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A LINEAR OPTIMIZATION APPROACH TO INVERSE KINEMATICS OF REDUNDANT ROBOTS WITH RESPECT TO MANIPULABILITY

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ABSTRACT

The solution of the inverse kinematics is required in many technical applications. In this contribution a concept is proposed which reformulates the inverse kinematics (IK) of kinematically redundant manipulators as a linear programming (LP) problem. This formulation enables the explicit consideration of technical constraints as for example mechanical end-stops, velocity and, if necessary, acceleration limits as linear inequality constraints. Besides that, automatic collision avoidance within the workspace of the manipulator can be included. The kinematic redundancy is resolved with respect to quadratic criteria. As the LP problem at hand belongs to the small-size problems, the optimal solution can be found numerically in appropriate time using standard algorithms such as the simplex algorithm or interior point methods. This article closes with a numerical example of the LP-IK of a planar 4-link manipulator.

KEYWORDS

redundancy, inverse kinematics, linear programming, manipulability, manipulator

1. INTRODUCTION

Many technical applications require the solution of the inverse kinematics (IK) problem of kinematically redundant manipulators, i.e. manipulators whose number of mechanical degrees of freedom (DOF) is greater than the DOF of the tool center point (TCP). Examples are construction machines, where the operator wants to move a tool attached to the end-effector to the area of operation. However, to this day the IK problem of choosing appropriate actuator configurations, for example of hydraulic cylinders, is often solved manually by experienced operators. Therein, the operator has to define a map from Cartesian end-effector coordinates to joint coordinates of the manipulator at any instant of time.

For an operator it is more intuitive and hence easier to command the end-effector than to command the joints of the manipulator individually. This benefit becomes more and more evident with an increasing number of joints, i.e. with increasing kinematical redundancy. On the other hand, redundancy is often required in order to account for obstacles in the workspace and therefore to avoid collisions.

To overcome this difficulty algorithmic solutions to the IK problem are desired. The concept proposed in this contribution formulates inverse kinematics as a linear programming (LP) problem. This formulation offers the explicit consideration of technical bounds in form of linear inequality constraints. Examples for technical constraints are mechanical end-stops as well as velocity and acceleration limits which are taken into account during the process of optimization. The main task of moving the end-effector from its actual to a desired position (and orientation) is realised by the concept of "affine manipulability", originally introduced by Schlemmer in [1]. The kinematical redundancy is resolved by minimizing a quadratic criterion using the Lagrange Function such that the solution delivers a feasible manipulator configuration. Moreover automatic collision avoidance can be included by applying potential fields according to [2].

2. RELATED AND PREVIOUS WORKS

This section shortly discusses common methods for solving the IK which constitute the basis for the concept explained in section 3.

In general, setting up the analytical forward kinematics of a serial-link manipulator

$$\mathbf{w} = \varphi(\mathbf{q}), \tag{1}$$

where $\varphi : \Re^n \to \Re^m$, $\mathbf{q} \in \Re^n$ is the vector of manipulator joint coordinates and $\mathbf{w} \in \Re^m$ is the vector of position and orientation of the end-effector is a trivial task.

2.1. Classical approach

As φ is generally a nonlinear vector function the IK is usually solved by approximating (1) using the Taylor expansion of φ and disregarding the terms of order higher than 1, such that

$$\mathbf{w} - \mathbf{w}_0 \approx \mathbf{J}(\mathbf{q} - \mathbf{q}_0) \tag{2}$$

with $\mathbf{w}_0 = \varphi(\mathbf{q}_0)$. Here $\mathbf{J} \in \Re^{m \times n}$ denotes the Jacobian matrix which has a rectangular shape for redundant robots and is therefore not invertible.

