

Integration of NDE Measurements and Current Practice In Bridge Deterioration Modeling

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Abstract –

Deterioration models are required and used in Bridge Management System (BMS) to predict the condition and performance of bridges. Effective maintenance of bridge structure relies on the accuracy of deterioration models used to predict bridge performance. Markov Model is a deterioration forecasting model that is widely used in BMS. However, research showed that Markov Model has many shortcomings. This research provides a review of bridge deterioration modeling with emphasis on accuracy improvement of the generated transition probability matrix used in the model. Dynamic Bayesian Network (DBN) technique is utilized to predict the future conditions of bridge decks. Variables such as, factors affecting deterioration process and inspection measurements from Non-Destructive Evaluation (NDE) methods are incorporated to increase the accuracy of the developed deterioration model. The impact of these factors is extracted from the literature and the DBN model is built using a numerical example. Measurements of NDE for years 2008 and 2013 for a case of a bridge deck are used to apply the model. The developed method is expected to improve current practice in forecasting bridge deck deterioration and in estimating the frequency of inspection.

Keywords –

NDE; Deterioration Modeling; Bridges; DBNs

1 Introduction

Deterioration models are used in Bridge Management System (BMS) to predict the future conditions and performances of bridges. Large number of historical data is required for deterioration modeling. The deterioration models are influenced by: 1-Bridge

age, 2-Bridge type, 3-Bridge environment, 4-Material properties, 5-Bridge design, 6-Bridge loading and 7-Bridge Capacity. Bridge deterioration rate is a decrease in condition rating per year. Bridge age and daily traffic load are the most critical factors that cause bridge deterioration. Bridge service life can be determined by defining the correlation between bridge age and condition rating. Therefore, the effective maintenance of bridge structure relies on the quality, accuracy of deterioration models that are used to predict bridge performance and service life [1; 2; 3].

Currently, there are two major types of deterioration models:

(i) Deterministic Models:

Deterministic models describe relationships between factors affecting bridge deterioration. However, it ignores random errors in prediction. Some of the limitations of deterministic deterioration models are as follows:

- 1- Deterministic models neglect uncertainty
- 2- They predict the average condition of group of (bridges) without focusing on individual bridge. These models provide less focus on current condition and the history of the bridge.
- 3- It is always difficult to estimate the impact of maintenance actions on deterioration when deterministic deterioration models are used.
- 4- These models neglect the interaction between bridge components.

(ii) Stochastic Models:

Stochastic models deal with deterioration process as random variables that incorporate uncertainty. Markov models are the most widely used deterioration models used to predict the condition of infrastructure facilities. It covers two limitations of deterministic models as it incorporates uncertainty and account for the current facility condition. Markov Chain Model forecasts bridge condition rating based on the concept of defining states of bridge condition from one to another during

transition period. Markov approach is a discrete time stochastic process that takes number of possible discrete states. Markov model has the following limitations:

- 1- Markov models assume discrete transition time intervals.
- 2- Future condition of a facility depends only on current facility condition and not on a history of the bridge, which is unrealistic.
- 3- Markov models assume that the condition of a bridge can stay the same or reduced to avoid the complexity to consider the treatment process and its impact.
- 4- Markov models cannot determine the interaction between different components of bridges.
- 5- In these models, transition probabilities require update when new information is available.

It has been suggested that integrating NDE methods into Markov model will reduce its limitation [4]. Also, the accuracy of Transition Matrix increases the accuracy of Markov-deterioration model [5, 6].

Another type of stochastic models available are Bayesian Networks (BNs). These models consist of a graph that includes nodes and arcs. The arcs connecting two nodes represent the dependences relationships between random variables nodes. BNs has many application in medicine diagnostics and in engineering predictions [7, 8, 9]. Few researchers have applied BNs in deterioration modeling. By using BN, dependencies among variables is easy to interpret. Variables are considered independent if there is no edge connecting those variables.

