Economic Model Predictive Control - A Review

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Abstract -

Leveraging the advanced estimation and control algorithms for power systems have always been associated with the renewable energy sources, rational power generation, consumer stimulus, emission reduction, as well as economically viable objectives. The model predictive control (MPC) strategies, that employ an economic-related cost function for real-time control, has lately proved a numerically efficient approach to managing the portfolio of energy usage in various residential and industrial projects. They are designated as *economic MPCs*, whose main endeavour is to cope with regularly changing energy prices. Unlike the traditional MPCs, economic MPCs optimize the process operations in a time-varying fashion, rather than maintain the process variables around a few desired steady states. The process may thus totally operate in the transient state with economic MPCs. This paper provides a rigorous review on the developed and progressive economic MPCs, as a contribution to the field while it is still in its infancy. In the second part of the paper, we briefly show the potential of applying the *quadratic dissipativity constraint*, previously introduced, to the closed-loop stability problem of an economic MPC problem.

Keywords -

Economic MPC; Quadratic dissipativity constraint

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1 Introduction

There are different methods in optimization and real-time control, direct or hierarchical, to deal with economic problems in the process and other industries. The articles [21, 31, 10] provides a clear overview of practical approaches to such economic problems in the process industry. The economic MPC has been found effective among these approaches, in which the real-time optimization (RTO) layer is not required for computing targets to the lower layer MPC as usually be the case in the process industry. Figure 1 depicts the differences between the typical hierarchical RTO plus MPC structure and the economic MPC. The objective function of EMPC consists of the economic objective and the MPC target tracking objective functions. According to Rawlings et Al. [31], an EMPC "directly and dynamically optimizes the economic operating cost of the process, doing so without reference to any steady state". EMPC has been successfully implemented for HVAC energy saving applications, and started flourishing in power systems, as has been found in recent works, including those from [17, 18, 15, 27, 1, 26, 30, 33, 5]. For chemical process systems, the review in [10] has also pointed out that "while steady-state operation is typically adopted in chemical process industries, steady-state operation may not necessarily be the economically best operation strategy". The review in this paper outlines concisely the main features of the developed economic MPC that will be useful for interested researchers and industrial practitioners.

Rawlings et Al. have first described the so-called unreachable set-point in MPC implementations in their paper [32], and shown that the set-point tracking MPC exhibit some advantages over the traditional target-tracking MPC [24, 23, 22], when the set-points are not reachable. It is recommended that, the approach "should also prove useful in applications where optimization of a systems economic performance is a more desirable goal than simple target tracking" [32]. A strictly convex cost function $L(x, u)$ is considered in this paper. For a traditional MPC that has a reachable steady-state target, the optimal (x^*, u^*) is tracked and the value of $L(x^*, u^*)$ is zero. In a set-point tracking EMPC, the cost function has the form of $L(x - x_{SP}, u - u_{SP})$, where (x_{SP}, u_{SP}) is usually different than (x^*, u^*). An apparent difficulty around the pre-existing MPC theory for unreachable set-points is that the controller cost function was established as a Lyapunov function for the closed-loop system, while the cost function for the unreachable set-point problem may not

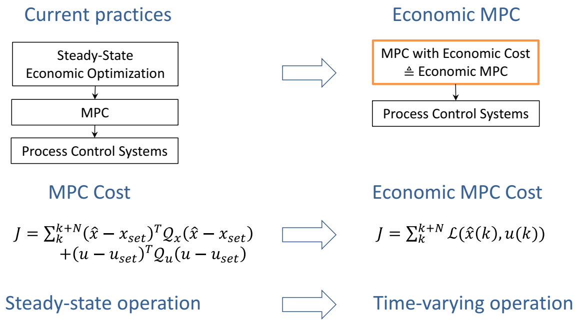

Figure 1. RTO plus MPC vs. Economic MPC [9].

be monotonically decreasing. By creating an auxiliary stage cost that "measures distance from the unreachable set-point" [32], and developing adequate terminal constraint for the finite-horizon problem, the closed-loop stability with unreachable set-points has been proved. The authors have not, however, called it economic MPC.

