Fuzzy-Chaos Framework for Analyzing and Quantifying Uncertainty in Labour Productivity in Complex Construction Environments

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Abstract –

Effective management of labour productivity in construction projects is important for ensuring project success. This paper introduces a novel approach to quantifying uncertainty in labour productivity through entropy calculations based on outputs from a fuzzy expert system. Two distinct scenarios are analyzed to demonstrate the practical application of the proposed methodology, one representing a week with high variability and the other a stable and predictable week. These scenarios highlight how entropy can be a powerful tool for identifying periods of high uncertainty and assisting project managers in making informed decisions about resource allocation, risk management, and strategic planning. The findings suggest that high entropy values indicate weeks requiring increased management attention and resources, whereas lower entropy values correspond to more stable conditions, allowing for standard operational procedures. This approach enhances understanding of dynamic project conditions and supports proactive project management practices. This study contributes to the body of knowledge by integrating fuzzy logic with entropy calculations, which offers a robust framework for managing the complexities of labour productivity in construction projects. Through real-world application and comparative analysis, this study validates the effectiveness of entropy analysis as a diagnostic and decision-making tool in the construction industry.

Keywords –

Construction management; Fuzzy expert systems; Chaos theory; Entropy

1 Introduction

Traditional linear and deterministic approaches for analyzing labour productivity in construction environments often fail to fully address the dynamic nature of construction projects [1]. Management strategies must evolve as projects become complex and are influenced by numerous interacting variables and unpredictable environmental conditions.

In this context, fuzzy logic offers ways to develop a methodological framework capable of handling the ambiguity and imprecision inherent in construction project management. Fuzzy logic can be used to more effectively deal with uncertain parameters, such as estimations of time and resources, by classifying them into degrees of truth rather than binary true-or-false categories [2]. This allows for better decision-making that aligns with the complexity of construction projects and expert knowledge.

Chaos theory also provides a valuable lens through which the unpredictable nature of construction projects can be understood and managed. It suggests that small variations in project conditions can lead to disproportionately large effects, a phenomenon often referred to as the “butterfly effect” [3]. Chaos theory has helped highlight the non-linear behaviours of project dynamics, thus emphasizing the need for project managers to anticipate and prepare for significant fluctuations in project trajectories that traditional methods might not predict [3,4].

Combining chaos theory and fuzzy logic addresses the dual challenges of unpredictability and imprecision by providing tools that predict potential chaos in project management while offering pragmatic approaches to handling complexities through adaptable and flexible decision-making frameworks [5]. Such an integrated approach is particularly effective in managing the interdependencies and dynamic interactions within construction projects, which traditional project management techniques often struggle to handle effectively.

In the context of this study, labour productivity is defined specifically as the measure of output produced per labour hour. This definition focuses on the efficiency of labour in delivering tangible results within a given time frame. In construction research, productivity can also encompass broader measures such as cost, duration, and overall project success.

This paper proposes a novel approach integrating chaos theory and fuzzy logic to enhance the understanding and management of labour productivity in construction environments. The objectives of this framework are to generate better understanding of the factors impacting labour productivity in complex construction environments and, ultimately, to facilitate more-informed management decisions. By understanding and applying the frameworks using these complex theories, project managers can enhance their ability to navigate the uncertainties of the construction field, leading to better project outcomes in terms of efficiency, cost, and time management. Thus, this approach advocates for a shift from conventional to more dynamic and responsive project management strategies that can better address the complexities of modern construction projects.

2 Background

Labour productivity in the construction industry is a critical measure of efficiency [6] and reflects the output per labour hour expended on construction tasks. It directly influences a project's cost, duration, and overall success. However, productivity in construction is uniquely challenged by the industry's inherent variability and complexity as well as the unpredictable nature of work environments and workforce dynamics [7]. It exhibits complex variability that is not normally distributed, which suggests the need to apply chaos theory to understand it [8].

Traditionally, construction productivity has been evaluated using time–motion studies, manual data collection, and direct observations of activities on-site [9]. Although useful, these methods often fail to capture the distinctions of complex environments where multiple interdependent variables affect productivity [10, 11]. Such methods are limited in handling construction projects' dynamic adaptations and stochastic nature, so more robust, adaptive analytical tools are required.

