

# Multiple Tower Crane Selection methodology utilizing Genetic Algorithm

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## Abstract –

Equipment selection for a construction project is a complex decision making task that impacts the project cost significantly. The literature highlights the increasing focus of recent studies on tower crane (TC) optimization, largely due to the shift from horizontal to vertical construction. Many studies tend to select a single TC model to be used throughout the site and assume the number of cranes to be used as a priori knowledge or is calculated using heuristics and schedule demands. Selection of a combination of TC models is relatively unexplored. Existing literature focuses on finding a single most optimum solution to a multimodal problem. Need to look for multiple optimums arises from the uncertainties of an integral but disjoint simulation process and the local and specific nature of un-modelled constraints. Thus to address this gap the study presents a selection approach, aiming to leverage the long reach of TC, formulated as a function of the rental cost and lifting requirements. The selection of the equipment is of prime focus in this research with implicit consideration for the positioning of the TC. A Bi-level optimization problem is formulated involving task allocation to different TC models and minimization of required crane count for each model. Genetic Algorithm (GA) has been employed to work with non-differentiable multimodal function and obtain the preferable TC combinations from the available model variations. The results derived from the proposed model included optimal yet dissimilar TC combination options, task allocation to the utilized TCs and feasible regions for crane placement. The major limitations were parameter setting for the adopted algorithm and the inability of the distance metric to robustly capture phenotypic differences.

## Keywords–

Genetic Algorithm; Tower Crane selection; Bi-level optimization; Multimodal optimization.

## 1 Introduction

Site Layout Planning (SLP) involves optimum space utilization for the resources required to aid construction. The equipment and machinery form an integral part of these required resources. Construction activities involve tasks like shifting of materials, lifting and hoisting along with holding up of loads in place for processing. Cranes being better suited for such tasks have gained the interest of site practitioners. Crane selection is one of the many critical decisions that construction managers have to make. As highlighted by Shawney and Mund [1], time, cost and safety pertaining to construction operations are significantly hinged to the selection of a suitable crane. Deployment of tower cranes typically demands the biggest investment for construction equipment on a site. On an average, major equipments amount to nearly 36% of the total procurement cost [2].

Crane selection consists of two components i.e. type selection and model selection. The former pertains to the selection of the crane type from the range of options available like Tower Cranes, Derricks, Wheel or Crawler mounted Cranes etc. This is highly dependent on the nature of work, geotechnical conditions of a particular construction site and limitations of crane type. Crane model selection is the next step which involves choosing the best combination of cranes of certain type. This choice is governed by a multitude of criteria like rental and operation costs, safety etc. Another challenge faced by construction practitioners is related to positioning of this heavy lifting machinery. This study is an attempt of TC model selection addressing certain limitations of present literature.

## 2 Crane Selection and Location Optimization Research

Extensive research has been undertaken on the topic of crane type selection in the existing studies. Alkass et al.[3] proposed a methodology utilizing object oriented programming providing solutions to the crane selection

problem using Rule-based and Case-based reasoning with project specific user inputs. Another Fuzzy logic approach to the problem was proposed by Hanna and Lotfallah[4] to incorporate qualitative factors like soil conditions, access road requirements etc. Sawhney and Mund[1] used Artificial Neural Networks in Intellicranes selection tool to tackle the subjectivity involved in decision making regarding the aforementioned factors. These studies laid down the drivers of crane type selection decision using codified expert knowledge. Simulation was relied upon in [3] & [1] for ensuring that the geometric constraints and productivity demands were met at possible crane locations for the suggested solutions. However no guided search algorithm was used in these efforts.

Many TC location optimization models have been developed. Tam et al.[5] used genetic algorithm to minimize the hook travel time by varying the TC and supply point locations around fixed demand points for sites using a single TC. Abdelmegid et al.[6] contributed in improving the travel time minimization model by incorporating the vertical velocity of the hook. Wang et al.[7] further linked this model to BIM and simulation modules to detect schedule conformance and clash detection. However these studies either considered a single TC ([5] and [6]) or assumed a single TC model by largest lift weight and task distances and number of TCs decided by heuristically derived crane efficiency ([7]). Shapira et al.[8] quantified a safety index for any construction site with TC. Safety related to the wind, operator proficiency, shift length, positioning of the cranes with respect to surrounding facilities etc. was captured.