According to Weber et al. [10], BN has the capability of modeling complex system. It makes prediction and diagnostics. It computes the probability of event occurrence. It updates beliefs based on new evidence. It integrates qualitative information and the quantitative ones. BN merges experience, past knowledge, impacting factors and measurements. So far, according to the literature review, BN has limited applications in maintenance and in bridge deterioration modeling. Dynamics Bayesian Network (DBN) is a class of BNs which represent stochastic process. DBN consists of sequence of slices; each consists of BN nodes. These slices are connected by direct arc from slice at time T1 to slice at time T2. DBN provides computational framework that allows accurate and efficient prediction of deterioration based on observations and deterioration parameters [11, 9]. These DBNs are expected to alleviate the main limitations of current Markov model.

In this research, the main objective is to develop a model for bridge deterioration using multiple NDE methods. The integration of NDE methods and current

practice is used to improve the accuracy of forecasting bridge condition.

2 DBN For Bridge Deck Deterioration

Dynamic Bayesian networks are a special class of BNs to analyze problems of bridge deterioration with time variation. It consists of a sequence of time slices (T1, T+1,, T+n). In each slice, there are one or more BN nodes. Time slices are connected with direct link, these links present probabilistic dependences.

In DBNs, bridge deterioration can be predicted from past experience. The knowledge from experts are used to build conditional probability table (CPT) directly. According to Wang et al. [13], this task is performed through 5 steps; expert selection, expert training, questions preparation, expert judgment, and results verification.

CPT can be determined directly from visual inspection or from NDE methods. In DBN discrete units of time is modeled, where each unit defines a time slice. These time slices are connected through links. The probabilities associated with links connecting these slices are defined as transition probabilities. In DBN, the basic network is repeated over time [7, 9, 14, 13]. DBNs utilize the Markov model process and allow taking into consideration the prior probability distribution of random variables considered in the deterioration process.

According to Rafiq et al. [14], deterioration of bridge element leads to reducing level of service and bridge safety level. In current practice, deterioration models are presented by Markov stochastic process. For simplicity in existing BMS, discrete time stochastic process is employed to model bridge deterioration at T+1 by prior knowledge about deterioration at T1. Rafiq et al. [14], applied DBN model for modeling the deterioration of masonry arch bridge. The authors utilized DBN to address the interdependency between main element and sub element.

Wang et al. [13] focused and studied BN. Wang et al. [13], used dynamic Bayesian Networks for prediction of structural reliability of steel bridge elements. The authors developed an approach that is able to update information from the observed measurements, and then corrosion process is modeled. Straub [9] Proposed DBN to model that updates variables based on information from inspection. Faddoul et al. [11] Presented DBN for maintenance action of roads taking into account updated information to improve the existing inspection, maintenance and rehabilitation actions.

3. Model Development of Bridge Deck Deterioration with DBNs and NDE Methods

The bridge deterioration model utilizes the inspection measurements acquired during bridge inspection. It includes the measurements from multiple NDE methods that usually used in the advanced inspection. These measurements are combined and their outputs are used to determine bridge deck condition rating. NDE measurements are used to detect bridge deck defected area. Bridge deck condition rating is assigned based on the percentage of the defected area. According to Minnesota department of transportation, 2013, there are five condition states used to assign bridge deck condition rating. These five condition states are defined as follows:

- Condition state 1: deck shows little or no deterioration
- Condition State 2: combined deterioration of deck areas are less than 2%.
- Condition State 3: combined deterioration of deck areas are between 2% and 10%.
- Condition State 4: combined deterioration of deck areas are between 10% and 25%.
- Condition state 5: combined deterioration of deck area are more than 25%.

Many attributes are impacting bridge deck deterioration. These factors are; bridge age, bridge design, environmental factors and excessive loading. The impact of these factors are stochastic. This impact is incorporated in the developed model.