The authors in [32] have also inferred the relevance of the infinite-horizon optimal control with unbounded costs in the economics literature, especially the *turnpike theory* [25, 7, 10], to the finite horizon MPC problem dealing with unreachable set-points. When the state diverges locally from the optimal steady state towards an improved economic cost, its trajectory is said turnpike. The authors in [10] explained that, "this property is referred to as a turnpike property since the state passes through the optimal steady-state until it finally moves away to achieve further economic benefit (like a vehicle getting on and then, off a turnpike or highway)".

The *economic optimizing MPC* phrase has later been used by the same authors in [6], wherein an innovative Lyapunov function has been presented. The economic MPC problem is explicitly defined to address the cases whereas there exits other points that make the cost function smaller than that of the steady state. The cost function is added with the storage function then minus the terminal penalty term to become the so-called rotated cost. The monotonic decreasing property of a Lyapunov function is obtained as a result of this addition and substraction. It is required to verify the *strong duality* of the steady state problem in this approach. A nonlinear chemical reaction model in the example that fulfilled the strong duality of the steady state has demonstrated the monotonic decreasing of the rotated cost function, but not necessarily of the economic cost function. An extension of this work has been presented in [2]. This choice of Lyapunov function has been deployed in a power system application in [8].

The *average performance* of economic MPC is proved not worse than the optimal steady-state operation in [4]. The optimally steady state operation herein means the stage cost at steady state is the upper bound of the average stage costs. A dissipation inequality with an adequately chosen supply function have been used in this work to extend the sufficient condition for the asymptotic stability of the steady state. The authors have chosen the storage and supply functions such that the dissipativity is also sufficient for the optimal steady-state operation, while the strict dissipativity assures the closed-loop stability. The distance from the current stage cost to the steady state one plays an important role in the supply function, but not in the Lyapunov function as in [6]. The rotated cost function is formed in a slightly different manner than those in [6]. Both equality terminal constraint and inequality terminal constraint with terminal penalty have been addressed in [31]. As a complement to the results in [4], the sufficiency of dissipativity for the optimal steady-state operation has lately been analyzed in [29].

Another approach to guarantee the closed-loop stability for EMPC is to enforce a stability constraint to the optimization. In this enforcing stability approach, the stage cost is modified directly. A rather critical view on the modified cost function is as follows: The stability enforcing problem is described as the one that lies somewhere between the two extrema, when the original stage cost is added with an appropriately chosen positive scalar function $\alpha(.,.)$. One extreme represents the original economic stage cost, when choosing $\alpha(.,.) \equiv 0$, which may leave the optimal steady state unstable. The standard tracking problem is viewed as the other extreme, assigning $\alpha(x, u)$ with the usual tracking objective while canceling the economic one. The asymptotic stability of the optimal steady state is obtained while sacrificing the economic objectives.

Periodic terminal constraint and average constraints presented in [4] are the two main extensions for applications having the pre-computed optimal periodic solutions and for the unsteady closed-loop processes. The average constraint is determined by having the average of a predefined auxiliary output variable $y = h(x, u)$ belong to a time-varying convex set. The periodic terminal constraint, obtained from a dynamic state feedback strategy, is introduced here for achieving a periodic optimal solution described in another work [3]. The notion of *sub-optimally operate off steady state* is also introduced herein. When the average constraint is satisfied, it is said that the "control system optimally operates at steady state on averagely constrained solutions". This statement also implies that the system sub-optimally operates off the steady state. The Lyapunov function for the periodic EMPC has recently been introduced in [42].

Along the line of enforcing stability, a self-tuning terminal weight version of economic MPC has been developed in [29]. This departed from the work in [11] which "shown that if the terminal weight is large enough, then the cost of the predicted terminal steady-state will be arbitrarily close to the cost of the best reachable steady-state." [29]. The main aim of this work is to eliminate the condition on having a sufficiently large terminal weight to keep up with the control performance, as well as to maintain the economic MPC objective. The optimal steady state cost is assumed zero herein. Under mild assumptions, the authors have proved that there exists an upper bound for the average stage costs of the closed-loop system, then determined an updating rule, which allows for a non-monotonic stage cost such that it minimizes that upper bound.