Selection and location of group of TCs has garnered only limited attention. While the primary hard constraint for TC model selection is to ensure the ability of the TC group to lift the prescribed material weights, divergent approaches have been used for location determination. Zhang et al.[9] optimized the safety and efficient operation by minimizing the number and extent of jib clashes and balancing workloads of cranes respectively. A major limitation included pre-determination of number and model of TC to be used. Irizarry and Karan [10] built on this work and displayed the selection of minimum number of TCs when a particular model was specified while claiming that the model being capable of finding the best combination with multiple models at disposal. Minimization of overlapping area of cranes among themselves and with facilities was used to reduce conflicts. Marzouk and Abubakr[11] used maximum site coverage for the same. Y. Ji and Leite[12] minimized the hook travel time and demonstrated the importance of doing so for the crane group as a whole rather than for each TC individually. However all these TC selection and location studies only find a singular

most optimal solution to a multimodal problem. The potential value addition in looking for the local optimums is described below.

TC location finalization is subject to it being free from spatial clashes and the arrangement possessing the ability to adhere to the schedule. Simulations have been widely proposed to test for such requirements. As pointed out in [11], the processes of location optimization and simulation in most research efforts have been disjoint, i.e. visualization for clash detection is done in a separate module by using outputs of the optimization module. Thus, the study reported, that a wide range of feasible solutions must be tested in simulation runs to find the near optimum. In case of a discovery of any issue through simulation, the knowledge of a favorable yet dissimilar solution to the one under consideration would add great value. Models integrating the optimization and visualization processes can be a viable option to tackle the issue. However, the gains, as stated by Einbu[13], of greater reusability, concealment of data and operations and higher adaptability that modularization provides to the software manufacturers and the service providers cannot be ignored. Moreover, Sepasgozar and Forsythe[14] highlighted how the studies up till now have focused on project specific factors while the organization based factors have remained largely unattended. The difficulty of factoring the complexity of maintenance and local availability of after sales services demands greater alternatives for decision makers to compare and choose from.

Thus to counter the unforeseen hindrances in simulations and the inability to model an exhaustive list of constraints in an optimization problem a TC selection model capable of giving multiple and varied sub-optimal solutions from a multitude of feasible options can provide flexibility. This study borrows from the framework adopted in [10] of a rental cost based TC selection model while the subsequent location determining objective not focused upon. The model uses Genetic Algorithm (GA) to minimize the rental cost of the group of TCs with an attempt to find local optimums has also been demonstrated. The scope of this study is TC selection and the aim is to supplement the currently proposed TC location methodologies by providing varied alternative solutions.

### 3 Optimization Model Description

This section contains a detailed description of the underlying logic used in the model. For a crane to successfully perform a task, i.e. shift a weight  $w$  from supply point (S) to demand point (D), it must be able to lift the prescribed weight at both the locations. Every crane can be characterized by their jib length (R) and

the maximum lifting capacity ( $W_m$ ). The lifting capacity of a crane varies along its jib, decreasing towards the tip, and is obtained from the load-radius curve provided by the manufacturer. Thus, for each weight  $w \leq W_m$ , a circular area of radius 'r' (obtained from the curve) exists for a particular crane inside which it can lift that weight. To perform a task, the crane must lie within the intersection area of the buffer zones centered about S and D points. This intersected region is called the feasible task area. No intersection implies the task cannot be performed by the crane. The size of the area is related to the distance between S and D, the weight of the load, and crane capacity. Larger the feasible area, more easily the task can be handled. A tower crane can handle two or more tasks, if it is located within the feasible areas of all those tasks (Fig. 1(a)), which is essentially the intersection of feasible areas of those individual tasks. If no common overlap exists, then a single TC is inadequate to handle all of those tasks (Fig. 1(b)).