The basic Bayesian network is modeled as illustrated in Figure 1. F_1, F_2, \dots, F_n are the deterioration factors impacting the bridge deck. Factors nodes are parents' variables contributing their impact to the condition states of the bridge deck. The bridge deck condition contribute the information to impact and cause the inspection measurements using NDE methods. Accordingly, the multiple measurements from NDE methods that are collected after the bridge deck inspection are child nodes of the bridge deck condition. These NDE measurements are considered parents nodes contributing and causing the information to their child node which is the bridge deck condition rating. Bridge deck condition is assigned based on the combined defected area. The qualitative part of the basic Bayesian network is illustrated as a graph as shown in Figure 1. The quantitative part is defined using conditional probability table (CPT) between each parent node and its child node. CPT measures the strength of the relationship between them. In this research, the CPT is defined by varying the impact and occurrence of factors on bridge deck condition. CPT between bridge deck condition rating and NDE measurements is defined by

Varying the measured areas through five states, each state has specific range of the % of the measured defected area.

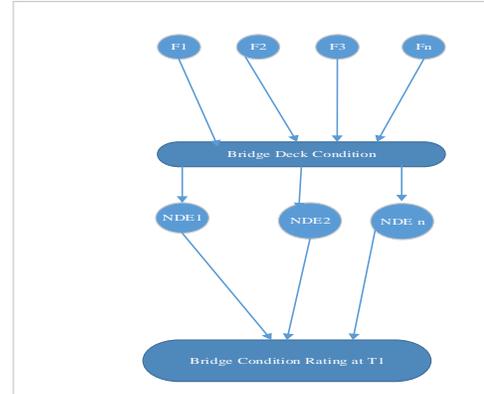


Figure 1. Bayesian Network for bridge deck deterioration modeling

The bridge deck condition state and the bridge deck condition rating are variables changing over the time. So, the basic Bayesian network is modeled as dynamic Bayesian network (DBN) through time slices. Each time slice includes the basic Bayesian network at specific time. Direct arrow is linking nodes of bridge deck condition at different times. The probabilities associated with links connecting the nodes of bridge deck condition at different time slices are defined as transition probabilities. As illustrated in Figure 2. The basic Bayesian network is repeated over the time. The bridge deck condition and condition rating are variables changing over the time T_1, T_2, \dots, T_n . Temporary arc is used to link the change of bridge condition rating over the time to build the transition probabilities of the bridge condition.

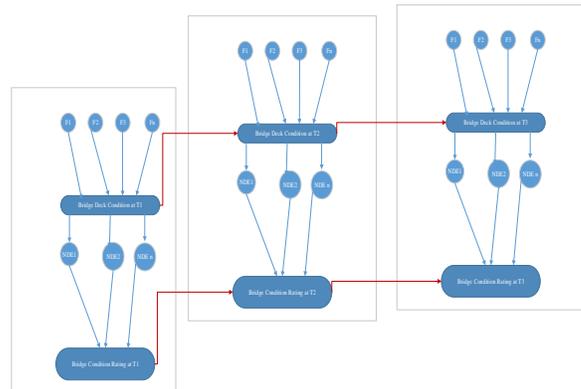


Figure 2. DBNs Model for bridge deterioration

The arc linking bridge deck condition at different times ensures that current bridge condition T_2 depends

on previous history of bride deck condition at T1. Modeling bridge deck deterioration this way, incorporates the maintenance action in previous time units. Incorporating the stochastic impact of deterioration factors at each time unit, helps to accurately forecast bridge deck condition. In the developed model, the experience, past knowledge, measurements from different sources of NDE and deterioration factors are combined. The model can be updated with new information from NDE measurements. It will be updated if more NDE methods are incorporated and their results are fused. More extra factors can be incorporated as well.

4. Numerical Example As Application of Dynamic Bayesian Networks Model

Factors impacting bridge deterioration are incorporated in Dynamic Bayesian network model. The impact of these factors are extracted from the literature review. Huang [16] identified 8 attributes that extracted from the inventory data of decks record from Pontis BMS. The author analyzed the five factors that has great impact on transferring bridge deck condition from state 1 to state 2, A12. These factors are: District, Design Load, ADT (Vehicle/Day), Environment, and Degree of Skew. The author analyzed the five factors that have great impact on transferring bridge deck condition from state 2 to state 3, A23. These five factors are: Design Load, Deck Length (m), Deck Area (m2), Environment, and Number of Spans. Huang [16] listed the 11 factors that did impact the bridge deck deterioration as illustrated in Table 1. Table 1 illustrates the impact of factors on the transition of bridge deck condition from state 1 to state 2 (A12), from state 2 to state 3 (A23) and from state 3 to state 4 (A34).