The research in smart grid and energy saving has started flourishing from economic MPCs, as has been found in recent works including those from [1, 15, 17, 18, 26, 30]. Unlike the rigorous developments in the previous three papers, these application-oriented papers simply prolong the predictive horizon for feasibility or applies an infinite-horizon approximation, while emphasizing the effectiveness of the economic MPC approach in power systems and energy saving problems. The existence of such prolonged predictive horizons for recursive feasibility under mild assumptions has lately been proved in [14]. The authors in [29] called the conditions developed in [14] as controllability.

Apart from the above, the work in [20] interpreted the economic problem as the one having cyclic steady state, which arose from the regularly changing energy price. The solution is to normalize the system around such cyclic steady state. A transformed system having a fixed steady state is obtained as a result of the normalization. The MPC objective is then constructed with this transformed model. Similarly to the work in [32], the chosen Lyapunov function is also the difference between the current stage cost and the steady state stage cost. The stability proof is supported by a Lipschitz continuous property, plus an assumption on the weak controllability of the cyclic steady state. Different strategies for the infinite-horizon EMPCs have also been proposed in [40, 19, 27, 26, 41, 30]. However, these are only the preliminary results or EMPC alike formulations, thus will not be given a review herein.

The chemical process control field has envisaged the importance of economic MPC. Recent research works in [16, 9, 43] have shown strong interests and beneficial results. These works extend the previously developed stabilizing method of Lyapunov-based MPC [28] to EMPC. The authors have criticized the method in [6] that, "it is difficult, in general, to characterize, a priori, the set of initial conditions starting from where feasibility and closed-loop stability of the proposed MPC scheme [6] are guaranteed." Nevertheless, the proposed scheme in this work consists of two artificially operational stages, which somehow resembles the idea of stability margins. The disadvantage of this proposed approach could possibly be at the difficulty to define such stability margins while not jeopardizing the optimality. Moreover, the periodic steady states and unreachable set-points have not been addressed formally. We are, therefore, not pursuing a detailed review for the Lyapunov-based EMPC herein. Interested readers can find a detailed review in [10] instead.

In the second part of this paper, we will show the potential of applying the *quadratic dissipativity constraint*, previously developed in [35, 38, 37, 39], to the economic MPC problem that is able to avoid the disadvantages of the above approaches. In this quadratic dissipativity constraint approach, two constraints on the initial control vector, one as a stability constraint, one as a recursive-feasibility constraint, for model predictive control are derived for implementation. Recursive feasibility and input-to-power-and-sate stability are simultaneously achieved as a result of imposing these two additional constraints into the MPC optimization. The MPC stage cost is not employed as a Lyapunov function in our approach, while the quadratic dissipativity constraint allows both non-monotonic decreasing storage function (of the dissipation inequality) and economic MPC stage cost. Developments for the dynamically optimal operating point or rotated steady state with the quadratic dissipativity constraint is underway.

This paper is organized as follows. Notation, system model and economic MPC problem formulation are outlined in Section 2. Section 3 is reserved for summarizing the results of the auxiliary stage cost for stability purpose, the proposed Lyapunov function and supply rate for dissipativity, terminal constraints and terminal cost, and asymptotic average performance in the works of Rawlings et Al. The potential of quadratic dissipativity constraint for economic MPC is briefly discussed in Section 4. Section 5 concludes this paper.

2 Preliminaries

2.1 Notation

Capital and lower case alphabet letters denote matrices and column vectors, respectively. $(.)^T$ denotes the transpose operation. $\|u_i\|$ is the ℓ_2-norm of vector u_i. $\|M\|$ is the induced 2-norm of matrix M. In the discrete time domain, the time index is denoted by $k,\ k \in \mathbb{Z}$. x^+ denotes $x(k+1)$ for conciseness. In symmetric block matrices, $*$ is used as an ellipsis for terms that are induced by symmetry. Bold typefaces are used to denote decision variables.

