings and trees. The difference between DSM and NDSM over same district can be distinguished through comparison between Figures 4b and 4c.

5 Results

To distinguish how various 3D patterns of urban neighborhoods can result in different numerical values of MI measurement, MI was applied on both 3D patterns in simulation study and 3D patterns of urban districts shown in Figures 2 and 3.

5.1. Simulation Results

Three different 3D patterns of a neighborhood are analysed where their difference comes from the change of elevation attribute of buildings and their layouts are exactly the same. Simulation results of MI over mono-centric (Figure 2a), polycentric (Figure 2b) and decentralized (Figure 2c) patterns are 0.13, 0.09 and -0.23, respectively. The results for more compact patterns of high rise buildings are positive whereas the result for decentralized pattern is negative. These results confirm the results obtained by Tsai [10] in simulation study. However, in that study the data type, layout pattern, number of cells and the attribute value of each pixel were different.

5.2. Implementation Results

As described before, two products of lidar point clouds are DSM and NDSM. The DSM object’s height contains terrain and the object height. For an object in NDSM the objects height is absolute elevation. The autocorrelation statistics of MI and GOG are applied to both DSM and NDSM.

5.2.1. MI Statistic Results

Table 2 indicates the results of applying MI to DSM. As it shows, the results of residential land use fabric range between 0.84 and 0.92. The MI values for the districts with fine fabric including large and tall buildings are higher than the fine fabric. The MI values for the districts in educational land use with dominant coarse fabric are obtained between 0.80 and
While the difference between the results of residential and educational land uses are not clear, the difference between the fine-low (b) and coarse-high (d) fabrics is considerable. MI for a fine-low fabric is achieved minimum among all the results which is 0.84 but the result for the coarse-high fabric is the maximum value among the results which is 0.97. The MI value for fine fabric of c is higher than the ones including high and large buildings (a and b). The results for a more compact form (d) in coarse fabric is higher than case (f) which is coarse-high but contains less buildings and the high buildings are dispersed.

<table>
<thead>
<tr>
<th>Land use</th>
<th>Urban fabric</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>a. Fine</td>
<td>0.89</td>
</tr>
<tr>
<td></td>
<td>b. Fine</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>c. Fine</td>
<td>0.92</td>
</tr>
<tr>
<td>Educational</td>
<td>d. Coarse</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>e. Coarse</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>f. Coarse</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Approximately similar pattern can be seen when we apply MI to NDSM. As can be seen in Table 3, fine-low fabric (b) has the minimum value of MI, (e) and (f) have lower levels of MI than case (d) which is more compact. The difference of the results of applying MI to DSM and NDSM is that in case of DSM, maximum value of MI was obtained for case (d) which is a fine-low fabric but in case of NDSM, maximum value is obtained for case (c) which is fine fabric including some large and tall buildings.

<table>
<thead>
<tr>
<th>Land use</th>
<th>Urban fabric</th>
<th>MI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>a. Fine</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>b. Fine -low</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>c. Fine</td>
<td>1.08</td>
</tr>
<tr>
<td>Educational</td>
<td>d. Coarse</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>e. Coarse</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>f. Coarse</td>
<td>0.65</td>
</tr>
</tbody>
</table>

All the z-scores and p-values are in significant part of a normal distribution curve. The z-scores higher than 1.65 are in significant area with 90 percent confidence level. All our achieved z-scores are higher than 2.58, which means that the level of confidence achieved for all cases is 99 percent. As described before, the null hypothesis here is Complete Spatial Randomness (CSR) which is rejected by 99% confidence in all of our results.

### 5.2.2. G Statistic Results

As discussed in section 3, G statistic can be used in conjunction with MI for complementary information; MI measures compactness of both location and numerical value of an attribute distributed overall the data set but GOG is a concentration measurement tool for high or low values [9]. Table 4 demonstrates the results of applying GOG to DSMs of the urban districts. As it indicates, maximum G value is obtained for district (b) and minimum G value is obtained for districts (a) and (f) followed by (c) with a partial difference.