Fuzzy logic offers a means to handle uncertainty and imprecision, characteristics that are common in construction data [12]. It offers the ability to develop flexible mathematical frameworks for modeling the vagueness associated with subjective human judgments and qualitative data. For instance, studies have successfully implemented fuzzy systems to enhance decision-making under uncertainty in project management, particularly in areas such as risk assessment, cost forecasting, and the mitigation of potential delays and cost overruns [13, 14, 15, 16, 17].

Chaos theory is pertinent to understanding construction projects' complex and dynamic nature [18]. It demonstrates how even minor variations in initial conditions can cascade into disproportionately large impacts, a principle especially relevant to the non-linear dynamics of construction management [19, 20]. Through use of sensitivity analysis and fractal geometry, chaos theory provides methods for identifying and addressing sensitive points in the project lifecycle and repeating patterns that may significantly influence outcomes [19, 20]. Although less commonly applied in project management than fuzzy logic, chaos theory has the potential to provide insights into labour productivity by modeling how small variations in work conditions or team compositions can lead to disproportionate impacts on productivity. This approach can be particularly valuable in planning and managing workflows in real time to mitigate the effects of emergent issues [21].

Entropy is a measure of disorder or randomness in a system. Entropy analysis has been extensively explored as a tool for quantifying uncertainty in construction projects, thus offering insights into the stability and predictability of project systems by measuring deviations in cost, schedule, and performance [22, 23]. Employing entropy calculations alongside traditional project management metrics, such as cost performance index and schedule performance index, allows managers to monitor project progress and proactively address emergent issues [1].

Recent studies have demonstrated the utility of entropy-based approaches in risk assessment, resource allocation, and schedule stability, showing the ability of such approaches to predict project disruptions and enable dynamic adjustments. For example, entropy analysis has been used in scheduling resource-constrained projects to measure disorder and forecast potential delays [22], and it has been integrated with Bayesian inference to refine predictions based on new data [23].

Integrating fuzzy logic and chaos theory in construction management is a novel approach that has seen limited exploration, especially in terms of enhancing productivity. This integrative approach offers a framework capable of managing the inherent uncertainties of construction projects and predicting and adapting to the non-linear dynamics that affect labour productivity. The proposed framework thus enhances traditional project management approaches by integrating advanced analytics specifically designed to navigate the complexities of modern construction environments, thereby significantly improving labour productivity.

3 Methodology

The methodology, presented in Figure 1, explains a systematic process that begins with factor shortlisting, where key labour productivity factors are identified and prioritized based on their relevance and impact on construction projects. Subsequently, these factors are evaluated through a fuzzy expert system (FES) that classifies each factor into qualitative states (*Poor*, *Fair*, *Good*) based on expert evaluation. This classification facilitates pattern analysis, enabling us to identify and interpret trends and patterns in the data, which provides insights into the factors' behaviors and interactions. The final step in this methodology is entropy analysis, where uncertainty and variability of the factors' states is quantified based on the FES results. This entropy analysis serves as a measure of disorder or randomness in the system, offering a deeper understanding of the project's complexity and better guiding strategic decision-making in managing construction project productivity.

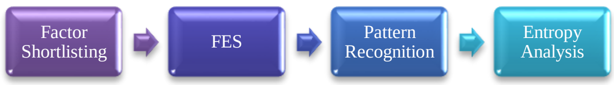

Figure 1. Proposed methodology

3.1 Labour Productivity Factors

A comprehensive list of factors comprising eight main categories and 137 sub-factors that can positively or negatively influence labour productivity is adapted from the framework proposed by Raoufi and Fayek [24]. The methodology proposed by Pourrahimian et al. (2024) is employed to shortlist this extensive list [25] and focus on the most relevant factors for the case study. The detailed selection process and criteria, however, are beyond the scope of this study. This framework was selected due to its proven applicability in similar contexts, where handling fuzzy, uncertain, and non-linear data is crucial. Notably, the framework's robustness in dealing with complex variability in labour productivity makes it particularly suitable for construction projects affected by a wide range of fluctuating conditions. Figure 2 illustrates the distribution of sub-factors across each main factor category.

