Variable Speed Limits Control for Smart Work Zone with Connected Vehicles

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Abstract

Freeway work zones with lane closures can cause traffic disruption and congestion with increased travel time, safety risks, and emissions. Variable speed limit (VSL) control has been widely studied to mitigate the negative impacts of work zone lane closures. To ensure effective VSL control, accurate detection of traffic conditions is important. However, stationary sensors from distant upstream or merging areas of a work zone can only provide location-based detections, which may not be sufficient for effective VSL control. Therefore, this study proposes a VSL control system for a smart work zone with a weighted traffic density estimation algorithm using data from stationary sensors and connected vehicles (CV). With location-based traffic detections from distant upstream and merging areas of a smart work zone, data from CVs in locations between the stationary sensors are used to obtain the weighted density. This weighted density is then fed to a feedback VSL controller, which can dynamically define the appropriate speed limits to reduce the negative impacts of the work zone closures. The proposed VSL controller was implemented and evaluated under a calibrated freeway work zone environment using the traffic simulator SUMO. The results show that the system can provide accurate traffic density estimation and effectively improve traffic mobility, safety, and environmental sustainability near the work zone area.

Keywords -

Variable Speed Limits Control; Stationary Sensors; Connected Vehicle; Weighted Traffic Density; Smart Work Zone

1 Introduction

Freeway work zones play an important role in maintaining and upgrading freeways. However, closing one or more lanes in a work zone area can cause bottlenecks and traffic disruption near work zone areas. This disruption can easily lead to congestion, increased travel time, safety risks to motorists and work zone workers, emissions, and fuel consumption. Therefore, developing an effective strategy and mitigating the negative impacts of work zones is essential.

Variable Speed Limit (VSL) control is one of the Intelligent Transportation Systems that has been studied to improve traffic conditions near work zone areas. By generating dynamic speed limits, VSL control can regulate the travel speeds of vehicles approaching a work zone to alleviate the impacts of capacity loss due to lane closures. Different VSL controllers have been developed using control strategies such as rule-based [1], feedbackbased [2], and reinforcement learning [3] approaches. These VSL controllers rely on accurate traffic condition detections to ensure the control effectiveness.

Stationary sensors can detect traffic conditions at fixed locations. Many VSL controllers have used traffic measurements by fixed sensors near work zone areas to achieve control objectives [4][5]. In practice, fixed sensors may be present at distant locations upstream of a work zone and may not provide accurate traffic measurements in work zone areas. However, sensors at a smart work zone's merging area can detect traffic conditions immediately upstream of a work zone. The density variations between locations of distant upstream and merging areas of a work zone can hardly be estimated using stationary sensors unless dense stationary sensors are installed, which is cost-prohibitive.

Connected vehicles (CVs) can provide real-time traffic state observations along their travel path. When a CV travels through work zone areas, the vehicle can become a probe sensor, sending the location and travel speed to fill the gap between the stationary sensors. Many approaches have used massive data from connected vehicles to estimate traffic states [6] [7]. However, work zones particularly short-term work zones exist for a limited period of time, which makes it challenging to collect a large amount of historical CV data for traffic state estimation.

This paper proposes a VSL control system with a weighted traffic density estimation algorithm for a smart work zone using stationary sensors and CV data. CV data is used to bridge the gap that exists between the data from sensors located at distant upstream and the ones in merging areas of a smart work zone.

The rest of this paper is organized as follows: studies on VSL control and density estimation are reviewed first. Then, the development of the weighted density approach and VSL controller are discussed in the Methodology section. Later, the system evaluation and results are discussed. Finally, the conclusion and future work are presented in the conclusion section.

2 Background and Literature Review

A hypothetical freeway smart work zone scenario in which one lane is closed is shown in Figure 1. Due to the capacity loss, traffic congestion occurs at the merging area of the work zone with a queue propagating upstream. The work zone is a smart zone with two sets of stationary sensors. Traffic Sensor (TS) 1 is a stationary sensor that is present upstream of the work zone, and TS 2 is the smart work zone stationary sensor at the merging area of the work zone. CVs capable of transmitting speed and location, along with conventional vehicles with no connectivity travel through the work zone area from free flow to congested and then recover to free flow once they pass the bottleneck area.



