Parametric or Non-parametric? Understanding the Inherent Trade-offs between Forms of Object Representation

Chris Rausch[a], Yinghui Zhao[a], Carl Haas[a]
[a]Department of Civil and Environmental Engineering, University of Waterloo
E-mail: chris.rausch@uwaterloo.ca , yinghui.zhao@uwaterloo.ca, chaas@uwaterloo.ca

Abstract –

The need for efficient computing in construction modelling and analysis workflows often requires making tradeoffs between the inherent advantages and disadvantages of different datatypes being generated and managed. This is notably observed in geometry reconstruction (e.g., scan-to-BIM, reverse engineering, etc.), where a tradeoff often occurs between computational efficiency and the choice between increased semantic enrichment or increased representational accuracy (often both cannot be achieved simultaneously). This dichotomy can be generalized as a choice or tradeoff between parametric and non-parametric object representation forms. This paper presents a simple conceptual model for characterizing the datatypes used for representing and defining construction geometry to understand key tradeoffs that exist. First, we survey existing literature across multiple domains to identify and distill key attributes used in characterization according to the terms parametric and non-parametric. Then, we develop and illustrate a conceptual model using an analogy to mathematical expressions vs. discrete digital approximations employed in computer vision (e.g., Gaussian kernel, Hough Transform, and Scale Invariant Feature Transform (SIFT) algorithm). Finally, we outline future research opportunities for improving the state of object representation.

Keywords –

Building Information Modelling; 3D Point Clouds; Data Managements; Geometric Accuracy; Digital Twin; Construction Geometry

1 Introduction

In construction workflows, the virtual representation of physical objects forms the basis for design, communication of product and process requirements, and as-built verification of constructed works. When objects are represented by a set of simple algebraic primitives such as curves, planes, and polysurfaces, an object is said to have a "parametric" form [1]. In contrast, when objects cannot be accurately represented by such primitives, and rely on implicit algebraic forms (e.g., surface normal histograms) or large, non-semantic data structures (e.g., point clouds and meshes), objects are said to be "non-parametric" [2,3]. Computer-aided design (CAD) and building information modelling (BIM) favor parametric representations [2,4], while workflows used for digitization of as-built objects (using tools such as photogrammetry and laser scanning) favor non-parametric representations [3,5]. Practitioners require specific object representations for different purposes across the construction life-cycle. As a result, across the lifecycle of any given project there can exist a wide range of construction elements, components, assemblies, and conditions exhibiting or being represented by parametric and non-parametric forms. This dichotomy exists due to (1) the nuance of methods employed for object representation, (2) inattention or inability to control dimensional variability during construction, and (3) the diversity of geometric forms and conditions that exist across various systems in the AEC industry.

1.1 Geometric Challenges in AEC Related to Parametric and Non-parametric Forms

The premise that geometry can be characterized by a mix of parametric and non-parametric forms has unique implications in Architecture, Engineering, Construction (AEC) that few studies have investigated in detail, but that continues to create challenges for mediation and resolution efforts. Many geometric and topological challenges can be traced to an inability to mediate effectively between parametric and non-parametric domains, as evidenced in the following examples:

- **Existing conditions characterization.** The challenge in these workflows is the process of fusing parametric and non-parametric data stemming from existing conditions and new components or assemblies. This is evidenced particularly in adaptive building reuse where data used to represent existing conditions is most accurately facilitated through non-parametric data. Parameterization of unstructured data can be performed, but often results in a loss of representational accuracy. Examples of as-built conditions which are difficult to parameterize include large concrete structures, beam camber, rail track levelness, or curvature of free-formed bespoke architectural features.