2.2 System Model and Economic MPC Problem

The general system model of the form

$$\Sigma: \; x^+ = f(x,u), \tag{1}$$

without any disturbances, and the variable constraints of $u \in \mathbb{U} \subset \mathbb{R}^m$ and $x \in \mathbb{X} \subset \mathbb{R}^n$ and the state transition map $f : \mathbb{X} \times \mathbb{U} \to \mathbb{X}$ are usually considered in the developed economic MPCs. f is normally Lipschitz continuous. The economic stage cost, which is not the target tracking one, is represented by $\ell(x(k), u(k))$. $\ell(.,.)$ is usually convex and continuous for linear systems. The optimisation problem is not normally convex for nonlinear systems. Similarly to the target tracking MPC problem, the optimization problem of economic MPC will be to minimize

$$V_N(x, \boldsymbol{u}) = \sum_{k=0}^{N-1} \ell(x(k), u(k)) \tag{2}$$

subject to the model (1), as well as point-wise constraints of $(x(k), u(k)) \in \mathbb{Z}, \quad k = 0, 1, \ldots, N-1$, for some compact and time-invariant set $\mathbb{Z} \subset \mathbb{X} \times \mathbb{U}$, the terminal constraint $x(N) = x_s$, $x(0) = x$, where the decision vector $\boldsymbol{u} := [u(0), u(1), \ldots, u(N-1)]$ is the MPC computed control sequence at each time step. This finite horizon optimization problem is solved recursively. Only the first element of the optimal control sequence $\boldsymbol{u}$ is applied to control the plant. The state measurement or estimate is fed back as the new initial state for solving the problem in the next time step. The admissible set $\mathbb{Z}_N$ is defined next. $\mathbb{Z}_N$ is a set of (x, u) pairs satisfying the constraints

$$\mathbb{Z}_N := \{(x, \boldsymbol{u}) \,|\, \exists\, x(1), \ldots, x(N) : x^+ = f(x,u),$$

$$(x_k, u_k) \in \mathbb{Z}, \forall k = 1, 2, ..., N-1, x(N) = x_s, x(0) = x\}.$$

The set of admissible states $\mathscr{X}_N$ as the projection of $\mathbb{Z}_N$ onto $\mathbb{X}$ is defined as

$$\mathscr{X}_N := \{x \in \mathbb{X} \mid \text{such that } (x, \boldsymbol{u}) \in \mathbb{Z}_N\}.$$

The control sequence $\boldsymbol{u}$ is called feasible with the initial state x if $(x, \boldsymbol{u}) \in \mathbb{Z}_N$. The so-called optimal steady state is defined as the pair (x_s, u_s) fulfilling such feasible condition, i.e.

$$\ell(x_s, u_s) := \min_{x,u} \big\{ \ell(x,u) \mid (x, \boldsymbol{u}) \in \mathbb{Z}, \; x^+ = f(x,u) \big\}.$$

The assumptions involved in this problem will be the continuity of f and ℓ; the admissible set $\mathscr{X}_N$ contains x_s in its interior; and the existence of finite state/input gains [31]. Unlike the traditional target tracking MPC, the optimal steady state stage cost $\ell(x_s, u_s)$ is not necessarily smaller than $\ell(x, u)$ in the economic MPC. In economic MPC, it is desirable to achieve the following *asymptotic average* limit:

$$\lim_{T \to +\infty} \sup \frac{\sum_{k=1}^{T} \ell(x(k), u(k))}{T+1} \le \ell(x_s, u_s).$$

The formal definition for asymptotic average is given in Section 3.1. The authors in [31] have proved that: "There exists at least one admissible control sequence that steers the state to x_s at time N without leaving $\mathscr{X}_N$ and the closed-loop system has an asymptotic average performance that is at least as good as the best admissible steady state". While asymptotic-average economic performance is guarantee, this problem (2) does not assure the closed-loop stability. The next section addresses the proposed solutions to the stability problem of economic MPC. One may expect that, the optimality will be, to some extends, compromised due to these stability measures.