Table 1. The impact of factors on the transition of bridge deck condition

States	P-value	ATTRIBUTES
A12	0.05	Degree of Skew
A34		Maintenance History
A34		Age
A34		Previous Condition
A12	0.0014	District
A12, A23		Design Load
A23	0.0158	Length
A23		Deck area
A12	0.0158	ADT
A12, A23		Environment
A23	0.0149	Number of Spans

Measurements for years 2008 and 2013 of GPR for a bridge deck are used in this study. This data were

extracted from condition mapping of bridge deck in years 2008 and 2013 [17]. The % of delamination for years 2008 and 2013 are calculated using generic model of Martino [18]. The amplitude values for years 2008 and 2013 of bridge deck are illustrated on Table 2. The amplitude values are extracted approximately from the deterioration mapping of previous research efforts [17] for years 2008 and 2013. Table 2 illustrates % of delaminated areas that are calculated using the generic model. % of delamination is calculated in the following steps because of lack of the raw data:

1- Dinh et al.[17] plotted two deterioration mapping for years 2008 and 2013. These deterioration mapping were built based on GPR signal attenuation. Amplitude values for the bridge deck are obtained from the deterioration maps.

2- Dinh et al.[17] plotted the results based on the rebar reflection amplitude and related the deterioration of bridge deck with this reflection.

3- The GPR amplitude ranges that appears as values of GPR in [17] are utilized to apply [18] generic linear model to get % of delamination in the bridge deck for each year.

4- Martino [18] indicated that the model can be used for bridge deck with moderate corrosion and with threshold -1.6dB by using equations (1), (2) and (3)

$$Y = 7.051725 * X + 1.78044 \tag{1}$$

$$Y = \% \text{ Delamination} \tag{2}$$

$$X = \text{Skew} * \text{Mean GPR Amplitude} \tag{3}$$

5- By Following this way, % delamination for each year (2008 and 2013 of the case study) are obtained.

Table 2. Amplitude values of Bridge Deck Years

2008	2013	2008	2013	
-9	-7	-18	-13	
-7	-7	-16	-14	
-10	-8	-7	-16	
-11	-9	-17	-8	
-16	-11			
-16	-11			
-17	-13			
-18	-18			
-10	-10			
-11	-14			
-7	-4			
-6	-5	-25.667	-22.222	Mean
-9	-9	-0.0608	-0.1694	SKEW
-12	-13	1.56151	3.7651	Mean *SKEW
-14	-10	12.7105	28.1357	%Delamination

Figure 3 illustrates the basic network that consists of factors impacting on the transition of bridge deck condition. NDE measurements are child node of A12, A23, A34 and A45. From NDE measurements at

different times, condition assessment nodes are determined with different time. Figure 3 is considered the qualitative part of the network as it shows the relationship between different nodes.