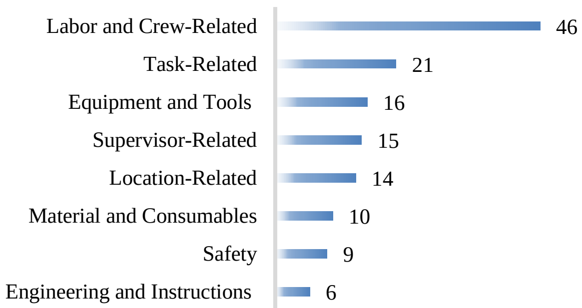

Figure 2. The number of sub-factors under each main factor

3.2 Fuzzy Expert System

FES is applied to manage the ambiguity and vagueness associated with these factors. Each main factor is represented as a fuzzy variable with degrees of membership ranging across relevant sets (e.g., *Safety* is categorized into *Poor*, *Fair*, and *Good*). Membership functions are defined for each fuzzy variable, quantifying the degree to which an element belongs to a particular fuzzy set. Fuzzy rules are constructed based on expert input and empirical data; these rules form the core of the fuzzy logic system, linking input variables to labour productivity.

To apply FES in labour productivity using a rule-based approach, it is essential to establish a set of rules within the model. Equation (1) is used to determine the number of fuzzy rules.

$$Number\ of\ rules = (m)^n \tag{1}$$

where m is the number of fuzzy sets and n is the number of input variables.

To reduce complexity and prevent an overwhelming number of rule definitions, rule blocks have been defined. Instead of directly using all sub-factors to evaluate the main factor, the model first assesses the impact of a group of sub-factors on their respective main factor and, subsequently, the influence of these groups on the main factor. For instance, with four input variables and three fuzzy sets, the required rules would be 81. However, by grouping these into two rule blocks, only 18 rules per block and an additional nine rules for combining the outputs of these blocks are needed. This method substantially reduces the number of rules, as Figure 3 illustrates.

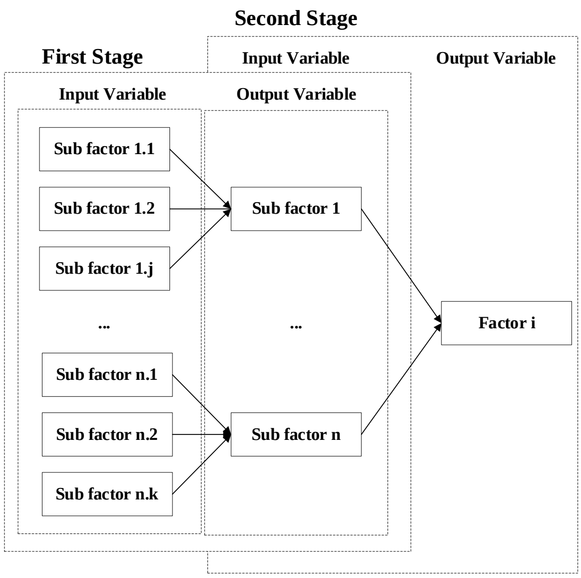

Figure 3. General structure of rule blocks

The input variables have the fuzzy set of x_i and the output variable of y. The input variables are measured on a scale of 0 to 10 and are divided into three categories, represented by the linguistic terms *Poor*, *Fair*, and *Good*. Membership values between 0 and 1 are assigned to each category. The output variable is on a scale of 0 to 100 and is also divided into three categories represented by the linguistic terms *Poor*, *Fair*, and *Good*. Table 1 tabulates the terms of assessment for x_i and y.

As the occurrence of each input variable is independent, FES rules were developed for each category. The outputs were used as input values for the next level. The standard operators *And: Min, Or: Max* implement the rules.