Figure 1. Layout of a Hypothetical Freeway Work Zone with Stationary Sensors and CVs

The traffic queue forms first at the merging area and then propagates upstream. Since it takes time for the queue to reach the location of TS 1, the traffic states detected by TS 2 could be heavily congested, while the traffic states from TS 1 could be free flow. The discrepancies in traffic detections from stationary sensors due to different sensor locations could impact the effectiveness of VSL Control.

VSL control has been studied to mitigate congestion near bottleneck areas with lane closures. The control strategies can be broadly categorized into rule-based [1], feedback-based [2], and reinforcement learning-based [3] controllers. Rule-based VSL controllers, such as the fuzzy-logic controller [8], post speed limits using predefined rules and thresholds of traffic flow, density, and speed. The control objective, such as the critical traffic density [2], is used by feedback-based VSL controllers to obtain speed limits. Reinforcement learn-based VSL controllers leverage learning algorithms and traffic measurements to train a model and generate dynamic speed limits to reduce travel time [3]. All these VSL controllers require accurate traffic state measurements. However, the types of traffic sensors, e.g. stationary sensors and probe vehicles, and the locations of sensors are commonly not investigated in such studies. The sensor factors could impact the accuracy of traffic state estimation near work zone areas and, consequently, the effectiveness of VSL control.

Dense stationary installation sensor could approximate the continuous measurements in space to provide higher accuracy in traffic state estimation. However, the high installation and maintenance costs are prohibitive for smart work zone systems. Instead of dense sensor installation, mathematical models, such as observers, are developed to provide traffic estimations using nearby sensors when sensor detections are missing [9][10]. For such models, the traffic fundamental diagram is calibrated to minimize the discrepancies between model-generated and real sensor data so that model-generated traffic state estimations can be used by applications such as VSL control [11]. However, traffic congestion can easily occur near work zone areas due to capacity loss, which leads to frequent traffic state transitions from free flow to congestion when a queue starts forming or from congestion to free flow when a queue starts resolving. This traffic state transition would make it difficult for mathematical approaches to estimate traffic state accurately between the locations of stationary sensors.

CVs travelling through work zone areas can be considered as probe sensors to estimate traffic transition between the locations of stationary sensors. Relying on the conservation law of traffic flow, the traffic state is estimated using the spacing and speed of CVs [12]. A deep convolutional neural network approach is studied to estimate traffic speed and congestion qualitatively and quantitatively using a large amount of historical probe vehicle data from different congestion scenarios [13]. [7] proposes a macroscopic model to estimate the traffic states of that segment based on the CV market penetration rate. These approaches use probe vehicles with either advanced onboard sensors like radar to track the headway between vehicles or a large amount of historical CV data to estimate traffic states. However, advanced onboard sensors may not be always available on CVs and lack of a large amount of historical CV data could occur due to the limited time periods of works zones, which would make it challenging to apply these probe sensors approaches and accurately estimate traffic states near work zone areas.

In this study, a VSL control system with a weighted density estimation algorithm was developed. Considering stationary sensors and CVs, weighted density is calculated and fed into a VSL controller. Specifically, sensors from distant upstream and merging areas of a smart work zone are used to obtain locationbased traffic densities. Then, the weight of two locationbased densities is derived using CV vehicles between the two sensor locations based on kinematic wave theory. The system proposed in this study mainly offers the following contributions:

- It investigates whether sensor locations have any impacts on VSL control. The effectiveness of VSL control is evaluated using fixed sensors at distant upstream and merging areas of a work zone.
- It provides weighted traffic density estimations for VSL control. The weight is calculated using CVs based on kinematic wave theory.
- It produces good VSL control performance, despite the impacts of stationary sensors locations at distant upstream and merging areas of a work zone. The system evaluation shows consistent improvement in mobility, safety, and sustainability.