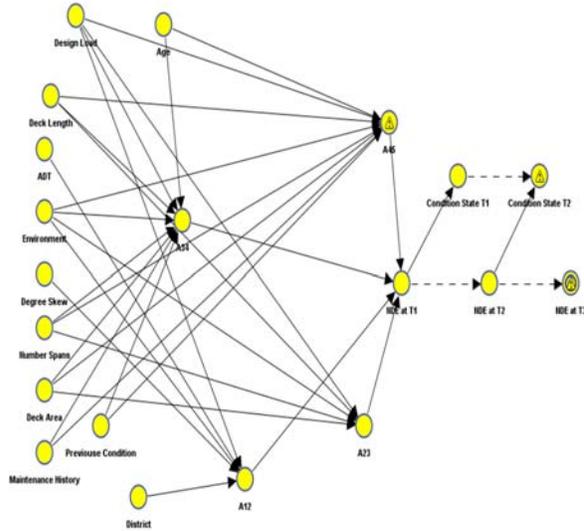


Figure 3. Dynamic Bayesian Network of Bridge Deck Deterioration Model

The relationship between nodes is quantified by defining the conditional probability table. Table 3 shows 32 conditional probabilities on node A12. Table 3 utilized to define the relationship between the factors and bridge state transition from state 1 to state 2 (A12). It measures the true and false percentage of A12 occurrence with high and the low impact of the factors. As illustrated in Table 3, the factors impacting A12 are District, Design Load, ADT, Environment and Degree of Skew. The 32 probabilities are generated based on varying the impact of the factors. For example, the first conditional probability is generated based on the low or false impact of factors District, Design Load, ADT, Environment and Degree of Skew. So, the first conditional probability is assigned true and false percentage values of 0% and 100% respectively. The second conditional probability is generated based on the false value of factors District, Design Load, ADT, Environment and the true value of the factor Degree of skew. So, the second conditional probability is assigned true and false percentage values of 50% and 50% respectively because Degree of skew factor is one of the significant factors with high impact comparing to the other factors. The other 30 conditional probabilities are assigned by the same way. Table 4 defines the conditional probability table of 32 conditional probabilities that measure the strength of the relationship between factors Deck Length, Deck area,

Number of Spans, Environment, Design Load and node A23.

Table 3. Conditional Probability table of transition from state 1 to state 2 (A12)

District	Design Load	ADT	Environment	Degree Skew	FALSE	TRUE		
FALSE	FALSE	FALSE	FALSE	FALSE	100	0		
				TRUE	50	50		
		TRUE	FALSE	FALSE	FALSE	95	5	
					TRUE	75	25	
		TRUE	TRUE	FALSE	FALSE	70	30	
					TRUE	20	80	
	TRUE	FALSE	FALSE	FALSE	FALSE	65	35	
					TRUE	15	85	
		TRUE	TRUE	FALSE	FALSE	FALSE	99	1
						TRUE	51	49
		TRUE	TRUE	TRUE	FALSE	FALSE	94	6
						TRUE	44	56
TRUE	FALSE	FALSE	FALSE	FALSE	49,286	50,714		
				TRUE	19	81		
TRUE	TRUE	FALSE	FALSE	FALSE	64	36		
				TRUE	14,286	85,714		
TRUE	FALSE	FALSE	FALSE	FALSE	86	14		
				TRUE	64	36		
		TRUE	FALSE	FALSE	FALSE	FALSE	19	81
						TRUE	31	69
		TRUE	TRUE	FALSE	FALSE	FALSE	44	56
						TRUE	80	20
	TRUE	FALSE	FALSE	FALSE	FALSE	51	49	
					TRUE	1	99	
		TRUE	TRUE	FALSE	FALSE	FALSE	85	15
						TRUE	35	65
		TRUE	TRUE	TRUE	FALSE	FALSE	80	20
						TRUE	30	70
TRUE	FALSE	FALSE	FALSE	FALSE	55	45		
				TRUE	5	95		
TRUE	TRUE	FALSE	FALSE	FALSE	50	50		
				TRUE	0	100		

Table 4 shows the relationship between factors and A34 through different probabilities and varying of incorporating the impact of the factors. It measures the true and false percentage of A34 occurrence with the high and the low impact of the factors. For example, the first conditional probability is generated based on the low or false impact of factors Deck Length, Deck area, Number of Spans, Environment and Design Load. So, the first conditional probability is assigned true and false percentage values of 0% and 100% respectively. The second conditional probability is generated based on the false value of factors District, Design Load, ADT, Environment and the true value of the factor Design load. So, the second conditional probability is assigned true and false percentage values of 28% and 72% respectively.