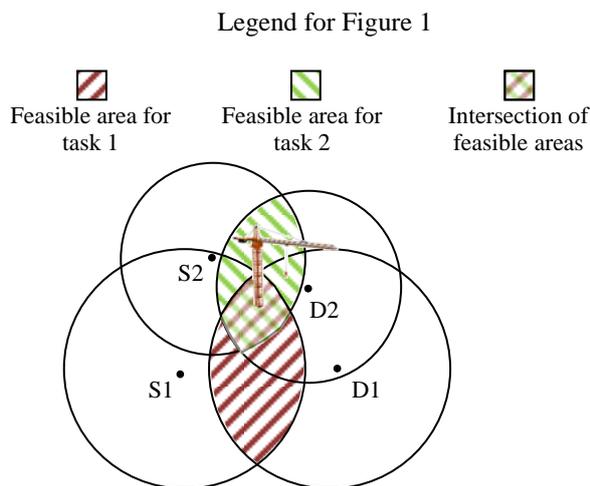


Figure 1(a). Single TC sufficient for two tasks if placed in the common feasible region

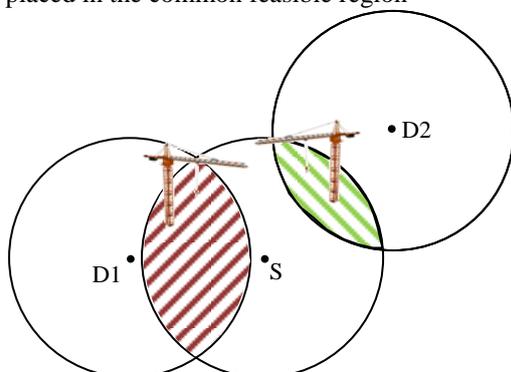


Figure 1(b). At least two TCs required for the tasks, one in each feasible region.

Assumptions of this study:-

1. Geometric Layout of all the S and D points is known along with the module weights for each S-D pair (task).
2. Only one TC is designated to perform any task.
3. For every task, there exists at least one crane in the database which is capable of performing it.

### 3.1 Bi-Level optimization

Two decisions are involved in finalizing the TC selection for a group of tasks - TC model that will be used to do certain tasks and the number of cranes of that model required to perform the allocated tasks. Thus for every task allocation, a minimization operation is required for each utilized TC model to find the respective number of cranes.

A bi-level optimization problem is a hierarchy of two optimization problems (upper-level or leader, and lower-level or follower). Although different objective is optimized at each level independent of each other, the decisions of each level have effects on one another [15]. The decision of the upper level (TC model allocation to tasks) determines the search space of the lower level (tasks for which minimum TC count is to be determined). The result of the follower contributes in the objective function evaluation of the leader.

#### 3.1.1 Upper Level :- Crane Allocation to Tasks

Ability of a crane to perform any task  $i$  depends on the distance between S and D points ( $Dist_i$ ) and the module weight to be carried ( $W_i$ ). Subject to them, the potential TC models for each task can be finalized. The constraint has been handled through a combination of two measures - appropriate structuring of population initialization and mutation operators and through exterior static penalty functions as summarized by Smith and Coit[16]. Initially, TC models capable of doing a task are filtered by comparing jib lengths and  $Dist_i$  and if they are rendered incapable for that task due to reduced reach owing to load-radius curves, a penalty is added. In addition, the aggregate feasible region after considering all the allocated tasks for a single TC of any model must be greater than a typical threshold value.

Let  $m$  be the number of tasks to be undertaken and  $n$  be the different TC models available for selection, an identification label is attached to each model. Number of optimization variables is equal to  $m$ . The variable  $x_i$  assumes the label value of the crane model being used for  $i^{\text{th}}$  task. Thus, the variables are of categorical type. Ordinal encoding has been preferred over one hot encoding to keep the dimensionality of the problem low, which in turn obviated handling of the constraint- every task must be allocated one and only one crane model.

$$\text{Minimize} \quad \sum_{TC=1}^n N_{TC}(x)R_{TC} \quad (1)$$

*Subject to constraints*

1.  $x_i=y, y \in \{z \mid 2^*(\text{reduced radius})_z \geq \text{Dist}_i\} \forall i,$
2.  $(\text{aggregate feasible area})_{y \geq} (\text{Threshold area})_y,$

where denotations are as follows

$i = i^{\text{th}}$  task,  $z =$  label given to the TC model,  $R_{TC} =$  rental cost of the TC model,  $N_{TC}(x) =$  minimum number of cranes of a model required according to the allocation  $x$ , obtained from lower level of optimization. Threshold area is set as the base area of crane.