3 Augmented Stage Cost, Strict Dissipativity and Stability

Details in the chosen Lyapunov function and the supply rate, average performance, strong duality and relaxation with dissipativity, as well as strict dissipativity and stability in the works of Rawlings et Al. [6, 4, 31] are presented in this section.

In traditional stability-guarantee MPCs, the optimal cost of $V_N(x)$, denoted as $V_N^o(x)$, is employed as a Lyapunov function for the closed-loop system. This optimal cost is monotonically decreasing along solutions of the closed-loop system, i.e. $V_N^o(x^+) \le V_N^o(x)$. In economic MPC, however, that is not necessarily the case, even when the system is stable. "More fundamentally, for general nonlinear systems and cost functionals, it is not even guaranteed that x_s is an equilibrium point of the closed loop system. Since there exists (x, u) such that $\ell(x, u) < \ell(x_s, u_s)$, it may be the case that the optimal trajectory from x_s at time 0 to x_s at time N is different than $x(k) = x_s$ for all $k = 0, 1, \ldots, N$" [31].

3.1 Storage function and supply rate

The auxiliary *rotated* stage cost $L(x, u)$ is used for achieving the monotonic decreasing instead of the actual stage cost in (2) of the economic MPC problem. The supply rate is chosen as

$$s(x, u) := \ell(x, u) - \ell(x_s, u_s). \tag{3}$$

And the rotated stage cost is of the form

$$L(x, u) := \ell(x, u) + V_P - V_P^+, \tag{4}$$

with a chosen storage function $V_P(x, P)$, P is the multiplier.

For any vector $v(k)$, the asymptotic average is defined as follows [4]:

$$\text{Av}[v] = \{\bar{v} \in \mathbb{R}^{n_v} \mid \exists\, \kappa_n \to +\infty : \lim_{n\to+\infty} \text{sub} \frac{\sum_{k=1}^{\kappa_n} v(k)}{\kappa_n + 1} = \bar{v}\}.$$

In [6], it assumed the *strong duality* of the steady state problem, known via

$$\min_{(x,u)\in\mathbb{Z}} \ell(x,u) + V_{\mathcal{P}} - V_{\mathcal{P}+} \geq \ell(x_s,u_s),$$

for achieving stability. And it is proved in [4] that the dissipativity w.r.t the above chosen storage function and an extended supply rate is a relaxation of strong duality, while strong duality is sufficient for dissipativity. According to [31] and [4], the dissipativity is sufficient for the *optimal operation at steady state*, defined as $\text{Av}[\ell(x,u)] \subseteq \big[\ell(x_s,u_s), +\infty\big)$, but not for stability. The closed-loop stability which is concluded by the strict dissipativity is proved in [4].

3.2 Closed-loop stability

There is a variety of formulations for achieving the closed-loop stability herein.

3.2.1 Terminal constraint

For the above economic problem (2) with equality terminal constraint, x_s is an asymptotically stable equilibrium point of the closed-loop system with region of attraction $\mathscr{X}_N$ if it is strictly dissipative with respect to the supply rate (3). The proof starts with creating an auxiliary augmented problem with rotated stage cost, then shown the feasible sets, $\mathscr{X}_N$, coincide with those of original problem, and so does the optimising result. By having the rotated cost-to-go as the Lyapunov function, the stability is achieved by the strict dissipativity accordingly.

3.2.2 Terminal cost and terminal region

The authors have also proved in [4] that the closed-loop stability is achievable by using an adequately chosen terminal cost with an inequality terminal constraint (instead of an equality one). The proof also used an augmented rotated stage cost, considered the rotated cost-to-go as the Lyapunov function, and employed the strict dissipativity property.