Table 4. Conditional Probability table of transition from state 2 to state 3 (A23):

Deck Length	Deck Area	Number Spans	Environment	Design Load	FALSE	TRUE		
FALSE	FALSE	FALSE	FALSE	FALSE	100	0		
				TRUE	72	28		
			TRUE	FALSE	86	14		
		TRUE		58	42			
		TRUE	FALSE	FALSE	77	23		
				TRUE	49	51		
	TRUE		FALSE	63	37			
	TRUE	FALSE	FALSE	FALSE	FALSE	90	10	
					TRUE	37	63	
				TRUE	FALSE	76	24	
			TRUE		48	52		
			TRUE	FALSE	FALSE	67	33	
					TRUE	39	61	
		TRUE		FALSE	53	47		
		TRUE	FALSE	FALSE	FALSE	FALSE	75	25
						TRUE	47	53
					TRUE	FALSE	61	39
				TRUE		33	67	
TRUE				FALSE	FALSE	52	48	
	TRUE				24	76		
	TRUE		FALSE	38	62			
TRUE	FALSE		FALSE	FALSE	FALSE	10	90	
					TRUE	65	35	
				TRUE	FALSE	37	63	
			TRUE		51	49		
			TRUE	FALSE	FALSE	23	77	
					TRUE	42	58	
	TRUE			FALSE	14	86		
	TRUE		FALSE	FALSE	28	72		
				TRUE	0	100		

Table 5 shows the conditional probabilities of GPR measurements at year 2008. Table 5 measures the strength of transition for condition states between A12, A23, A34 and A45 and the probability of existence of defected areas. For example, the first conditional probability is generated based on the true occurrence of A12, A23, A34 and A45. So, the first conditional probability is assigned 0 values if there is no area defected. The first conditional probability is assigned 0.25 if the defected area is less than 2%, less than 10%, more than 10% and more than 25 %. The second conditional probability is generated based on the true occurrence of A12, A23, A34 and the false occurrence of A45. So, the second conditional probability is assigned 0 value if there is no area defected or the area defected is more than 25%. It is assigned a value of 0.1 if the defected area is less than 2%. It is assigned a value of 0.45 when the defected area is less or more than 10%.

Table 5. Conditional Probabilities of node NDE measurements

A12	TRUE				
A23	True				
A34	True	False	True	False	True
A45	True	False	True	False	True
No Area Defected	0	0	0	0	0
Area Less 2%	0.25	0.1	0.1	0.5	0
Area Less 10%	0.25	0.45	0.45	0.5	0.333
Area More 10%	0.25	0.45	0	0	0.333
Area More 25%	0.25	0	0.45	0	0.333

The dynamic Bayesian network of the basic network at different times T0, T1 in 2008 and T2 in 2013 is built. The basic Bayesian network is repeated within the time and at each time slice the networks are connected thorough temporary arcs. Modeling deterioration this way, ensures that future condition depends mainly on current condition, previous condition and related factors.

The final results of the developed NBNs show the probability of condition states at different times. As illustrated in Figure 4, the vertical axis represents the probability of different condition states. The horizontal axis represents the time steps. The spacing between time steps is 5 years. From time t0 to time t1, the bridge deck falls under condition state 1. From time t1 to time t2 (year 2008) and from time t2 (year 2008) to t3 (year 2013), the bridge deck fall under condition state 3. After 5 years (year 2018), the bridge deck will fall under condition state 5.

The results also show the probability of the existence of the defected areas measured by NDE at different time steps, from t0 to t1, the defected area is falling under the category of no area defected. From t0 to t2, the defected area is falling under the category of area defected more than zero and less than 2%. From t1 to t4, the defected area is falling under the category of area defected is more than 10% and less than 25%. From t3 to t4, the defected area is falling under the category of area defected is more than 25%.