<table>
<thead>
<tr>
<th>Land use</th>
<th>Urban fabric</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>a. Fine</td>
<td>$43 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>b. Fine</td>
<td>$98 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>c. Fine</td>
<td>$47 \times 10^{-6}$</td>
</tr>
<tr>
<td>Educational</td>
<td>d. Coarse</td>
<td>$52 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>e. Coarse</td>
<td>$64 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>f. Coarse</td>
<td>$43 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Table 5 contains the results of applying GOG to NDSM. The maximum G value is achieved for district (b) and minimum G value is obtained for district (d) followed by (a). Maximum G value is obtained for same district (b) and district (a) is in lowest level of G values in both Tables 4 and 5.

<table>
<thead>
<tr>
<th>Land use</th>
<th>Urban fabric</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residential</td>
<td>a. Fine</td>
<td>$828 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>b. Fine</td>
<td>$1328 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>c. Fine</td>
<td>$1188 \times 10^{-6}$</td>
</tr>
<tr>
<td>Educational</td>
<td>d. Coarse</td>
<td>$667 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>e. Coarse</td>
<td>$1078 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>f. Coarse</td>
<td>$1088 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

In all results for $G$ statistic, the z-scores remain in significant part of a normal distribution curve and the null hypothesis (CSR) can be rejected by 99% level of confidence.

The districts where their MI value is maximum have minimum $G$ value and the districts with lowest MI value are obtained maximum $G$ value. This happens because MI measures compactness of features and values location whereas $G$ measures the compression of high or low values. Districts (a) and (c) where we obtained maximum value of MI and minimum $G$ are fine fabric in residential land use and the urban objects are compact in layout so their MI are higher than other districts. In this residential fine fabric, low objects are concentrated compared to cases (d) and (e) where higher buildings are clustered.

### 6. Discussion

We promoted the bottom-up approach (from structure to process) of studying urban patterns using 3D remote sensing data and by proposing 3D metrics. The questions remain on how to improve the top-down approach to explain the 3D pattern of urban areas. It is expected that the drivers and factors influencing on 3D pattern and growth of urban areas be explored by urban planners and economists. In detail, these questions are: 1) How the top-down approach can characterise 3D urban growth? 2) Which factors, drivers and processes influence on making different
3D urban patterns? 3) How urban modelling and spatial urban theories can be developed to consider both urban layouts growth and various vertical patterns?

While the proposed 3D metric of MI is capable of distinguishing among different 3D patterns of urban neighbourhood caused by variation of only height attribute in simulation study, it needs to be integrated with the information derived from G statistic in the implementation step. Indeed, using MI in conjunction with GOG in each district details our derived information about each district.

As the sample size and study areas are different in the implementation step, the MI results have to be compared within their context characteristics such as the number and size of urban objects, the area of study extent and the distribution of objects’ height pattern.

Among 3D urban fabric categories (Table 1), the categories of fine-low, fine including some large and tall buildings, coarse-medium, coarse-medium with decentralised objects and coarse-high are analysed in this paper and still the compactness values from MI for other patterns need to be explored.

The 3D compactness analysis in the implementation step could measure the compactness of all urban objects on a DSM including natural and man-made objects. For further assessment of compactness of man-made objects (buildings), we need to apply the statistics to the classified building points. For studying the compactness of natural objects such as trees and vegetation class we have to apply MI to only the classified vegetation points.

7. Concluding Remarks

In this paper, we aimed to improve 3D urban metrics for 3D pattern recognition of urban neighbourhoods. MI was firstly applied to simulation study over 3 urban neighbourhood patterns where their difference come from the difference in height attribute only. In the simulation study, a compact form achieved a positive value of MI whereas a decentralized pattern obtained a negative value of MI. The results from the implementation of MI over urban districts indicate that the general idea of clustering works and can differentiate the 3D compactness level among the districts. However, as the urban areas are more complicated than the simulation patterns, we need further information to enhance our understanding of different patterns. Therefore, in the implementation step we added G statistic in our data processing and found that it could enhance our understanding through assigning a lower value to the patterns with shorter urban features than the patterns including taller objects.

It is recommended to apply the proposed 3D spatial metrics to other urban fabric types for a comprehensive study of 3D compactness of urban neighborhoods. For future work, applying MI to the classified buildings and vegetation is suggested to find the level of compactness of the built-up areas and vegetation compactness, respectively.

References