3.2.3 Enforcing stability with modified stage cost

The stability is also achievable by directly modifying the stage cost, which is considered as a tuning activity in [31]. For

$$h(x,u) = V_P - V_{P+} + \rho(x) - \ell(x,u) + \ell(x_s,u_s),$$

in which $\rho(x) \leq L(x,u) - L(x_s,u_s)$, the modified stage cost of the form $\ell_m(x,u) := \ell(x,u) + \alpha(x,u)$, with $\alpha(x,u) \geq h(x,u)$, will help achieve the asymptotically stable equilibrium point of x_s with region of attraction $\mathscr{X}_N$. The chosen value of $\alpha(x,u)$ is given in Theorem 4 in [31]. The last paragraph of Section VI in [4] explains the modification to the stage cost in economic MPCs as an intermediary value lies between the original economic stage cost and the traditional target tracking MPC stage cost, adequately chosen.

3.2.4 Extensions to unsteady closed-loop processes

Periodic terminal constraint is considered for systems having pre-computed optimal periodic solutions, while *average constraints* are introduced to apply to state and input averages of unsteady closed-loop processes. The periodic terminal constraint, obtained from a dynamic state feedback strategy, is introduced in order for achieving a periodic optimal solution described in a previous work [3]. Reasoning for the average constraints is clearly given in [4]. The authors stated that, "shifting the focus from convergence to average performance leads naturally to the consideration of constraints on average values of variables (typically inputs and states), besides point-wise in time hard bounds as discussed in the previous sections and customary in MPC" [4] .

According to [3], when

$$\text{Av}[\ell(x,u)] \subseteq \big[\ell(x_s,u_s), +\infty\big),$$

the system is said optimally operated at steady state. And if, in addition, one or both of the conditions of

$$(1).\ \text{Av}[\ell(x,u)] \subseteq \big(\ell(x_s,u_s), +\infty\big),$$

$$\text{or } (2).\ \lim_{k\to\infty} \inf\ ||x(k) - x_s|| = 0$$

hold, it is said sub-optimally operated off steady state.

The economic MPC problem with average constraint is to obtain the following:

$$\text{Av}[\ell(x,u)] \subseteq \big(-\infty, \ell(x_s,u_s)\big], \tag{5}$$

$$(x,u) \in \mathbb{Z}\ \forall k \geq 0, \text{ and } \text{Av}[y] \subseteq \mathbb{Y},$$

where $y = h(x,u) \in \mathbb{Y}$ is an auxiliary output. $\mathbb{Y}$, the chosen average constraint set, is defined as $\mathbb{Y} \subset \mathbb{R}^p$ and required to contain $h(x_s,u_s)$, be closed and convex. On the ground of (5), the economic MPC with average constraint is thus sub-optimally operated off steady state.

In the next section, we show that the previously developed quadratic dissipativity constraint [35, 38, 37, 39] can be conveniently apply to the economic MPC problem.

4 Quadratic Dissipativity Constraint for Economic MPC

The quadratic dissipativity constraint for economic MPC is presented in this section. In this proposed stabilisation approach, the storage function and supply rate of the dissipation inequality are independent to the stage cost of economic MPC.

Define a parameterised quadratic supply rate for Σ (1), as follows:

$$\xi\big(u, x \,|\, Q, S, R\big) := u^T R u + 2x^T S u + x^T Q x, \quad (6)$$

where Q, S, R are multiplier matrices with symmetric Q, R. The input-state pair (u, x) of Σ is said to satisfy the *quadratic dissipativity constraint* (QDC) if there exists a function α of class $\mathcal{KL}$, and $k_0 \geq 0,\ k_0 \in \mathbb{R}$, such that

$$|\xi_{(k)}| \leq \alpha(|\xi_{(1)}|, k-1)\ \forall k \geq k_0. \quad (7)$$

For implementation, it is of interest to consider $\xi_{(k)}$ in its half plane. The following inequalities are used instead:

$$\text{For } \xi_{(1)} > 0, \quad 0 \leq \xi_{(k)} \leq \beta\, \xi_{(k-1)}\ \forall k \geq 1,\ 0 < \beta < 1. \quad (8)$$

$$\text{For } \xi_{(1)} < 0, \quad 0 \geq \xi_{(k)} \geq \beta\, \xi_{(k-1)}\ \forall k \geq 1,\ 0 < \beta < 1. \quad (9)$$