A widely utilized fitness sharing method for multimodal optimization has been used. It is based on evolution of different species in separated niches of nature. The search space is divided into niches and search for local optimums in them occurs in parallel. Fitness of closely resembling chromosomes is decreased to maintain diversity in population. Thus convergence to single optimum is prevented since the presence of a high number of similar individuals is discouraged. In this study, similarity between two chromosomes has been measured using genotypic distance which is the number of string positions by which they differ. Greater the distance, lesser is the similarity. As outlined by Deb and Goldberg[17], setting the dissimilarity threshold (the minimum distance between two chromosomes above which they don't affect each others' fitness) must be done carefully. Method proposed by them for calculating the parameter for binary string is as follows

$$\frac{2^l}{q} \leq \sum_{i=0}^k l$$

where  $l$  stands for the string length,  $q$  for the number of optimal/suboptimal solutions and the lowest integer value of  $k$  for which the inequality holds gives the parameter value. The parameter is denoted as  $\sigma_{\text{share}}$ .

The LHS denotes the average volume of search space occupied by each niche and the RHS denotes the number of possible different strings if at most  $k$  bit differences are allowed. Same logic was applied to get the value of dissimilarity threshold although with modification to the expression since the categorical variables of proposed model are not necessarily binary but can take multiple values. The  $q$  is an input from the user to be decided arbitrarily when no prior information is known about the problem.

Following is a summary of how the fitness of individuals is altered according to the fitness sharing method as described by Deb[18]. The value of sharing function is defined for a pair of individuals with  $d$  as the distance between them.

$$Sh(d) = \begin{cases} 1 - (\frac{d}{\sigma_{\text{share}}}), & d < \sigma_{\text{share}} \\ 0, & \text{otherwise} \end{cases}$$

The summation of sharing function values for an individual paired with every other individual gives the scaling factor ( $m_i'$ ) for that particular individual.

$$m_i' = \sum_{j=1}^N Sh(d_{ij})$$

The fitness value is divided by the scaling factor to get the shared fitness value.

$$f_i' = \frac{f_i}{m_i'}$$

Greater the population density in a certain search space area, greater is the scaling of fitness of those individuals. .

### 3.1.2 Lower Level :- Minimum Crane Count

Once the crane model has been allocated to each task, determination of the minimum number of cranes of each model required to perform them remains, i.e. finding the values of  $N_{TC}(x)$  for evaluating function (1).

Let  $m_1$  number of tasks from the total  $m$  be allocated to TC model with label 1. The variable encoding is similar to the upper level with the exception that all the cranes here are exactly identical. Earlier, two variables assuming different label values of TC models implied they had been allocated to different models of TC whereas at this level, two variables assuming different label values implies that they will be performed by two physically different cranes of the same model. Thus a maximum number of cranes of each model must be fixed to limit the search space. Let this number be  $n_1$ .

The objective function value is the number of cranes used which is equal to the number of unique label values taken by the variables. The task variables with the same label value are said to be grouped together. Feasibility of these grouped tasks to be performed by a single TC is tested. For every such infeasible task group, a constant penalty equal to  $n_1$  is added. The objective function is minimized and its value, representing the number of cranes is fed into the upper level as  $N_{TC}$ .

Derivation of  $n_1$  is empirical. Lower level optimization model is run for each TC model separately assuming all the tasks satisfying the constraint (1) and (2) of upper level are allocated to that TC model. A large  $n_1$  translates to a huge search space for the algorithm which might result in inability to find the least sufficient crane count. Initially,  $n_1$  is kept large and its value updated after every algorithm run, changed to the output of the previous run until there's no difference in the value of  $n_1$  and result. Hence  $n_1$  is the minimum number of cranes of model 1 required if it gets assigned all the tasks it can perform.

