# Optimizing the trajectory of the painting robot

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**Abstract –**

**The main objective of this research work was to create a technological and mathematical model of the robotic arm movement for conducting painting work. For experiment was chosen model of the robotic arm, which corresponds in its construction robots commonly used in industrial production. This model of the robot was adapted to the conditions of carrying out painting work. One of the objectives was to create a mathematical model, which is designed to find the optimal trajectory for moving of the robot arm. Thanks to the mathematical model, which allows to optimize the trajectory of the painting robot, it was possible to deal with technological aspects. One of the main aspects was the consumption of material, in this case the method of applying colour to the surface. The path of the endpoint of the robotic arm must apply uniform colour layers with minimal overlap of previously painted surfaces. This is related to the fluidity of movement of the robot, which is determined by the functionality of the articulated joints. In the model was monitored load of servomotors that are placed always in contact with the arm pads, arm with next arm and arm with the endpoint. The optimal load for the servomotor, in particular the total energy consumption during the movement, was designed to avoid overload and subsequent collision. The aim of optimizing the technological model of painting robot is shortening the period for which the robot, with defined consumption of materials and energy, is capable to make the optimal trajectory.**

**Keywords –**

**Mathematical model; Robotic arm; Painting work; Trajectory for moving; Energy of servomotors**

# 1 Introduction

The topic of the given project goes hand in hand with the effort of implementing robotics and automation into the construction process. The process is completely different from automation of manufacturing due to the amount of performed operations, mobility and fluctuating work environment. Thanks to robotics in civil engineering we can achieve higher rate of safety, increase in productivity and maintaining the quality of the final product.

The project is divided into several phases. The first step is choosing the robotic machine (industrial robot with six axes) alternatively the construction of corresponding industrial robot (see Figure 1). The next step is a simulation of the movement itself, which has to be as economic as possible. The emphasis is put not only on ideal trajectory but also the final consumption of energy of every servomotor in the model. The last part of the research is suitable choice of color applicator with emphasis on fittingly chosen technology of painting in such a way that used material is saved.

Figure 1 – First model of the painting robot

# 2 Optimal trajectory of spraying movement

The main objective is finding the optimal movement course of the painting robot's arm. To fulfill the main objectives it is necessary to accomplish following partial goals.

- Creation of mathematical model of the robot's movement;
- Choice of the applicator and nozzle;
- Specification of the optimal movement for color application,
- Identification of the optimal layer overlapping

## 2.1 Mathematical model of the robot's movement

For the correct choice of the model it is needed to perform a movement simulation which will abide by the following procedure:

- choice of mathematical method of optimization;
- suggestion of mathematical model (determination of the inputs, parameters and outputs);
- verification of mathematical model by physical model of the painting robot;

Suggested course of action has to be verified with the whole physical model of the robot.

To solve the task of robotic arm's optimal movement a model using a TSP (travelling salesman problem) algorithm was designed [1]. TSP algorithm is one of the NP-difficult (nondeterministic polynomial time) discrete optimization problems (heuristic algorithms giving approximate solution were used to solve this). Solution of algorithm mathematically displays and generalizes the task of finding the shortest possible path going through all the given spots on the map or knots in network graph. The goal of the task was to find the shortest path for the „business traveler" or for closed Hamilton's circle with the lowest edge rating. The task is one of the very important and widespread in graph theory. The difficulty of the task drastically rises with the increasing number of knots.

Solving methods:

- **Solving with brute force** – the whole area of solving is being investigated. The time needed for finding solution increases with factorial, so it is not possible to use this method in practice even for the small areas of possible solutions. For 20 knots of our task for painting robot it was necessary to evaluate 1,21 x $10^{17}$ variants of solutions [2].
- **Random solution** – does not guarantee any results, because the task is not convergent to any value [3].
- **Hunger algorithm** – choice of minimal closer value, often goes to a dead end [4].
- **Simulated annealing** - is a probability optimization method for searching of state area. Algorithm does not guarantee finding of global minimum [5].
- **Finding of minimal frame** (Jarnik's algorithm) – entry graph is not closed [6].
- **Method of branches and bounds** - the principle of this method is in systematic scanning of all potential solutions. The estimations get more precise during the calculation [7].
- **Genetic algorithm** - the principle of work is subsequent forming of generation of various solutions of given the problem [8].
- **Ant Colony Optimization** – algorithm uses set of agents, ants during calculation. ACO is a meta-heuristic technique used in artificial intelligence field for search of approximate solutions of combinatorial problems [9].