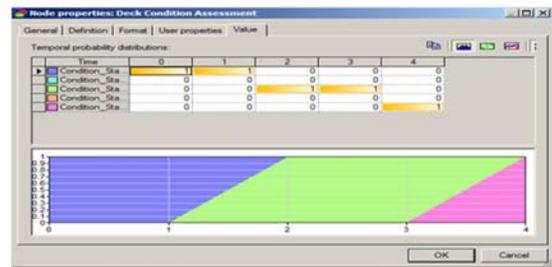


Figure 4. The Result of bridge deck condition

5. Prediction of Bridge Condition Using Markov Model

Markov Model is used to predict the future condition of the bridge deck for the same case study. So, condition state C0 at time 0 is taken at 1978 year, In order to predict the future condition of the deck at different times every 5 years Equation (4) is applied, where t is the number of transactions

$$C(t) = C(0) * TPM^{power\ t} \dots\dots\dots(4)$$

$$= [1 \ 0 \ 0 \ 0 \ 0] * [0.513 \ 0.487 \ 0 \ 0 \ 0]^{Power\ t}$$

0	0.492	0.507	0	0
0	0	0.5	0.5	0
0	0	0	0.513	0.487
0	0	0	0	1

At year 1993 which means 15 years after the initial condition and the time interval between inspection measurements are 5 years. So, the number of transactions are 3
**Condition States = [0.135005697 0.368965323
 0.274562808 0.220489737 0]**

At year 2008 which means 30 years after the initial condition and the time interval between inspection measurements are 5 years. So, the number of transactions are 6
**Condition States = [0.018227 0.093755 0.23468386
 0.3171046 0.3345]**

At year 2023 which means 45 years after the initial condition and the time interval between inspection measurements are 5 years. So, the number of transactions are 9
**Condition States= [0.0024607 0.01756 0.0698756
 0.16798167 0.708455]**

The results of the probabilities of the five condition states at different years are summarized in Table 6.

Table 6: Probability of bridge condition states at different transactions

Years	1978	1993	2008	2023
Condition 1	1	0.135	0.01823	0.0024
Condition 2	0	0.3689	0.09375	0.0175
Condition 3	0	0.27456	0.2347	0.06988
Condition 4	0	0.22049	0.3171	0.1679
Condition 5	0	0	0.3345	0.70846

6. Comparing The results of DBNs and Markov Model

Table 7 illustrates the final results of modeling bridge deterioration using Dynamic Bayesian Networks and Markov model techniques. Although, transition probabilities matrices for Markov model were built using some of the information from Bayesian networks, it doesn't consider the impact of the deterioration factors. Also, it doesn't take into consideration the previous condition at previous time. It is very clear from the result of Markov model, that it doesn't consider the impact of maintenance action. So, at years 1993 and 1998, the bridge was deteriorated faster to reach condition 2. Starting from year 2008, the bridge deteriorated faster to reach condition 5. In the DBNs model, factors impacting bridge deterioration are incorporated. It is very clear from the results that bridge deck will start to deteriorate and reach condition state 5 at year 2018.

Table 7. The Results Comparison between Markov and DBNs models

Years	Markov Model	DBNs Model
1993	2	1
1998	2	1
2003	3	3
2008	5	3
2013	5	3
2018	5	5
2023	5	5

7. Conclusion

This research provides a method to predict bridge deck condition states using Dynamic Bayesian Networks. The model is built using limited inspection records for two years at 2008 and 2013. The model incorporates the impact of deterioration factors extracted from the literature. Modeling bridge deck deterioration this way, ensures that future condition depends mainly on current condition, previous condition and factors impacting bridge deck deterioration. The model circumvents the limitations of current practice which is based on traditional Markov model. The final results of Dynamic Bayesian Networks are compared with the results of Markov model. These results show that incorporating deterioration factors improve the forecasting accuracy and its impact on forecasting inspection frequency and maintenance action required. The main contribution of the developed model lies in building an advanced deterioration modeling for bridge deck by using measurements of NDE methods and incorporating related factors. The model is generic, it can be updated when new observations are incorporated.