To overcome this redundancy, a quadratic cost function

$$Z(\mathbf{q}) = \tfrac{1}{2}(\mathbf{q} - \mathbf{q}_0)^T \mathbf{M}(\mathbf{q} - \mathbf{q}_0) - \mathbf{f}^T \mathbf{q}, \tag{3}$$

is introduced [3]]

where $\mathbf{M} \in \Re^{n \times n}$ is a positive definite weight matrix and $\mathbf{f} \in \Re^{n \times 1}$ is a weight vector. Together with the kinematical constraint according to (2)

$$N(\mathbf{q}) = \mathbf{J}(\mathbf{q} - \mathbf{q}_0) - (\mathbf{w} - \mathbf{w}_0) \tag{4}$$

the Lagrange function

$$L(\mathbf{q}, \boldsymbol{\lambda}) = Z(\mathbf{q}) + \boldsymbol{\lambda}^T N(\mathbf{q}) \tag{5}$$

can be set up. The necessary condition for a local minimum is

$$\frac{\partial L}{\partial \mathbf{q}^T} = \mathbf{0} \text{ and } \frac{\partial L}{\partial \boldsymbol{\lambda}^T} = \mathbf{0}. \tag{6}$$

This leads to the linear system of equations

$$\begin{bmatrix} \mathbf{M} & \mathbf{J}^T \\ \mathbf{J} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{M}\mathbf{q}_0 + \mathbf{f}^T \\ \mathbf{J}\mathbf{q}_0 + (\mathbf{w} - \mathbf{w}_0) \end{bmatrix}, \tag{7}$$

which can then be solved efficiently as the matrix on the left hand side is usually sparse.

One choice of **M** could be the mass matrix but usually is chosen as a diagonal weighting matrix. According to Komainda [3] the vector **f** can be regarded as an artificial generalized force which can be used for influencing the null space motion. If **M** is the identity matrix, **f** the zero vector and (7) is resolved for **q**, the IK equation

$$\mathbf{q} - \mathbf{q}_0 = \mathbf{J}^+ (\mathbf{w} - \mathbf{w}_0) \tag{8}$$

is obtained where

$$\mathbf{J}^+ = \mathbf{J}^T (\mathbf{J}\mathbf{J}^T)^{-1} \tag{9}$$

is the well-known Moore-Penrose pseudo-inverse matrix.

2.2. Nonlinear programming approach

However the method shown above does not guarantee the provision for technical constraints. In fact, the motion of a joint near its limits strongly depends on the potential function **f**, if an artificial force is engaged. For this reason in **Error! Reference source not found.** and [4] IK is formulated as a nonlinear programming problem (NLP). Besides taking the technological constraints into account it is possible to apply the nonlinear forward kinematics according to (1) as side conditions. Hence, the IK can be solved numerically exact as the forward kinematics is not linearized according to (2).

In the following Schlemmer's [1] concept of "affine manipulability" is discussed because it forms the basis of the approach presented here.

Firstly, a quadratic criterion as cost functions to be minimized subject to the nonlinear forward kinematics is presented. While the quadratic criteria resolve the redundancy, the affine manipulability criterion (Fig. 1) moves the end-effector from $\mathbf{w}_0$ to a similar, i.e. affine feasible position and orientation $\mathbf{w}_{feasible}$. Thereby, $\mathbf{w}$ may contain position and/or orientation of the end-effector.

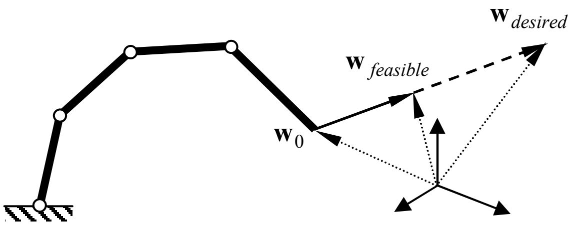


Figure 1. Affine Manipulability according to [1]

The relationship between $\mathbf{w}_{feasible}$ and $\mathbf{w}_{desired}$ can then be formulated as

$$\mathbf{w}_{feasible} - \mathbf{w}_0 = p(\mathbf{w}_{desired} - \mathbf{w}_0)\,, \tag{10}$$

where p is the affine manipulability indicator or dexterity. Thereby, it is reasonable to restrict p such that $0 \le p \le 1$, in order to prevent the end-effector from moving in the opposite or beyond the desired direction. It is obvious that the optimum, i.e. maximum is reached if $p = 1$, so that $\mathbf{w}_{feasible} = \mathbf{w}_{desired}$.

The nonlinear problem can then be solved by a Sequential Quadratic Programming (SQP) algorithm [1]. However, solving NLP problems is computationally expensive.