- **Updating BIM to reflect the as-built status**. The process of updating BIM during construction is complex and subject to varying degrees of accuracy for geometric representation. When objects deviate outside of allowable tolerance thresholds and cannot be accurately or easily represented using parametric primitives, updating the as-designed BIM becomes time-consuming and error-prone.
- **Abstraction of geometric deviations**. While the comparison of 3D sensed data (e.g., point clouds) with BIM has been relied upon in industry for many uses cases, the challenge that still exists is localization, interpretation, and abstraction of discrepancies. While a 3D point cloud can be overlaid on a 3D model to produce a map of geometric deviations, this analysis by itself does not translate into distinct kinematic (i.e., parametric) deviations or non-kinematic (i.e., non-parametric) deviations. The challenge of sensing and interpreting geometric deviations is especially important in repetitive assembly workflows, such as modular construction, where parametric corrections to assembly geometry configuration (i.e., location, orientation, size of objects, etc.) can have a profound impact on manufacturing efficiency and reduction of rework due to dimensional variability.
- **Mixed-form object assembly planning**. Challenges in object assembly planning arise due to varying levels of object regularity, manufacturing processes and dimensional variability. For instance, the assembly of manufactured products into "stick-built" buildings can be particularly challenging since manufactured products are often characterized as being highly parametric with low dimensional variability, while site interfaces are often non-parametric with a high degree of variability [6]. Another example of mixed-form object assembly planning is the optimal packing of irregularly shaped objects into containers, which occurs in nuclear waste disposal. In this case, irregularly shaped objects exhibit non-parametric representations while regularly shaped objects (containers) exhibit parametric representations [7]. These types of packing scenarios are challenging to derive optimized solutions for.

1.2 Problem Statement and Contribution

In light of the challenges with having a mixture of parametric and non-parametric representations in various AEC workflows, this research provides a better understanding into the trade-offs that exist. In some cases, parametric representations might be preferred, while in other cases non-parametric representations might be necessitated. Current classifications and definitions for parametric vs. non-parametric datatypes are verbose, and as a result of the different requirements for datatypes in AEC, navigating this delineation is challenging.

This contribution of this paper is as follows. First, a review of existing classifications for parametric vs. non-parametric entities is explored from a systems-, geometric schema-, semantic information-, and associative modelling-based standpoint. Then, a simple conceptual model is proposed, which captures the intrinsic trade-offs between these representation forms. A demonstration is used to demonstrate how such a model is efficacious for describing these trade-offs. Finally, the implications of this model are discussed, to provide a better understanding into decision making regarding the handling of geometric data in AEC.

2 Delineating between Parametric and Non-Parametric Entities

2.1 Systems Context

The distinction between parametric and non-parametric systems is verbose and multi-variate. In mathematical modelling of systems, the distinction between parametric models and non-parametric models lies with fixity and immutability of system parameters. Parametric models have fixed non-changing parameters for system characterization, while non-parametric models assume that a fixed set of parameters cannot be used for proper system characterization [8]. This notion also appears in robust parameter design [9] where systems are characterized in terms of controllable design aspects, or parameters, and noise variables which are much more challenging and sometimes not feasible to control.

In some cases, non-parametric systems are viewed as having an infinite number of parameters, and since only a finite number of these parameters are used for modelling these systems, the output can change with the same input [10,11]. It should be noted that a clear association of, and connection between the terms *parametric*, *non-parametric*, *stochastic* and *deterministic* for codifying systems cannot be made. For instance, parametric models can be stochastic if the data being observed fits a known distribution with constant variance, and variables are numerical and continuous [12].

Systems can also comprise both parametric and non-parametric attributes, and thus can be considered semi-parametric. The use of *semi-parametric* for domain classification also appears in the context of regression analysis, where a combination of linear and non-linear regression can be useful for statistical inference [13]. The use of localized variables and global parameters define model components that do not change (i.e., global parameters of an entire population), and model components that do experience variation (i.e., local variable instances of a sample from a population). In addition to describing system certainty, these classes also describe system scale and provide a distinction between design vs. observational attributes.

In summary, the distinction between parametric and non-parametric systems is multi-variate across domains and is based on several factors including modelling approach, system certainty, design vs observational data, and system scale (Figure 1).

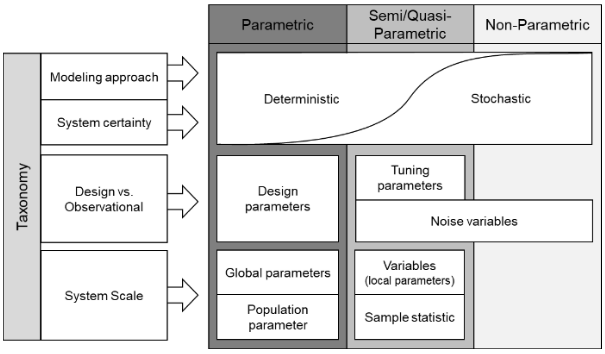

Figure 1. Taxonomy of systems classification in terms of modelling approach, system certainty, design vs. observational data, and system scale.