3 Methodology

The effectiveness of VSL control relies on the accuracy of traffic state estimation using traffic sensors near work zone areas. A VSL control system with a weighted density algorithm is illustrated in Figure 2. The weighted traffic density is obtained using stationary sensors and CVs to approximate traffic density at a work zone area. By using this weighted density, a VSL controller is designed to regulate traffic speeds. The framework of the proposed system is discussed, followed by traffic density estimation and VSL control in the subsequent subsections.



Figure 2. VSL Control System Framework

A feedback-based VSL controller is developed with critical traffic density as the control objective. The VSL controller processes the critical density and weighted density error to calculate speed limit. These speed limits regulate vehicle travel speeds and influence traffic conditions in the vicinity of the work zone area. This study assumes a regulatory VSL system with the full driver compliance.

The weighted density component of the framework

effectively approximates the traffic density within the work zone area. The baseline traffic density is detected from sensors at the distant upstream and the merging area and then is adjusted by weights that are calculated using the speed and location of CVs. The weighted density is obtained by applying the weight to traffic density from stationary sensors.

Comparing the error between the critical density and weighted density, VSL control will generate dynamic speed limits to minimize the error. This proactive approach ensures that the traffic flow can sustain the maximum work zone capacity, thereby enhancing traffic conditions without inducing a capacity drop [14]. The capacity drop phenomenon occurs when a queue forms at the bottleneck area leading to increased capacity loss [15]. Details of the estimation of the weighted traffic density and VSL controller are presented in the subsequent sections.

3.1 Traffic Density Estimation

Traffic sensors at a work zone's distant upstream and merging area can detect location-based traffic density. However, the two sensor locations may have a density discrepancy, as illustrated in Figure 1. A weighted density is calculated using CVs based on kinematic wave theory to address this discrepancy.

The traffic state transition is shown in the triangular fundamental diagram in Figure 3.



Figure 3. Traffic States from Fundamental Diagram

The triangular fundamental diagram has the free flow branch with densities smaller than the density ρ_r and the congested branch with densities larger than the density ρ_r .States *a*, *d*, and *c* lie on the free flow branch while States *b* and *b'* lie on the congested branch. State *a* represents high traffic demand, which is higher than the work zone capacity *b'* at the congested status. Because the demand is higher than the road capacity, congestion occurs at the merging area and leads to the capacity drop phenomenon. Traffic flow further drops to State *b*. State *c* represents low traffic demand, lower than the work zone capacity so that the queue forms due to congestion can be resolved. State *d* represents the same work zone capacity at the free flow status. v_b and v_f represent travel speeds at the State *b* and *a* respectively. State *d* and *c* share the same free-flow speed v_f with State *a*. Traffic density ρ_a under free flow status and ρ_b under congested status for State a and b are also illustrated in Figure 3. The VSL control objective is to maintain traffic flow at the work zone capacity State *d* without a capacity drop. The work zone critical density ρ_{cr} as the VSL control goal corresponding to State *d* is shown in the figure as well, which will be discussed in detail under the VSL Control section below.

Figure 4 illustrates the traffic state transition from the space-time diagram under the kinematic wave. The bottleneck location is bold in blue. Two traffic sensors, as shown as grey blocks in Figure 4, are located in two areas, one close to the bottleneck at the merging area and the other at the distant upstream of the bottleneck. When the high traffic demand reaches the bottleneck, a queue forms at the bottleneck area and transfers the traffic state from *a* to *b*. This queue propagates upstream at the shockwave speed ω_{ab} . When the low traffic demand *c* appears and meets at the queue, this queue starts resolving and propagating downstream towards the work zone at shock wave speed ω_{cb} . Once the queue is resolved, the traffic state *c* will prevail along the work zone area.



Figure 4. Traffic States from Space-time Diagram

From Figure 4, traffic states at two traffic sensors are different until the queue propagates upstream and reaches the distance upstream of the work zone. Consequently, the traffic densities $\rho_{ma}(k)$ and $\rho_{mb}(k)$ from measurements of sensors located distant upstream and merging area of the work zone respectively, likely face discrepancies before the queue reaches the sensor location distant stream. Therefore, a weighted density ρ_w is calculated using Equation (1) below.

$$\rho_w(k) = (1 - \alpha(k))\rho_{ma}(k) + \alpha(k)\rho_{mb}(k) \qquad (1)$$

The discrete time step is represented by k. The weight $\alpha(k)$ is applied to traffic density $\rho_{ma}(k)$ and $\rho_{mb}(k)$ detected by traffic sensors at time step k to calculate the weighted density $\rho_w(k)$.