Choice of method: To find the optimal (minimal course) movement of the robotic arm the method of branches and bounds was used. This method was developed in 1960 by a group of mathematicians (A. H. Land, A. G. Doig) for searching optimal solutions of different optimization problems, especially in discrete optimization and combinatorial optimization.

Method of branches and bounds is used for solving optimization tasks and it searches the tree of solutions. Set of possible solutions of task T is marked by symbol S(T). By that the optimization task T with objective function $f$ is expressed. The task T can be solved by creation of tasks $T_1,\ldots,T_n$ with the same objective function so that (see Equation (1)):

$$S(T) = S(T_1) \cup \ldots \cup S(T_n) \quad (1)$$

Optimal solution of task T can be obtained by choosing the best of all optimal solutions of subtasks, $T_1,\ldots,T_n$. Individual subtasks could be considered as peaks of the root tree, that were called tree of solutions. During the calculation the tree grew larger and only the leaves of the root tree were considered interesting for the calculation. The leaves were divided into so called alive and dead. Solutions that were either optimal, did not have acceptable solution or a better solution existed were considered dead. The calculation ended when all the leaves were dead. To simplify the calculation for each minimization task the bottom bound $Pre(T_i)$ was set. During branching graph was searched in width or depth.

The shortest Hamilton cycle can also be searched for by method of branches and bounds. The task would be reformulated as follows:

Searching for the cheapest subgraph H of given graph G so that:

- Edges of the subgraph H would form a system of apex disjunctive cycles that would pass through all the apexes of the graph G;
- The subgraph H was coherent;

Subsequently a cycle where all the apexes were marked as a set M was determined. This led to creation of subtasks for each apex $v_i \in M$ (all the edges that led from apex $v_i$ to set M were banned) [10].

After choosing suitable mathematical method, the starting entry conditions and parameters were set:

- placement of robot $P_0$ (point „zero") and $_0$ (vector „zero"): coordinates [$x_0$, $y_0$, $z_0$] in global coordinate system;
- definition of working plane: a 3-point method for dextrorotary orthogonal coordinate system (determination of the basis for robot „KUKA");
- decisive determination of the working plane (surface for painting without holes or permeation): matrix of critical points in working plane [x, y];
- setting of technical parameters of the robot's nozzle (effective stripe/width of slit of the nozzle, overlapping of color);

In the next phase the model was realized by the following steps [11]:

- Layout (division) of the working plane to active and passive rectangles (vertical, from the ceiling towards the floor, see Figure 2): rectangles (R) were decisively determined by points $P_{i0}$ [x, y] – upper left corner, $P_{i1}$ [x, y] – bottom right corner. Two matrixes were constructed (active $R_a$ size 2 x $n_a$; passive $R_p$ size 2 x $n_p$, where $n_a$ and $n_p$ is the number of active and passive rectangles);

- Setting of the robot's course during painting of the active rectangles (number of vertical stripes).
- Drawing of oriented network graph (see Equation (2) and (3)):
  - Number of knots:

$$\mathrm{N}(k) = 2 \,\mathrm{x}\, \mathrm{n}(a) + 1 \qquad (2)$$

  - Number of edges:

$$\mathrm{N}(e) = 3 \,\mathrm{x}\, \mathrm{n}(a)^2 + 2 \,\mathrm{x}\, \mathrm{n}(a) \qquad (3)$$

- Construction of the matrix of oriented network graph containing the coordinates of all knots, edge orientation, edge length, determination of active and passive areas (respectively input/output points of active rectangles);
- Inspection of robot's arm reach (comparison of distance between point zero and points of working area with the robot's arm reach).
- Finding of the shortest path (closed Hamilton path) by several means, subsequent comparison and choice of optimal course.

In the final phase of the mathematical model of robotic arm, the optimal path (the shortest trajectory of movement of the painting robot's arm respectively Hamilton path - see Figure 3) was chosen and verified.