2.2 Geometric Schema Context

The representation of object geometry using distinct geometric schemas is also verbose and oftentimes unclear. Classification of these schemas are often based on *ambiguity*, *directness*, and *compactness*.

Unambiguous (or complete) representations are used to describe the entirety of a physical object, and underlying primitives or descriptors can be inverted to recreate the exact same object being represented. Unambiguous approaches can be used to provide a one-to-one mapping between objects and their representation [14,15]. Ambiguous representations are often used to distinguish between objects in a very efficient manner [15], even though the descriptors used cannot provide a one-to-one mapping between objects and their representation. Within unambiguous representations, methods can be further classified into implicit and explicit representations. The key difference between these two approaches is the directness for computing objects. Indirect representations use intermediate geometric descriptors such as histograms of normals or curvature to describe objects [1] as compared to explicit methods which directly describe objects using surface (e.g., polysurfaces, meshes), or volumetric (e.g., constructive solid geometry) descriptors [1,5].

A final division between approaches is parametric vs. non-parametric representations. While explicit methods can be either parametric or non-parametric, implicit methods are distinctly not of a parametric form [16]. Parametric representations are more algebraically refined than non-parametric methods, however, they can be more computationally intensive to perform operations on [2]. While this may not be the case for simple primitives such as lines, circles and planes, it is especially true for representing complex geometry, using formats such as polysurfaces, Bezier curves, B-Splines, Non-Uniform Rational Basis Spline (NURBS), and piecewise functions. Non-parametric representations, in contrast, are more computationally efficient, but are more difficult to achieve exact geometric representation. A non-parametric representation can be implicit as in the case of differential properties of a surface of a given location [1], or can also be explicit as in the case of polygonal meshes. The conversion between parametric and non-parametric representations is discussed in [16]. The conversion from parametric to non-parametric is defined as *implicitization* and it is possible to perform for any rational parametric surface of curve. The reverse process, *parameterization*, is not as easy to execute and is not always possible to perform for higher-order descriptors. Parametric representations have become the “quasi-standard” for CAD modelling [2] due to algebraic topology capabilities, while non-parametric representations are preferred in machine vision and as-built modelling systems [5,15] due to computational efficiency.

Despite the existence of classifications for representation schemas such as the one shown in Figure 2, exact definitions and distinctions are at times fuzzy and inconsistent. For instance, some studies state that explicit representations can be either parametric or non-parametric [1,3], while other studies state parametric approaches are always based on implicit methods of describing geometry [17]. This fuzziness stems from the fact that some classifications refer to the datatype itself as the entity being characterized, while other classifications refer to the process taken to derive a datatype as the entity being characterized. In addition, it is difficult to understand clear distinctions between the nature of datatypes with respect to parametric vs. non-parametric attributes. For example, it is well regarded that a representation scheme that allows for modifications of its underlying variables (e.g., control points in NURBS) is parametric. However, the same argument could theoretically be made for meshes (i.e., control vertices can be modified), yet these are clearly defined as being non-parametric schemas. Perhaps there is a theoretical limit to the number of control variables in a representation scheme whereby it starts to become non-parametric. However, no such definitive limit has been (or perhaps can be) defined, which further adds to the existing fuzziness of classification.