To obtain the weight $\alpha(k)$, the speed and location transmitted by CVs are utilized. In Figure 4, at time step

k, the length of the queue from the bottleneck is l_1 and the total distance between two traffic sensors is l_2 . Because of the queue propagating upstream, at the congested area within the queue l_1 , vehicles travel at the speed v_b while vehicles outside the queue but between two traffic sensors l_2 travel at the speed v_f . Therefore, the ratio of l_1 and l_2 is used to calculate the weight as

$$\alpha(k) = 0.5 + 0.5 \, l_1(k) / l_2(k) \tag{2}$$

From Equation (2), the weight $\alpha(k)$ equals 0.5 when the queue length is 0, meaning the same weight is applied to traffic densities detected by sensors at distant upstream and merging areas of a work zone when there is no queue or congestion. Only traffic density from the merging area is used when the queue reaches the sensor located distant upstream. Therefore, more weights are assigned to the sensor at the merging area when the queue becomes longer.

To determine the location of a queue $l_1(k)$, the speed and location of CVs are used. We assume the speed of CVs is similar to conventional vehicles, as studied in [7]. Then the length $l_1(k)$ can be approximated in Equation (3) as the distance between a CV, which is at the furthest upstream of the bottleneck travelling below a speed threshold v_{tr} , and the location of the bottleneck. $l_{i,CV}$ represents the distance between a CV and the bottleneck. This CV satisfies the requirement in Equation (4) where the CV $l_{i,CV}(k)$ is not larger than l_2 and speed $v_{i,CV}(k)$ is not higher than v_{tr} .

$$\begin{cases} l_1(k) = \max(l_{i,CV}) \\ l_{i,CV}(k) = distance \ between \ a \ CV \ and \\ bottleneck \end{cases}$$
(3)

$$\begin{cases} l_{i,CV}(k) \leq l_2 \\ v_{i,CV} \leq v_{tr} \end{cases}$$
(4)

The speed threshold v_{tr} is obtained using Equation (5).

$$v_{tr} = \frac{v_f + v_b}{2} \tag{5}$$

The free flow speed v_f and speed v_b at state *b* can be obtained from the fundamental diagram in Figure 3. It should be noted that there may be cases where no CVs travel below the speed threshold v_{tr} or no CVs are available between the two traffic sensors located at distant upstream and merging areas due to a low market penetration rate. In those cases, the same weights are

assigned to traffic densities detected by the two stationary sensors at different locations.

3.2 Variable Speed Limit Control

3.2.1 Controller

A feedback VSL controller [16] can generate dynamic speed limits by controlling traffic density at a work zone area towards the control objective, which is the critical density ρ_{cr} as shown in Figure 3. When the traffic state can be maintained at State *d* in Figure 3, the flow rate can be preserved at the maximum work zone capacity without causing a capacity drop.

The VSLs can be calculated using Equations (6) and (7) below.

$$\beta(k) = \beta(k-1) + K(\rho_{cr} - \rho_w(k)) \tag{6}$$

$$v_{vsl}(k) = \beta(k)v_{vsl}(k-1) \tag{7}$$

The factor $\beta(k)$ is calculated using the factor $\beta(k - 1)$ from the last time step and the error between the weighted density $\rho_w(k)$ and the critical density ρ_{cr} . The parameter *K* is the gain applied to the density error. By applying the factor $\beta(k)$ to the speed limit $v_{vsl}(k - 1)$ from the last time step, the current speed limit is obtained.

The setup of this feedback VSL control system consists of a VSL control zone, acceleration zone and work zone. VSLs are posted at the VSL control zone to regulate traffic speed while vehicles passing the VSL control zone into the acceleration zone can accelerate to free flow speed before entering the work zone. Readers can refer to [13] for more details of this VSL control zone and accelerate zone setup.