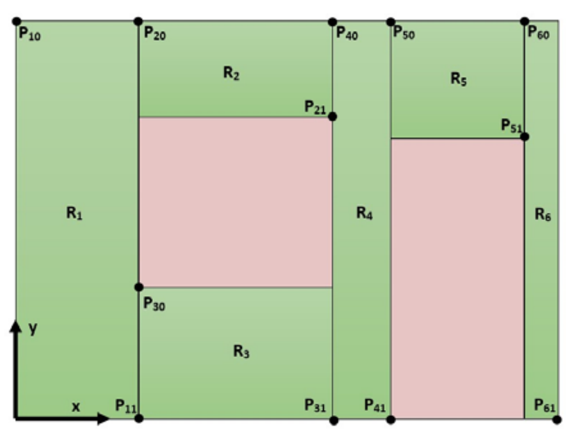


Figure 2 – Separation of the working surface to active and passive rectangles

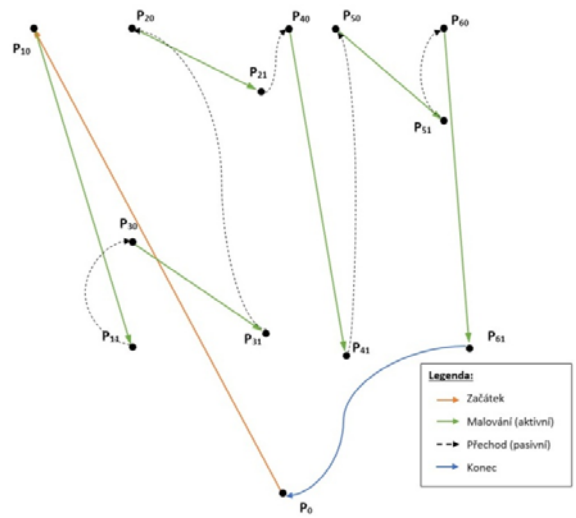


Figure 3 – Hamilton path

## 2.2 Choice of applicator and nozzle (pattern)

The spray gun was chosen from two technologies of spray (by air and by pressure) [12]:

- **By air:** air spraying is classic and so far most widespread. Compressed air from a compressor gets through a hose to the spray gun, where it mixes with the color from a container, which is located on the gun. The air pressure is depending on the type of the compressor in range from 2 to 7 bars.
- **By pressure:** less widespread (but gaining popularity) is so called airless spraying. It works without air, it means without using a compressor. It uses a pump. Strainer of the machine and a powerful pump, which depending on the type reaches pressure up to 230 bars is plunged to a container with the paint. It forces the paint or the glaze through the hose to the pistol. It eliminates the need for setting the air/paint ratio and complications caused by wrong adjustments. The only thing that is being adjusted is the pressure which is highly important. This way spraying is not only economical but also guarantees high quality of the spraying.

Subsequent choice of the nozzle and its pattern geometry is really important because it determines its shape and intensity of the spraying. For our applicator it was possible to choose from following patterns (see Figure 4) [13].

- **Solid stream** – One or more solid points (streams). Application with low pressures. The nozzles used for this type are exceptionally simple;
- **Flat spray** – Thin, long spraying. Almost unified pattern in its whole application. Useful for large areas;
- **Hollow cone** – Spraying is distributed in circle from the nozzle;
- **Full cone** – Spraying is applied in the whole circular area. The most widespread pattern;

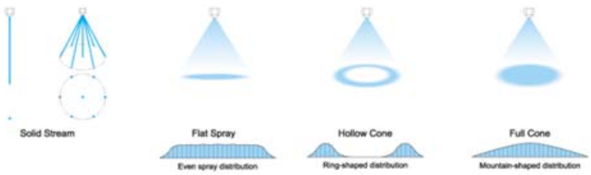


Figure 4 – Nozzles geometry [13]

## 2.3 Overlapping of layers

During the application of spraying the surface the most important criterion is keeping the suggested distance from the surface. The applicator should be in optimal distance of 30 centimeters (cca 12 inches) and it should always impact the surface perpendicularly. In order to secure equal amount of paint on the surface, the overlap ratio is suggested to be 50% and more in manual spraying. However, this solution is not economical and it causes unnecessary loses, which would have negative impact on the price when painting larger areas.

When using flat pattern (see Figure 5) we can achieve desired decrease in losses mainly thanks to reducing the overlapping which we modify from 50% to approximately 10%. The *a* area has in 90% of the spraying the same properties of overlapping and the two remaining areas *b* each occupying approximately 5% minimization their overlapping properties. It is due to the mutual overlapping of areas *b* while applying the paint we achieve economies.