The presented case in this research targets selection of TC from a pool of options available to the construction practitioner to choose from. This underlying assumption of availability of multiple options of TC represents the market scenario and thus providing a single solution will make the decision making task stringent. Also, the fitness function is highly sensitive to the underlying decision variable values. Due to traditional optimization algorithms' reliance on derivative or slope information and their ability to reach a single optimum solution only, they are not suitable for this problem. Therefore a nature inspired algorithm capable of providing a set of optimal solutions is adopted in this study. Moreover, the ability of GAs to move from highly fit lower order schemata to higher order ones [19] is of particular interest as it translates to grouping of geometrically closer tasks in the phenotypic space. Fig.2 explains the flow process of the adopted algorithm. At every fitness evaluation step, lower optimization function is called for each TC model used.

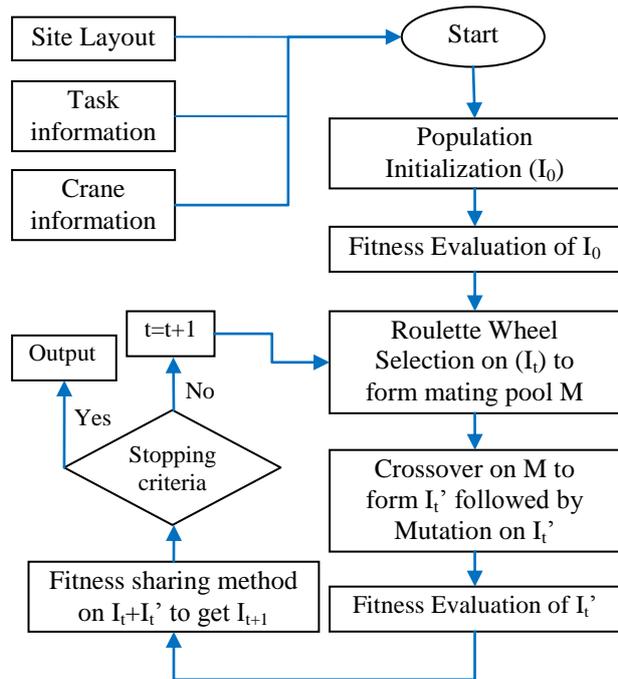


Figure 2. Flow process of the GA fitness sharing model

#### 4 Case Study

A hypothetical site layout was used to test the proposed selection model. The boundaries of both temporary and permanent facilities in the layout are considered to be enclosing the areas not allowed for TC placement. These boundaries also include the minimum clearance from entities required for placing a TC. The

inputs to the model include geometric information of temporary site facilities and the details for coordinates and module weights for each task, as highlighted in Table 2. Load charts and rents of available TC models are uploaded. As mentioned in Table 3, four models of varying jib lengths and lifting capacity were used. Table 1 summarizes the input parameters used for the upper level problem.

Table 1. Algorithm parameters

Parameter	Value
Population Size	100
Maximum Generation	150
Number of Variables	17
Crossover Rate	0.8
Mutation Rate	0.06
q (Number of peaks)	3
$\sigma_{share}$	12

Setting of GA parameters for the lower level problem like population size and  $n_1$  as discussed before is empirical and requires fine tuning with multiple runs to ensure correct answers and also to keep run time in check.

The final population of chromosomes produced by the algorithm contained different optimal solutions with varying task allocation to different crane models and hence varying combination of crane requirements. The results display a combination of TCs of dissimilar jib lengths and lifting capacity can result in lower rental costs as against the common practice of a common TC model usage across the site. Table 4 gives the total rental cost for the solutions and the number of tasks performed by each utilized crane. The feasible areas for TC combinations for the obtained solutions along with the temporary and permanent facilities of the site have

Table 2. Task information:-S&D coordinates, lift weight

ID	Supply ( in m)		Demand ( in m)		$W_i$ (t)
	Abcissa	Ordinate	Abcissa	Ordinate	
1	46.25	116.25	41.25	141.71	2.25
2	46.25	116.25	25	100	2
3	15	30	25.25	60.71	2.25
...	...	...	...	...	...
17	91.84	90.62	25.25	127.71	1