Table 1. Input and output variables of FES

Variable	Numerical scale	Linguistic descriptor	Membership function
x	1–10	Poor	Trapezoidal (0,0,1,4)
		Fair	Triangular (2.5, 4.5, 6.5)
		Good	Trapezoidal (5,8,9,9)
y	0–100	Poor	Trapezoidal (0, 0, 30, 50)
		Fair	Triangular (30, 50, 70)
		Good	Trapezoidal (60, 80, 100, 100)

3.3 Pattern Recognition

Chaos theory is utilized to identify patterns and conduct sensitivity analysis within the productivity system, highlighting its dynamic and highly responsive nature to minor variations in input conditions.

The model's pattern identification process examines productivity measure's non-linear responses to alterations in task characteristics or human factors, demonstrating how these changes can disrupt usual productivity patterns. Run charts can help identify these patterns. A run chart is an effective tool for monitoring data trends over time, displaying data points along a timeline on the x axis and their values on the y axis. It is used primarily to detect shifts identified by six or more points consistently above or below the median and trends indicated by five or more points consecutively rising or falling. Additionally, a run chart helps identify non-random variations by analyzing the number of runs (sequences of points on one side of the median) against expected ranges. It also shows outliers and unusual data points that merit further investigation.

Figure 4 shows an example run chart for the defuzzified evaluation of FES for a specific factor across various weeks. Tracking a variable over a series of weeks makes it easier to visualize trends, shifts, and patterns. Two distinct trends are indicated in differently coloured boxes. The section in a green box shows an increasing trend where the data points consistently rise over several weeks, suggesting a positive development in the monitored factor. The section in a red box illustrates a decreasing trend where the data points decline over time, signalling a reduction in the analyzed factor.

Another key feature shown in the chart is a "Shift," highlighted in a yellow box, where the data points remain on one side of the median over consecutive weeks. This could point to a significant event or intervention that impacted the process, and it merits closer examination to understand its causes and implications.

Additionally, the chart includes an "Astronomical" point, indicating an outlier. This particular data point is substantially different from others and could represent an anomaly that may skew the overall analysis or a unique event that requires separate consideration.

This run chart exemplifies how visual data analysis tools can help monitor ongoing processes. Identifying increasing and decreasing trends provides insights into the performance dynamics over time.

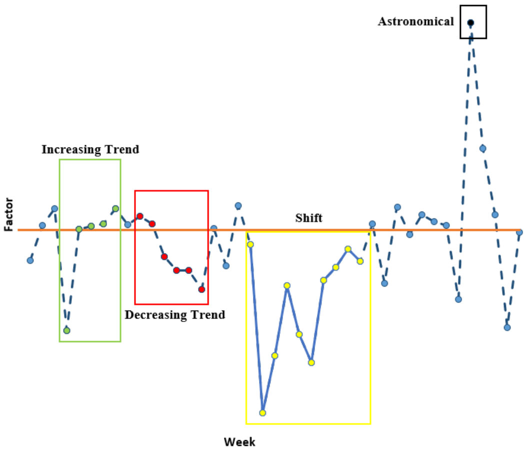

Figure 4. Pattern identification example

3.4 Entropy Analysis

Following identification of potential chaotic patterns, entropy calculations are integrated to measure the degree of uncertainty and disorder within the productivity factor states. This step enhances researchers' understanding of the variability in productivity outcomes and provides a quantitative measure of system complexity.

By applying FES, each productivity factor is evaluated and categorized as *Poor*, *Fair*, or *Good*; the next step entails quantifying the uncertainty embedded in these assessments using entropy calculations. The FES provides membership values for each state of a factor, which are inherently probabilities that sum to one.

Therefore, no additional normalization is required. These membership values directly reflect the system's confidence in each productivity state for each factor considered.

Entropy is calculated for each factor to measure the level of uncertainty or disorder associated with the fuzzy state distributions. Shannon's entropy formula is applied, given by:

$$H = -\sum_{i=1}^{3} p_i \log(p_i) \tag{2}$$

where p_i represents the probability of each state for a particular factor. Where a state's membership value is zero, its contribution to the entropy calculation is also zero, as $0\log(0)$ is mathematically treated as zero. This process is repeated for each factor, and the results provide a measure of the uncertainty associated with each aspect of labour productivity. To gain a holistic view of a project's uncertainty in terms of labour productivity, the entropy values are aggregated across all factors. This can be done by averaging all individual entropy values:

$$H_{average} = \frac{1}{8}\sum_{j=1}^{8} H_j \tag{3}$$

where $H_{average}$ is an overall measure of the uncertainty in the project's labour productivity and indicates the complexity and variability within the project environment.