3.2.2 Speed Limit Constraints

Constraints are considered when VSLs are applied in practice. The following constraints are included in the VSL control system:

- Speed limits are within the range of the minimum v_{min} and the maximum v_{max} Speed limit.
- Speed limits are discrete values rounded to the close integer speed limits at the incremental speed Δv .
- Speed limits between two consecutive time steps are within the maximum incremental speed. Δv_{max} .
- Speed limits control time interval is Δt , so frequent speed limit changes can be avoided.

4 Experiment and Results

4.1 Experimental Setup

The proposed system was evaluated using a calibrated

freeway work zone segment on SR99 northbound in California. The freeway segment has a maximum speed limit of 105 km/h (65 mi/h). A work zone with a closure of one of the three lanes was observed starting from State Postmile (PM) 19.7 on May 3, 2018, as shown in green in Figure 5. Traffic congestion occurred in the presence of the work zone. A 2.2 km freeway segment upstream of the work zone and a 650 m two-lane work zone was built in the simulation environment under the traffic simulator SUMO. The traffic network was calibrated using travel speed and flow from loop detectors in California Freeway Database at work zone areas under the static speed limit [17].

Two loop detectors were added in the simulated freeway segment to study the impacts of stationary sensors and CVs. One set is located 500 m upstream of the work zone [18] and acts as the stationary sensor at a distant upstream of the work zone. The other set was added 50 m upstream of the work zone and acted as the stationary sensor at the merging area. Both loop detectors can detect traffic density at respective locations every 15 s. The market penetration rate of CVs was set as 10% to transmit speeds and locations as probe sensors. The length of the VSL control zone and acceleration zone were set as 850 m and 550 m, respectively. The lengths of the VSL control zone and acceleration zone could range from 500 m to 1 km, as studied in [14]. Further studies on more selections of sensor locations near work zones could be assessed in future work.



Figure 5. Freeway Work Zone on SR99 Northbound in California

The fundamental diagram was calibrated with free flow speed v_f , critical density ρ_{cr} , the threshold speed v_{tr} , as 105 km/h (65 mi/h), 50 veh/km, and 65 km/h. The gain K was selected as 0.01 via extensive tests. The maximum speed limit v_{max} , minimum speed limit v_{min} , incremental speed Δv , and the maximum incremental speed Δv_{max} and control time interval Δt were selected as 105 km/h (65 mi/h), 24 km/h (15 m/h), 8 km/h (5 mi/h), 8 km/h (5 mi/h) and 60 s.

4.2 **Results Analysis**

To demonstrate the effectiveness of the proposed

VSL control system, four scenarios are considered in the simulation: 1) no VSL control, in which the traffic conditions were collected as the baseline; 2) VSL control with only the stationary sensor at distant upstream of the work zone; 3) VSL control with only stationary sensors at the merging area of the work zone and 4) VSL control with stationary sensors at distant upstream and merging area of the work zone and CVs. The calibrated freeway work zone segment was first built using the traffic simulator SUMO. An interface TraCI was used to communicate with SUMO to retrieve simulated vehicle and network data and update speed limits [19]. Traffic demand for the simulation was generated as follows. The simulation was run for 600 s as the warm-up period. Simulation data from this warm-up period was discarded. Then, the simulation was run for 5400 s. The low traffic demand of 3000 veh/h was generated for the first 1200 s, and gradually increased to 3700 veh/h at 1800 s and maintained at this high demand for 1500 s. Following the high demand, traffic demand gradually reduced to 2500 veh/h at 3900 s and dropped to 2000 veh/h until the end of the simulation.

The traffic density within the acceleration zone upstream of the work zone under the four scenarios can be found in Figure 6.

The black dash line refers to the traffic density collected using the traffic simulator, which is used as the density reference. The detected density refers to the density outputs from the stationary sensors while the weighted density is derived from outputs of stationary sensors using Equation (1).





Figure 6. Traffic Density Comparison under (a) no control (b) VSL control with stationary sensors at distant upstream (c) VSL control with stationary sensors at merging area (d) VSL control with both stationary sensors and CVs

Figure 6 shows that the density under no control increases to around 150 veh/km when traffic demand is high and reduces to around 20 veh/km when the low traffic demand appears at the end of the simulation.