Table 3. Tower Crane Model information

Label	1	2	3	4
Rent ( $\times 10^3$ /day)	15	20	27	35
Jib Length	25m	40m	50m	60m
Max. lift capacity(t)	3.5	8	9	12

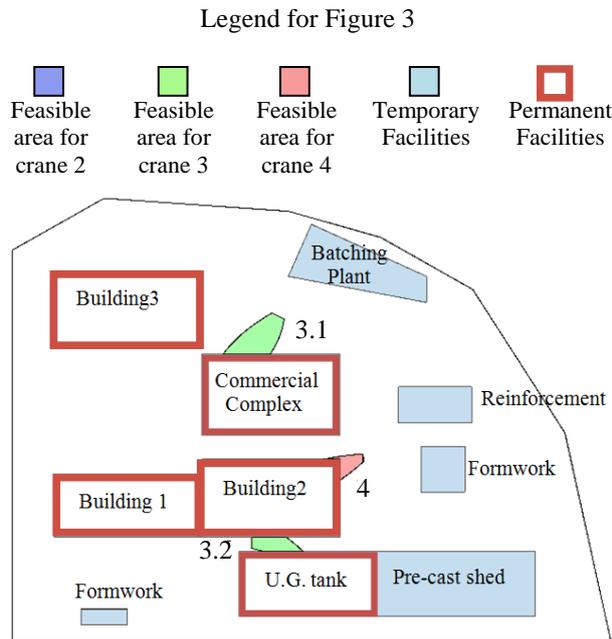


Figure 3(a)

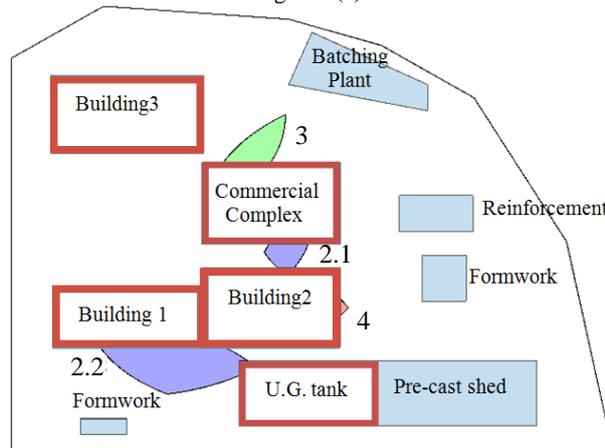


Figure 3(b)

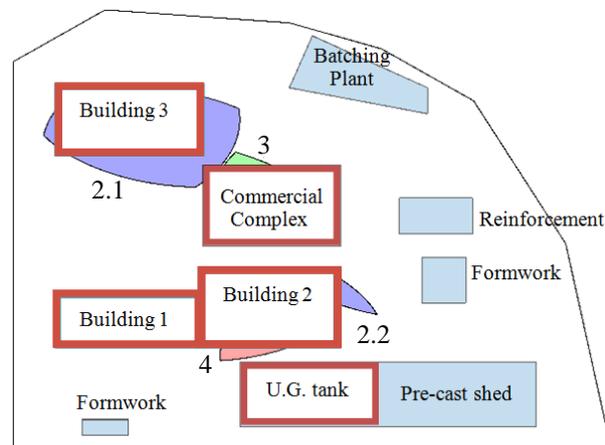


Figure 3(c)

been shown in Figure 3. Legend indicates what the shaded regions represent. One TC model of the indicated type is required to be placed in each of the shaded feasible region. The neighboring number to the shaded region points to the corresponding entry in Table 4 to get the number of tasks it handles. For example two TCs of model 3 are required in solution of Figure 3(a) with the crane placed in the region corresponding to 3.1 performs four tasks.