2.3. Linear programming approach

It is worth noting that Schlemmer already proposes the following *LP1* problem with respect to affine manipulability:

$$LP1 = \max_{\mathbf{q},p}\{p\} \tag{11}$$

subject to

$$\mathbf{J}(\mathbf{q} - \mathbf{q}_0) - p(\mathbf{w} - \mathbf{w}_0) = \mathbf{0} \tag{12}$$

$$0 \le p \le 1\,,\ \mathbf{q}_{\min} \le \mathbf{q} \le \mathbf{q}_{\max} \tag{13}$$

and further bounds with respect to velocity such as

$$\dot{\mathbf{q}}_{\min}\Delta T \le \mathbf{q} - \mathbf{q}_0 \le \dot{\mathbf{q}}_{\max}\Delta T\,, \tag{14}$$

where ΔT denotes the discrete time interval.

This proposal delivers an effective alternative for the IK at velocity level as the LP problem can be solved effectively with the well-known simplex algorithm or interior point methods. However, the motions that are calculated suffer from jittering as can be seen in Fig. 3. Another approach of solving the IK with LP can be found in Ho [5]. Here the main idea is to define the cost function as the sum of absolute values of the elements in $\Delta\mathbf{q} = \mathbf{q} - \mathbf{q}_0$, i.e. $\min\|\Delta\mathbf{q}\|_1$ which then has to be minimized subject to (2).

3. LP APPROACH

Motivated by Schlemmer's affine manipulability, here an LP formulation which considers the minimization of the quadratic cost function stated in (3) in order to overcome the jittering of *LP1* is proposed. For this purpose the constraint in (4) is modified according to (12) which leads to

$$N(\mathbf{q}) = \mathbf{J}(\mathbf{q} - \mathbf{q}_0) - p(\mathbf{w} - \mathbf{w}_0) = \mathbf{0}\,, \tag{15}$$

where p is the affine dexterity and is regarded as a parameter, not a variable. In the next step the Lagrange function is set up, differentiated and set to zero as described in section 2.1. Hence, the linear system of equations

$$\begin{bmatrix} \mathbf{M} & \mathbf{J}^T \\ \mathbf{J} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{M}\mathbf{q}_0 + \mathbf{f}^T \\ \mathbf{J}\mathbf{q}_0 + p(\mathbf{w} - \mathbf{w}_0) \end{bmatrix} \tag{16}$$

is obtained. From now on p is considered to be a variable that has to be maximized and by rearranging (16) we finally the following *LP2* problem

$$LP2 = \max_{\mathbf{q},\boldsymbol{\lambda},p} \{p\} \tag{17}$$

subject to

$$\begin{bmatrix} \mathbf{M} & \mathbf{J}^T & \mathbf{0} \\ \mathbf{J} & \mathbf{0} & -(\mathbf{w} - \mathbf{w}_0) \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \boldsymbol{\lambda} \\ p \end{bmatrix} = \begin{bmatrix} \mathbf{M}\mathbf{q}_0 + \mathbf{f}^T \\ \mathbf{J}\mathbf{q}_0 \end{bmatrix}, \tag{18}$$

$0 \le p \le 1$, $-\infty \le \boldsymbol{\lambda} \le +\infty$, $\mathbf{q}_{\min} \le \mathbf{q} \le \mathbf{q}_{\max}$

and further constraints at velocity level is obtained.

3.1. Controlling smoothness of motion

One can additionally influence the manipulator's null space motion by adding the quadratic function

$$Z_2(\mathbf{q}) = \tfrac{1}{2}(\mathbf{q} - \mathbf{q}_{ref})^T \boldsymbol{\Lambda}(\mathbf{q} - \mathbf{q}_{ref})$$

to the cost criterion such that the following side conditions hold

$$\begin{bmatrix} \mathbf{M} + \boldsymbol{\Lambda} & \mathbf{J}^T & \mathbf{0} \\ \mathbf{J} & \mathbf{0} & \mathbf{w}_0 - \mathbf{w} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \boldsymbol{\lambda} \\ p \end{bmatrix} = \begin{bmatrix} \mathbf{M}\mathbf{q}_0 + \boldsymbol{\Lambda}\mathbf{q}_{ref} + \mathbf{f}^T \\ \mathbf{J}\mathbf{q}_0 \end{bmatrix}.$$

Here $\boldsymbol{\Lambda} \in \Re^{n \times n}$ is a diagonal weighting matrix with positive diagonal elements and $\mathbf{q}_{ref}$ is a reference point in joint space, which may be defined as the midpoint between the joint limits [1]. This definition leads to joint configurations, which lie far apart from the joint limits and consequently increase the manipulability.