References

- [1] Agrawal, A., Kawaguchi, A., and Chen, Z. "Deterioration Rates of Typical Bridge Elements in New York." *J. Bridge Eng.*, 15(4): 419-429, 2010.
- [2] Cesare, M., Santamarina, C., Turkstra, C., and Vanmarcke, E. "Modeling Bridge Deterioration with Markov Chains." *Journal of Transportation Engineering*, 118(6): 820-833, 1992.
- [3] Robelin, C. and Madanat, S. "History-Dependent Bridge Deck Maintenance and Replacement Optimization with Markov Decision Processes." *J. Infrastructure System*, 13(3):195-201, 2007.
- [4] Frangopol, D. M., Kallen, M. J., and Van Noortwijk, J. M. "Probabilistic models for life cycle performance of deteriorating structures: review and future directions". *J. Progress in Structural Engineering and Materials*, 6(4): 197-212, 2004.
- [5] Madanat, S., Mishalani, R., and Ibrahim, W. H. W. "Estimation of infrastructure transition probabilities from condition rating data". *Journal of infrastructure systems*, 1(2), 120-125, 1995 .
- [6] Roelfstra, G., Hajdin, R., Adey, B. and Brühwiler, E., "Condition evolution in bridge management systems and corrosion-induced deterioration." *Journal of Bridge Engineering*, 9(3): 268-277, 2004.
- [7] Murphy, K. P. "Dynamic Bayesian Networks: representation, inference and learning" (Doctoral dissertation, University of California, Berkeley), (2002).
- [8] Jha, M. (2009). "Dynamic Bayesian Network for Predicting the Likelihood of a Terrorist Attack at Critical Transportation Infrastructure Facilities". *Journal of Infrastructure Systems*, 15(1): 31-39, 2009.
- [9] Straub, D. "Stochastic modeling of deterioration processes through dynamic Bayesian networks". *Journal of Engineering Mechanics*, 135(10): 1089-1099, 2009.
- [10] Weber, P., Medina-Oliva, G., Simon, C., and Jung, B. "Overview on Bayesian networks applications for dependability, risk analysis and maintenance areas". *Journal Engineering Applications of Artificial Intelligence*, 25(4): 671-682, 2012.
- [11] Faddoul, R., Raphael, W., Soubra, A. H., and Chateaneuf, A. "Incorporating Bayesian Networks in Markov Decision Processes". *Journal of Infrastructure Systems*, 19(4): 415-424, 2013.
- [12] Jensen, F. V., and Nielsen, T. D. "Bayesian network and decision graphs" Springer-Verlag. NY, USA, 2001.
- [13] Wang, R., Ma, L., Yan, C., and Mathew, J. "Condition deterioration prediction of bridge elements using Dynamic Bayesian Networks (DBNs)". In *Proceedings of the conference Quality, Reliability, Risk, Maintenance, and Safety Engineering (ICQR2MSE)*, International Conference pages. 566-571. IEEE, 2012
- [14] Rafiq, M. I., Chryssanthopoulos, M. K., and Sathanathan, S. "Bridge condition modelling and prediction using dynamic Bayesian belief networks". *Journal of Structure and Infrastructure Engineering*, 11(1): 38-50, 2015.
- [15] Wang, R., Ma, L., Yan, C., and Mathew, J. "Structural reliability prediction of a steel bridge element using Dynamic Object Oriented Bayesian Network (DOOBN)". In *Proceedings of the conference, Quality, Reliability, Risk, Maintenance, and Safety Engineering (ICQR2MSE)* pages. 7-12, International Conference on). IEEE, 2011.
- [16] Huang, Y. H. (2010). "Artificial neural network model of bridge deterioration". *Journal of Performance of Constructed Facilities*, 24(6): 597-602, 2010
- [17] Dinh, K., Zayed, T., Romero, F., and Tarussov, A. "Method for Analyzing Time-Series GPR Data of Concrete Bridge Decks". *Journal of Bridge Engineering*, 20(6): 04014086-1- 04014086-8, 2014
- [18] Martino, N. Quantifying reinforced concrete bridge deck deterioration using ground penetrating radar (Doctoral dissertation, Dissertation, Northeastern University), 2013