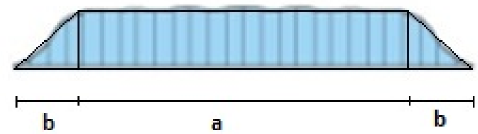


Figure 5 – Areas of coverage (*a*-constant, *b*-descending)

## 2.4 Delimitation of optimal movement for applying the paint

The last studied factor was the choice of appropriate movement of the trajectory of the end point where an emphasis on minimization of the energy consumption was made. For this part of the project a program presenting percentual workload of the servomotors was created. Basic trajectories (vertical and horizontal) were compared and each one of them was composed from three movements. The movement 1 of vertical course shifted from upper left downwards by distance x. Movement 2 followed from left to right by distance y. Furthermore, via movement 3 it returned back to the top by distance x. Horizontal movement started by shifting from right to left by the distance x, continued downwards by distance y and backwards from left to right by distance x (see Figure 6)

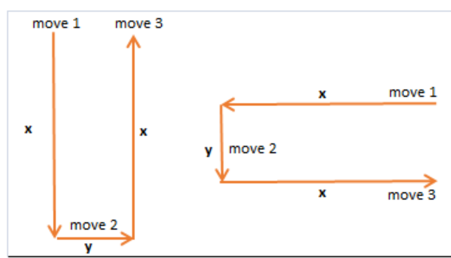


Figure 6 – Scheme of vertical and horizontal movement

Subsequently the times of each servomotors for both trajectories of robotic arm's movement were measured. Because of this the total energy consumption of each movement and their economic efficiency were expressed.

In the following Tables the servomotors (servo6, servo5, …) are presented with their percentual workload while performing given movement. Due to percentual workload and measured time on servomotors, the individual work of both movements could be determined and compared.

Table 1 - Horizontal trajectory

| servo | Position | Post 1 | Post 2 | Post 3 | Post 4 | Move 1 | Move 2 | Move 3 |
|---|---|---|---|---|---|---|---|---|
| servo6 | Rotate | 225 | 132 | 124 | 225 | 40% | 3% | 44% |
| servo5 | Bottom | 149 | 149 | 175 | 175 | 2% | 11% | 2% |
| servo4 | Middle | 115 | 115 | 127 | 127 | 4% | 5% | 3% |
| servo3 | Top | 127 | 127 | 113 | 113 | 3% | 6% | 3% |
| servo2 | Rotate | 200 | 200 | 200 | 200 | 2% | 2% | 2% |
| servo1 | Gripper | 39 | 124 | 132 | 39 | 33% | 3% | 36% |

Table 2 - Energy consumption (partial and total) for horizontal movement

| *W1[Ws]* | *W2[Ws]* | *W3[Ws]* | *U[V]* | *I[A]* | *t1[s]* | *t2[s]* | *t3[s]* |
|---|---|---|---|---|---|---|---|
| 43,67 | 3,76 | 47,43 | 12,00 | 2,00 | 1,82 | 0,16 | 1,98 |
| 2,16 | 12,21 | 2,16 | 12,00 | 2,00 | 0,09 | 0,51 | 0,09 |
| 4,32 | 5,63 | 3,24 | 12,00 | 2,00 | 0,18 | 0,23 | 0,14 |
| 3,24 | 6,57 | 3,24 | 12,00 | 2,00 | 0,14 | 0,27 | 0,14 |
| 2,16 | 2,16 | 2,16 | 12,00 | 2,00 | 0,09 | 0,09 | 0,09 |
| 36,00 | 3,39 | 39,39 | 12,00 | 2,00 | 1,50 | 0,14 | 1,64 |
| ***TOTAL*** | 222,886 | **Ws** | | | | | |
| | 6,19E-05 | **kWh** | | | | | |

Table 3 – Vertical trajectory

| servo | Position | Post 1 | Post 2 | Post 3 | Post 4 | Move 1 | Move 2 | Move 3 |
|---|---|---|---|---|---|---|---|---|
| servo6 | Rotate | 217 | 217 | 200 | 200 | 1% | 7% | 1% |
| servo5 | Bottom | 147 | 202 | 202 | 147 | 24% | 1% | 24% |
| servo4 | Middle | 145 | 75 | 75 | 145 | 30% | 2% | 30% |
| servo3 | Top | 154 | 56 | 56 | 158 | 43% | 1% | 44% |
| servo2 | Rotate | 200 | 200 | 200 | 200 | 2% | 2% | 2% |
| servo1 | Gripper | 50 | 45 | 61 | 61 | 2% | 6% | 1% |