3.2. Singularity

Near singularities the joint velocities tend to have high values in spite of relatively small end-effector velocities. Even though the joint velocities are limited, it still causes jittering (Fig. 8). Komainda [3] proposes a modified side constraint where (4) is extended by $k\mathbf{I}\boldsymbol{\lambda}$, such that (7) changes to

$$\begin{bmatrix} \mathbf{M} & \mathbf{J}^T \\ \mathbf{J} & k\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{M}\mathbf{q}_0 + \mathbf{f}^T \\ \mathbf{J}\mathbf{q}_0 + (\mathbf{w} - \mathbf{w}_0) \end{bmatrix}. \tag{19}$$

Due to this extension the matrix on the left hand side of (19) cannot become singular for $k \neq 0$. For this reason [3] refers to this as the robust inverse where k is the manipulability measure due to Yoshikawa [6]. However the effort for calculating k is high.

It is fact that the Lagrange multiplier $\boldsymbol{\lambda}$ tends to have high values near singular configurations. For this reason limiting $\boldsymbol{\lambda}$ by

$\boldsymbol{\lambda}_{\min} \le \boldsymbol{\lambda} \le \boldsymbol{\lambda}_{\max}$

may also prevent the jittering effect as can be seen in Fig. 9. The choices for $\boldsymbol{\lambda}_{\min}$ and $\boldsymbol{\lambda}_{\max}$ are subject to further research.

4. SIMULATION RESULTS

The following section evaluates the proposed LP-IK applied to a redundant 4-link planar manipulator (Fig. 2).

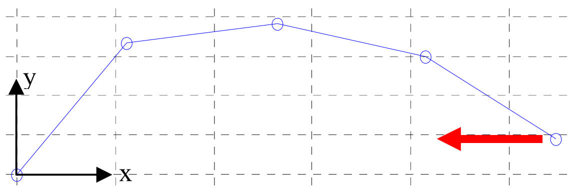


Figure 2. Redundant 4-link planar manipulator

From the initial configuration as shown in Fig. 2 the main task is to pull in the end-effector horizontally. Furthermore, to demonstrate the engagement of joint limits, the first joint is limited to $q_{\max,1} = 95°$.

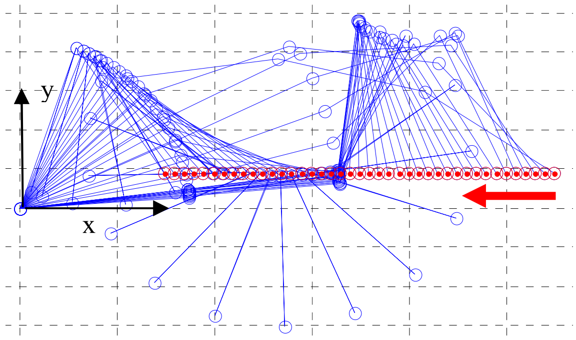
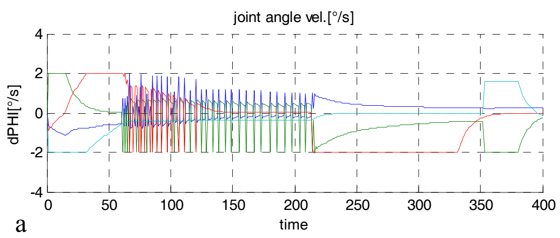


Figure 3. x-y-motion of the manipulator according to *LP1*

It can be seen that the main task, i.e. pulling in the end-effector, can be realized. However, the trajectories in joint space are jittering (Fig. 3 and 4a).



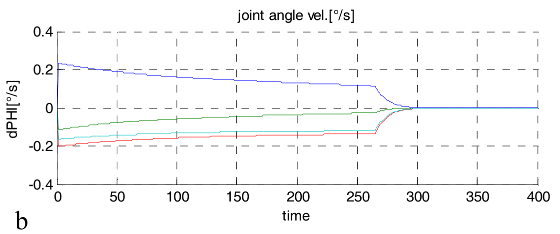


Figure 4. a) Joint space motion according to *LP1*, b) Joint space motion due to *LP2*

In Fig. 4b one can see, that using *LP2* due to the quadratic cost function, the jittering vanishes and technically reasonable motions are generated besides fulfilling the main task (Fig. 5). Moreover, the constraint of the first joint is maintained such that the end-effector cannot move in any further.