An entropy value close to 1 indicates greater unpredictability and variability among the factors, which implies a more complex scenario where multiple interdependent factors influence productivity. Conversely, a lower entropy value, closer to 0, indicates more stability and predictability. Understanding these entropy values helps project managers and decision-makers identify periods or conditions of high uncertainty, which enables them to take pre-emptive measures or adjust project management strategies accordingly. This approach aids in better resource allocation, risk management, and scheduling, ultimately leading to more efficient project execution. For a more in-depth discussion, including specific entropy thresholds and their implications across various project phases, the authors refer readers to the detailed journal article by Pourrahimian et. al (2024), which is citation [1]

Figure 5 maps the assessment process of various productivity factors and their sub-factors within a construction project using the proposed framework. Each factor group is evaluated using fuzzy logic to determine each factor's state (*Poor*, *Fair*, *Good*). Subsequently, the distribution of these states across each factor is analyzed to calculate entropy, providing a measure of uncertainty or variability.

3.5 Objectives and Expected Outcomes

The primary objectives of this framework are twofold. First, by integrating fuzzy logic and chaos theory, the framework aims to better understand the factors impacting labour productivity in complex construction environments. Second, it is designed to support decision-making processes by understanding the variability in productivity outcomes and providing a quantitative measure of system complexity. This dual focus enhances

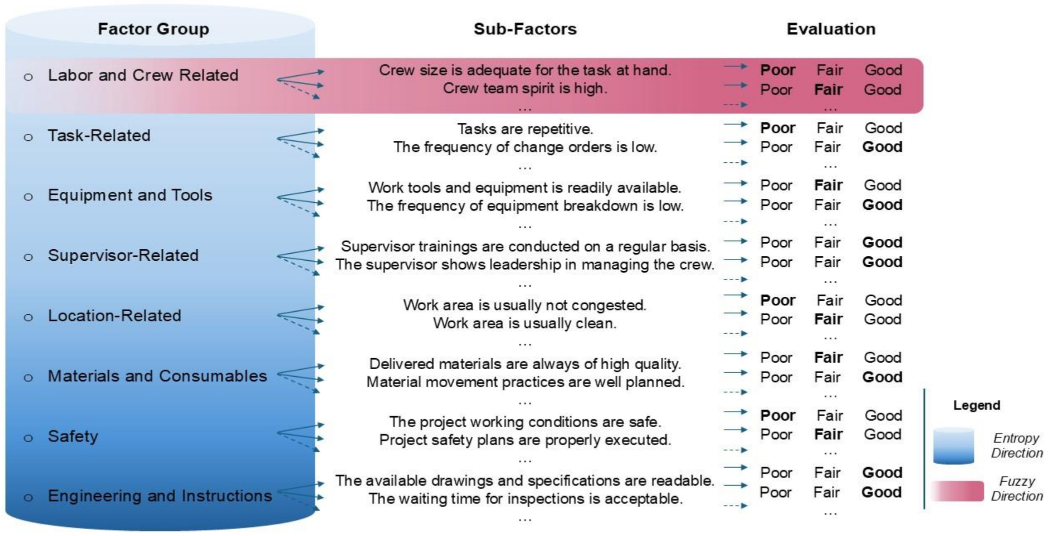

Figure 5. Integrated analysis of labour productivity factors using fuzzy logic and entropy

comprehension of dynamic productivity issues and facilitates more-informed management decisions.