Table 4 - Energy consumption (partial and total) for vertical movement

| *W[J=Ws]* | *W[J=Ws]* | *W[J=Ws]* | *U[V]* | *I[A]* | *t1[s]* | *t2[s]* | *t3[s]* |
|---|---|---|---|---|---|---|---|
| 1,08 | 7,98 | 1,08 | 12,00 | 2,00 | 0,05 | 0,33 | 0,05 |
| 25,83 | 1,08 | 25,83 | 12,00 | 2,00 | 1,08 | 0,05 | 1,08 |
| 32,87 | 2,16 | 32,87 | 12,00 | 2,00 | 1,37 | 0,09 | 1,37 |
| 46,02 | 1,08 | 47,90 | 12,00 | 2,00 | 1,92 | 0,05 | 2,00 |
| 2,16 | 2,16 | 2,16 | 12,00 | 2,00 | 0,09 | 0,09 | 0,09 |
| 2,12 | 6,78 | 1,08 | 12,00 | 2,00 | 0,09 | 0,28 | 0,05 |
| ***CELKEM*** | 242,22 | **Ws** | | | | | |
| | 6,72E-05 | **kWh** | | | | | |

When abiding by all conditions of the spraying (optimal distance, ideal pattern of the nozzle and applicator) with the combination of optimization of movements (for spraying horizontal movement and for shifting the arm between the active areas by Hamilton path) the desired savings were achieved. Active width of spraying will be approximately 35 centimeters and the passive width on both sides around 2,5 centimeters. Sprayed stripe with total width around 40 centimeters can be put in area R1 y/37,5 times (2,5 centimeters is mutual overlap of the stripes). After finishing area R1 the robotic arm will move to another area based on the optimizing algorithm of ideal trajectory and here it will continue in horizontal movement for application of the paint again.

This option seems to be the most economical regarding the energy and paint consumption. When facing the request for multi-layer spraying, the very same method would have been used, the difference being that the new spraying would overlap with the old one by a half of the total width of applied paint.

## 3 Conclusion

The optimizing algorithm managed to quickly find at least partially optimal trajectory of the robotic arm's movement which saved engine time and wearing of the robot's servomotor. Algorithm can be extended by other optimizing parameters. The working area could be divided into rectangles also by using mathematical algorithm. The calculation could be extended also by optimizing moves of minimization of the paint and energy consumption. Evaluating edge would be evaluated not only by the distance but also by the time of robotic arm's travel and energy consumption respectively number of revs of servomotors.

The most suitable paint applicator was pressure spray gun. This method of spraying was not only economical but it guaranteed high quality of the spraying. The spraying was more comfortable also due to the pistol not having the container with paint. The arm could have moved without restrictions and spray from various positions. On top of that when used correctly the losses are minimal.

As the ideal pattern of the nozzle was chosen the flat pattern which was suitable for large areas too because of its geometry and coverage. Thanks to robotics the exact position of applied paint could be set and bound of coverage, which was reduced from 50% to approximately 10% determined.

The ideal trajectory of painting robot's end point was horizontal movement where the total energy consumptions 222 Ws was lower than the vertical one with 242 kWh. The differences were minimal but during the spraying of larger areas and applying the ideal movement, the energy savings will be increased too.

In the Figure (see Figure 7) the painting robot's model is depicted. Its paint transport to the final point is not yet completely solved. Next, the problems of tidiness and fluency of movement will be investigated.

In this case it was not taken into consideration because it would be unnecessarily complex due to limited software of the model.

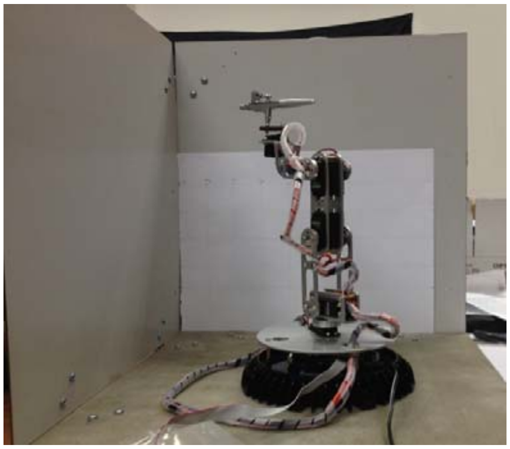

Figure 7 - Model of the painting robot at work

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