Despite VSL control being in effect under scenarios (b) and (c), traffic density has reached above the critical density and caused some control failure when traffic demand is high. This control failure is mainly due to impacts of sensor locations. Under scenario (b), because stationary sensors are located at distant upstream of the work zone, sensors cannot detect congestion in a timely manner before 1800 s when a queue has already formed at the work zone at around 1500 s. This congestion detection delay causes a delay for VSL control, and consequently, the queue is not effectively resolved. Under scenario (c), although traffic congestion is detected once the queue forms at the work zone area, it overestimates traffic density between 1500 s and 2200 s, which shows that the detected density is significantly higher than the density reference. This overestimation restricts traffic heavily and causes more traffic to enter the work zone, and congestion cannot be effectively resolved. Under scenario (d), the weighted density can provide relatively more accurate density estimation for VSL control. Accordingly, VSL control is more effective under scenario (d) with density remains around the critical density.

The density estimation error analysis of scenarios (b), (c), and (d) can also be found in Table 1. RMSE of Traffic Density. The results show that scenario (d) has the lowest RMSE (root mean square error) of density.

Table 1. RMSE of Traffic Density

Scenario	Density (veh/km)		
(b)	23.1		
(c)	15.0		
(d)	8.2		

The speed profile upstream of the work zone, the work zone, and downstream of the work zone under all four scenarios can be found in Figure 7. The time-space diagram of Figure 7 is color-coded by travel speeds. The speed color-coded bars on the right of the sub-figures use different colors representing different travel speeds from 0 (dark red) to 110 km/h (dark blue). Different vehicle travel speeds are shown on the time-space diagram of Figure 7. Traffic congestion occurs under no control scenario and the queue propagates towards upstream first. Then, when the traffic demand reduces, the queue propagates downstream until it is fully resolved. Under scenario (b), the queue forms only at the beginning of the congestion around 1800 s due to the delay of traffic density, while under scenario (c), the queue forms when higher traffic demand reaches the work zone due to traffic density overestimation.





Figure 7. Speed Profile under (a) no control (b) VSL control with stationary sensors at distant upstream (c) VSL control with stationary sensors at merging area (d) VSL control with both stationary sensors and CVs

Scenario (d) shows limited congestion occurs in the work zone area, and vehicles can remain at high travel speed once passing the control zone.

The performance of the VSL control system in terms of travel time, NOx, CO2 emissions, fuel consumption and time-to-collision risks is also assessed for the four scenarios as shown in Table 2. The travel time, NOx, CO2 emissions, and fuel consumption were obtained using the built-in libraries with default models [19]. The time-tocollision risks were calculated using the minimum 1.5 s as the safe distance [13].

Table 2. VSL Control Performance

Scenario	Travel Time (min)	NOx (kg)	CO2 (t)	Fuel (l)	Time-to- Collision (%)
(a)	3.0	1.0	2.6	882.8	11.5
(b)	2.7	0.9	2.4	775.8	2.5
(c)	3.2	1.0	2.6	821.1	2.5
(d)	2.6	0.9	2.4	767.6	0.7

Scenario (d) shows the consistent improvement in travel time, NOx, CO2 emissions, fuel consumption and time-to-collision risks. However, scenario (c) causes longer travel time compared to the no-control scenario, while the lower fuel consumption and time-to-collision risks than no control scenario are shown. Scenario (b) also demonstrates consistent improvements, which are not as significant as the scenario (d).

5 Conclusions

A VSL control system for a smart work zone with CVs is proposed in this study to address the discrepancies in traffic detections from different stationary sensors. Traffic densities from stationary sensors at the distant upstream and merging area of the work zone are weighted by applying kinematic wave theory and using speed and location transmitted from CVs. The weighted density approximates traffic density at work zone areas with lower estimation error. While the discrepancies caused by the locations of stationary sensors negatively impact the VSL control performance, the weighted density ensures the effectiveness of VSL control. The results show the proposed VSL control system can consistently improve traffic mobility, safety, and sustainability near work zone areas.

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