Table 4. Total rental cost and number of tasks performed by each crane

Solution	Rental cost per day	TC model 2		TC model 3		TC model 4
		2.1	2.2	3.1	3.2	
Fig. 3(a)	89000	-		4	5	8
Fig. 3(b)	102000	3	3	3		8
Fig. 3(c)	102000	3	5	3		6

The algorithm was successful in maintaining sub-optimal solutions through the generations. Also, the feasible regions produced by them showcase a certain degree of variety in terms of the potential TC locations. Such provision can endow the decision makers with flexibility while making decisions about TC model selection and location. Difficult to encode constraints like soil conditions or to account for intangibles like TC maintenance, availability of options can prove helpful. Moreover, the flexibility of multiple solutions can provide options if clashes are detected in simulations. It must be noted that for a certain ownership cost, more than one Task Distributions can be possible. The ones shown above have been chosen from the solution set based on more equitable task distribution among TCs.

## 5 Limitations and Future Scope

K-means clustering was used to separate the final population into 3 clusters, equal to the set value of  $q$ . Relatively low silhouette values suggest weak clustering in the population. The genotypic distance between the optimal solutions from different clusters was lesser than the value of  $\sigma_{share}$  used which points towards a revision to a lower value. A smaller  $\sigma_{share}$  implies sustenance of more number of solutions which demands higher population levels leading to impractical processing times. Moreover, the existence of individuals in a niche with fitness values lower than the local optimum indicates a highly rugged landscape which leads to survival of less fit but different individuals even within

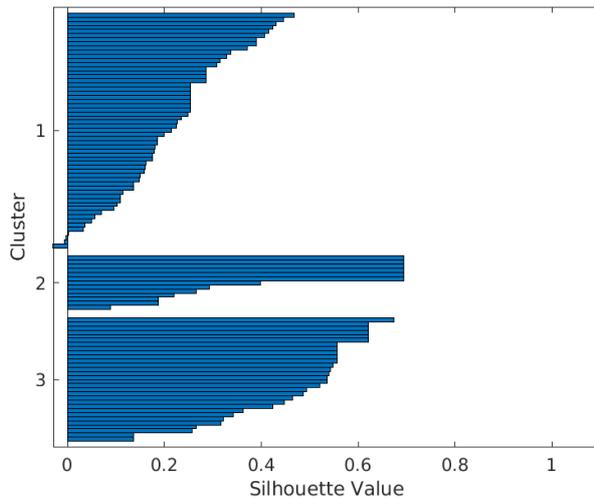


Figure 4. Silhouette plot for k-means clustering

a niche.

The root cause of this limitation of the study lies in the distance metric. As outlined in [17], phenotypic distance can perform better than genotypic distance in certain problems. The problem with this approach is the limited translation of genotypic distance to the physical space. Consider two instances of task distribution for 5 tasks using the scheme described above

$$\begin{aligned} X &= [4 \ 4 \ 4 \ 3 \ 3] \\ Y &= [4 \ 4 \ 4 \ 2 \ 2] \\ Z &= [4 \ 4 \ 4 \ 1 \ 1] \end{aligned}$$

The genotypic distance between X&Y and X&Z is equal to 2. But if a single TC of model 2 and 3 is sufficient in X and Y, it is plausible two TCs of model 1 are required in Z. Thus a measure to capture phenotypic information i.e. the feasible area for each task, for distance calculations between two individuals can result in better results. The presented approach in this study is part of an ongoing project and the developed code to select TC is yet to be validated on a real construction project. The code will be made available in public interest but only after validation. Till then any request in this regard can be made directly to the authors.

## 6 Conclusion

Fitness sharing method for Genetic Algorithms was used for multimodal optimization problem of TC selection. The proposed methodology in the study was particularly aimed at finding cost effective combinations of TC from the available models without restricting to finalization of a single model based on aggregate site demands or heuristics. The results produced alternative solutions for TC selections, which provided varied feasible solutions for the user to choose for location

optimization and subsequent simulations. The process of planning site utilization involves intertwined tasks. This calls for addressing interdependencies between these tasks. Therefore as part of an ongoing project, the presented approach is sought to be integrated with site layout planning problem where positioning of temporary facilities would be dealt. These positions would be taken up as input for the demonstrated approach in this study and is expected to result in a much robust solution.

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