4 Case Study, Results, and Discussion

The case study examines data from a $100-million industrial project, from February 2022 to October 2023, involving approximately 400,000 direct labour hours. The project scope included electrical and instrumentation (E/I) work, which made up 13% of the project, and piping tasks, which comprised 40% to 45%. The project was expected to employ a peak workforce of 280 individuals, comprising 250 workers and staff, plus 30 subcontractors, with 125 workers dedicated specifically to piping tasks. An expert panel featuring structural, electrical, piping, and mechanical engineering professionals was assembled to identify critical productivity factors. The panel included a structural construction manager with 22 years of experience, an electrical superintendent with 15 years, a piping superintendent with 34 years, a mechanical engineer operations manager with 19 years, and an electrical foreperson with 12 years. Following extensive analysis, this panel unanimously identified the essential state variables impacting project productivity.

Table 2 presents two distinct scenarios with the FES for each factor, each representing the labour productivity conditions across two different weeks of the project, to illustrate how varying project environments can significantly impact the entropy calculations and, consequently, the perceived stability and uncertainty within a construction project.

The first scenario explores a week characterized by high variability in project conditions. This includes disruptions in supply chains, unforeseen changes in project specifications, and fluctuations in workforce availability. The FES's evaluations reflect this instability, assigning a mix of Poor and Fair states across most factors, with very few factors rated as Good. This scenario illustrates a week for which entropy calculations reveal high uncertainty, indicating a need for proactive management interventions to stabilize the project environment. Conversely, the second scenario depicts a week where the project environment is stable and predictable. Factors such as weather conditions, material supply, and crew performance are optimal, leading to predominantly Good evaluations from the FES. This week demonstrates a lower entropy value, suggesting a well-managed project with minimal uncertainty and high predictability in labour productivity.

The integration of fuzzy logic and chaos theory within construction management can provide a promising framework for addressing the industry's inherent complexity. Beyond the immediate applications demonstrated in this study, insights from related fields further emphasize the potential of these methodologies to provide an alternative way to enhance the performance of construction projects.

Table 2. Membership value for each factor in two scenarios

	Scenario 1			Scenario 2		
Factor	Poor	Fair	Good	Poor	Fair	Good
Engineering and instructions	0.6	0.4	0	0	0	1
Safety	0.5	0.5	0	0	0.1	0.9
Material and consumables	0.7	0.3	0	0	0.1	0.9
Location-related	0.5	0.5	0	0	0.1	0.9
Supervisor-related	0	0.5	0.5	0	0.1	0.9
Equipment and tools	0	0.6	0.4	0	0.1	0.9
Task-related	0	0.4	0.6	0	0	1
Labour- and crew-related	0.5	0.5	0	0	0	1
Entropy ($\boldsymbol{H_{average}}$)		0.97			0.25	

The authors hypothesize that the fuzzy-chaos model will exhibit superior adaptability and accuracy in forecasting labor productivity due to its unique ability to manage construction projects' inherent uncertainty and non-linearity. This capability stems from the integration of fuzzy logic, which allows for handling ambiguous and imprecise data, and chaos theory, which addresses the sensitive dependence on initial conditions.

The fuzzy chaotic neural network proposed by Zhang and Wang (2012) presents a combination of fuzzy logic, chaos theory, and neural network dynamics to tackle highly non-linear and uncertain systems. This approach mirrors the core challenges faced in construction labour productivity management, where complex interdependencies and persistent fluctuations dominate [26]. Incorporating elements of such neural networks into construction project models could significantly enhance their ability to simulate issues, such as workforce dynamics or unforeseen disruptions. For instance, chaotic feedback loops could be employed to anticipate cascading delays or cost overruns, providing a proactive management tool.

Similarly, the chaos-adapted multi-verse optimizer algorithm demonstrates the power of integrating chaotic maps and fuzzy logic in optimization tasks. Although applied in benchmark optimization problems, the principles underlying these chaotic adaptations could be valuable for construction resource management [27]. For example, replacing traditional deterministic approaches with chaos-driven algorithms could improve scheduling robustness and enable construction managers to dynamically allocate resources in response to fluctuating conditions.

The use of chaotic maps to replace stochastic components in optimization algorithms offers an appealing analogy for managing uncertainties in construction projects. Just as the algorithm balances exploration and exploitation of solution spaces, construction managers could apply similar principles to balance short-term project needs with long-term strategic goals, which might ultimately enhance efficiency and resilience.