The convex quadratic constraint w.r.t. $\boldsymbol{u}$ (10) below is equivalent to (8) when $R \succ 0$:

$$\boldsymbol{u}^T R \boldsymbol{u} + 2S\boldsymbol{u} + \psi \leq 0, \quad (10)$$

where $\psi = x^T Q x - \delta_{(k-1)}, \delta_{(k-1)} = \beta\xi_{(k-1)},\ \beta < 1.$

The constraint (10) will be imposed on the economic MPC optimisation (the original one) as an enforcing stability constraint. For the closed-loop stabilisability of Σ, the input u is required to be bounded by the above quadratic dissipativity constraint (10), in association with the quadratic dissipativity of the open-loop Σ (1) w.r.t. the supply rate $\xi(.,.)$.

Σ (1) is said to be *quadratically dissipative* w.r.t the quadratic supply rate $\xi(u, x)$, if there exists a nonnegative storage function $V(x)$ such that for all $x(k)$ and all $k \in \mathbb{Z}^+$, the following dissipation inequality is satisfied irrespectively of the initial value of the state $x(0)$:

$$V(x_{(k+1)}) - \sigma V(x_{(k)}) \leq \xi(u, x),\ 0 < \sigma < 1. \quad (11)$$

$V(x) = x^T P x,\ P \succ 0$ is considered herein.

Proposition 1: $\Sigma\ :\ x^+ = Ax + Bu$ is quadratically dissipative w.r.t the quadratic supply rate $\xi(u, x)$ if the following LMI is feasible in P, Q, S, R, see, e.g. [12]:

$$\begin{bmatrix} P & PA & PB \\ * & \sigma P + Q & S \\ * & * & R \end{bmatrix} \succ 0,\ P \succ 0. \quad (12)$$

The stabilisability condition for unconstrained system Σ is then stated in below proposition.

Proposition 2: Let $\xi(0){>}0$, and $0{<}\sigma{<}1$. Consider Σ without control and state constraints. Suppose that the following optimisation is feasible:

$$\min_{P,\ Q,\ S,\ R} \quad x_0^T Q x_0$$
$$\text{subject to} \quad (12), \quad Q \prec 0, \quad R \succ 0;$$

Then any $u(k)$ feasible to (10), employing the resulting multiplier matrices Q, S, R, stabilise Σ.

The stabilisability theorem for nonlinear input-affine system Σ (1) with an extended supply rate has been developed in [38]. On the ground of this stabilisability condition, the economic MPC in association with the stability constraint (10), i.e. the MPC optimisation (2) is added with a new inequality constraint of (10), will assure the stability of the closed-loop system. The feasibility of (10) for constrained problems has been presented in previous works, see, e.g. [34, 36]. Since the MPC stage cost is not employed as a Lyapunov function in our approach, the quadratic dissipativity constraint allows both non-monotonic decreasing storage function $V(x)$ and economic MPC stage cost $\ell(x, u)$.

From the dissipativity perspective, the supply rate is different to those in [31]. The distance stage cost is not associated with the supply rate in this QDC approach, but the classical quadratic function w.r.t input and output. Its use is justified by the fact that the inclusion of the product $x^T u$ in quadratic supply rates is perceived as less conservative than the small-gain type supply rate, see, e.g. [13]. Nevertheless, the average performance and constraint feasibility will need to be addressed thoroughly in future developments. Developments for the dynamically optimal operating point or rotated steady state with the quadratic dissipativity constraint is underway.

5 Conclusion

A review on the economic MPC (EMPC) was given in this paper. EMPCs have been found effective in various energy-efficient applications. The unreachable setpoints and periodic steady states, as well as modified stage cost for stability purpose and average performance are the essences of these emerging EMPCs. The potential of applying the quadratic dissipativity constraint to the economic MPC has also been shown in the last section. From this review, we can, indeed, conclude that the economic MPC has merely started progressing both theoretically and practically.

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