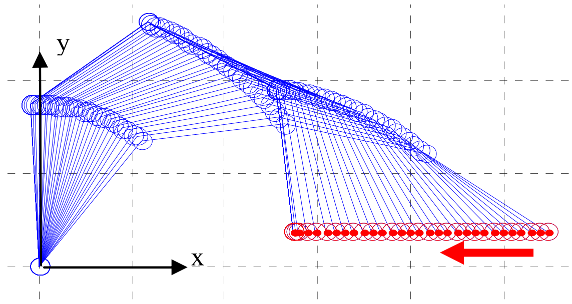


Figure 5. x-y-motion of the manipulator according to the proposed approach *LP2*

Accordingly the affine manipulability indicator p shows, that it is not possible anymore to maintain the desired end-effector motion as p nearly reaches zero (Fig. 6).

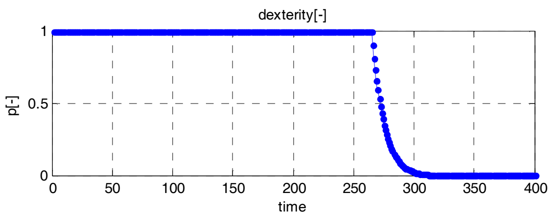


Figure 6. Affine dexterity according to the proposed approach

In the following the TCP is supposed to be moved from the initial configuration (Fig. 2) to an edge singularity (Fig. 7).

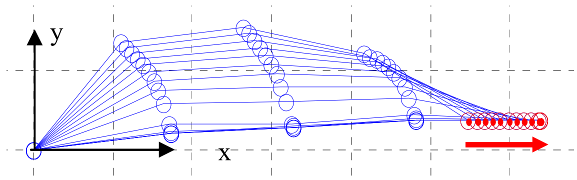


Figure 7. Displacement to an edge singularity

In this simulation one can see the influence of the Lagrange multipliers. Near edge singularities the Lagrange multipliers tend to have high values which cause a jitter effect (Fig. 8 at t=100).

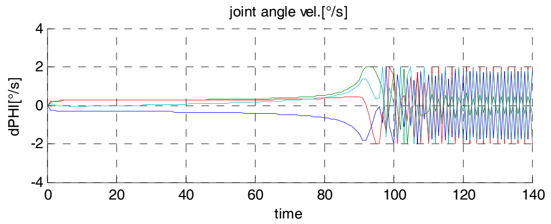


Figure 8. joint motion near singularity

By limiting the multipliers it is possible to prevent the jitter effect as can be seen in Fig. 9. Furthermore, the affine dexterity tends to zero.

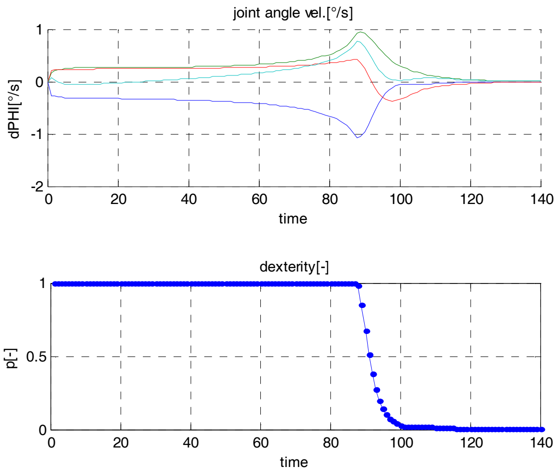


Figure 9. Dexterity and joint motion near singularity due to limited Lagrange multipliers

5. CONCLUSION

In this paper a concept for solving the inverse kinematics of redundant manipulators by formulating a linear programming problem is presented. It was shown that the solution of this LP problem leads to technically feasible results as quadratic criteria are taken into account. The affine dexterity is a possible concept to characterize the current manipulability of the manipulator. It can provide a beneficial tool in terms of human-machine interfaces (HMI). By limiting the Lagrange multipliers the numerical instability in the proximity of singular configurations can be overcome. This leads to smooth motions near the singularities. But finding adequate values for λ_{min} and λ_{max} is still part of current research. Finding the solution of such LP problems can be done in appropriate time, as for example by the simplex algorithm, because the problems at hand belong to the small size problems.

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