These external applications provide evidence of the power of chaos theory and fuzzy logic across domains, which can be successfully adapted to the construction industry.

This study contributes critical advancement by contextualizing and empirically validating these theories within the construction industry. This not only enhances the understanding of complex project dynamics, but also offers actionable strategies that enhance productivity management and project resilience.

5 Practical Implementation

Project managers can follow a structured approach to facilitate the integration of the fuzzy-chaos model into daily decision-making processes. This includes the initial setup of the model within a decision support system (DSS), regular updates of project data, and interpretation of output for proactive management. Specifically, the authors suggest incorporating the model into existing project management software platforms that support modular customization. This allows for seamless integration without requiring managers to switch between different systems.

Practical steps for adopting the fuzzy-chaos model in construction project management include:

- Initial setup: Integrating the model with existing project management tools or DSS
- Data entry: Regular input of project data into the model to reflect current conditions
- Analysis and interpretation: Using the model's outputs to make informed decisions based on the calculated entropy values, which indicate the level of uncertainty and potential risk
- Adjustments and re-evaluation: Continuously adjust project strategies based on model feedback and re-evaluate the results for improved decision-making

To further enhance the practical utility of the fuzzy-chaos framework, the authors propose its extension to include longitudinal analysis capabilities. This adaptation would allow project managers to track labour productivity trends over weeks and across entire project lifecycles. Continuous monitoring can be facilitated by integrating the framework with real-time data collection tools and advanced analytics platforms.

Such an approach would enable the identification of long-term trends and patterns in labor productivity, which are essential for strategic planning and resource allocation. It would also allow for the adjustment of project management strategies in response to observed trends, thereby enhancing the adaptability and effectiveness of the management process.

6 Conclusions

This study presents a novel methodology that integrates fuzzy logic and entropy calculations to evaluate labour productivity factors in construction projects. The proposed approach utilizes the strengths of FES to classify productivity-related factors into three distinct states: Poor, Fair, and Good. This classification provides an understanding of various productivity factors, enabling project managers to pinpoint specific areas that require attention or demonstrate strength within the project.

Further, entropy calculations are applied to quantify the uncertainty inherent in these assessments. Findings indicate that factors characterized by high entropy values typically reflect significant variability, influenced by external uncertainties such as supply chain disruptions and workforce fluctuations. These high entropy values signal areas requiring closer managerial attention and possibly implementing adaptive management strategies to mitigate potential risks. Conversely, factors with low entropy values exhibited stability and predictability, suggesting that existing management practices were effective and could be maintained to ensure ongoing project efficiency.

This methodology has implications for project management. By identifying factors with high entropy, project managers can better allocate resources, pre-emptively implement risk mitigation strategies, and dynamically adjust project plans to address emerging challenges. This approach also supports continuous monitoring of labour productivity, enabling project teams to respond swiftly to changes and maintain control over project delivery.

While the fuzzy-chaos model is anticipated to provide significant insights into labor productivity, it is important to acknowledge its limitations. One such limitation is the model's reliance on the quality and availability of input data. Inaccurate or incomplete data can significantly affect the model's predictions and outputs. Additionally, the complexity of integrating chaos theory and fuzzy logic may present a steep learning curve for users unfamiliar with these concepts. Furthermore, the model's novel approach, while robust in handling project complexities, has not yet been tested across a broad spectrum of construction environments, which may limit its generalizability.

Looking ahead, further research could explore the integration of this methodology with advanced predictive analytics and machine learning techniques to enhance the accuracy of productivity assessments and entropy predictions. Additionally, applying this methodology across different construction projects, including residential, commercial, and infrastructure, would help validate its effectiveness and adaptability. Investigating the impact of incorporating real-time data feeds into the fuzzy logic assessments could further refine this tool, offering a more dynamic and responsive resource for project management.

In conclusion, this paper contributes an analytical framework to the field of construction project management, bridging the gap between qualitative assessments and quantitative analytics. It can be used to enhance project managers' ability to understand and manage labour productivity, fostering more resilient and efficient project environments. The methodology informs current project management practices as well as sets the stage for future innovations in project analytics.

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