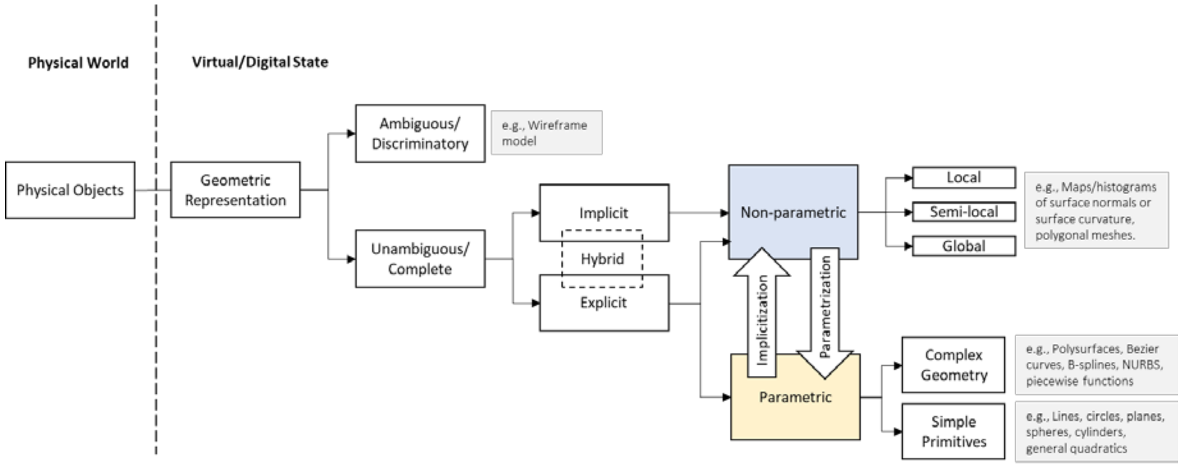

Figure 2. Classification of geometric schemas used to represent physical objects in AEC according to ambiguity, directness, and compactness (adapted from [3] using [1,2,5,14-16]).

2.3 Semantic Information Context

From a building information modelling standpoint, the classification of objects as being parametric extends beyond just representational form and has unique attributes to facilitate use across the entire construction lifecycle. Parametric BIM objects must also include semantic information (parameters) in the form of associated data, rules, topology and material-specific data [4]. Parameters are also used to classify objects into categories, families, types and instances, which is stored as data in the form of text, integers, numbers, area, volume, angles, URLs, or binary data [18]. Across this wide definition of parametric objects in BIM, attributes related specifically to geometry and topology are investigated in this research since they directly affect geometric mediation between objects. Topology describes the spatial relationships between elements that do not change based on changes to geometric parameters of those elements [19]. Topology can relate to the relationship between features of an element (e.g., face to edge), the relationship between elements (e.g., beam to column) or relationship between groups of objects or spaces (e.g., room to room). Topology plays a key role in the way architects and engineers understand the function and expected behavior of building elements. Topology, geometric representation and material properties are distilled into the “semantics” of an object, which can be interpreted as the form, function and behavior of objects and systems of objects [20]. Current modelling practice in construction emphasizes the creation and preservation of semantics by explicitly outlining that building information models must contain parametric intelligence, topological relationships and object attributes [21], otherwise, they are considered no more than 3D geometric models.

2.4 Associative Modelling Context

A fourth context can also be used to describe the delineation of parametric vs. non-parametric entities in AEC. Parametric modelling (or parametric design) involves the use of geometric rules and constraints to embed explicit domain knowledge into BIMs and provides a way for automated design regeneration via an “associative” model [22]. While any CAD modelling system can contain a parametric representation of an object, the following characteristics differentiate parametric modelling systems: users can define custom relationships between features and objects, parameters between objects can be integrated into a system (i.e., a parameter of one object can be used for defining parameters on other objects), parametric definitions are compatible in a system or are otherwise mutually exclusive such that no two parameters create conflicting relationships; geometry should be object or feature-based [22]. This approach to modelling has become very popular in the manufacturing industry and is seen in software such as SolidWorks®. Constraint-based design requires all dimensions of features and parts to be parametrically defined, constrained and related to other features [23], or it is otherwise considered under-constrained, and cannot be realized [24]. While this degree of parametric constraints is not employed in most AEC workflows, the use of associative model does provide a source of additional parametric properties that can be exploited.

2.5 Summary

Exploring a range of contexts reveals a verbose and multi-variate landscape for describing the delineation between parametric and non-parametric entities. Selecting a suitable object representation (whether parametric or non-parametric) extends beyond just the geometric schema employed, includes semantic information requirements, and might include associative relationships to other objects. In a systems context (2.1), the delineation between parametric and non-parametric is fuzzy (with an intermediate category sometimes being used), and this also translates down to the object level. However, it is clear from the examples of cases where both datatypes exist in AEC, that assessing trade-offs is useful for understanding the inherent benefits and constraints to each datatype. In some cases, parametric object representations are preferred, while in other cases, non-parametric object representations might be required.

3 Proposed Conceptual Model

This research presents a conceptual model to help summarize the key trade-offs between parametric and non-parametric object representations. This model is based on the distinction between analytical expressions and digital approximations that appear in computer vision for pattern and shape recognition processes. As depicted in Figure 3, this model classifies parametric object representations as having many relations between individual datapoints or entities, and are by nature more abstract, while non-parametric object representations have a greater number of unstructured datapoints and are by nature more discrete or approximate.

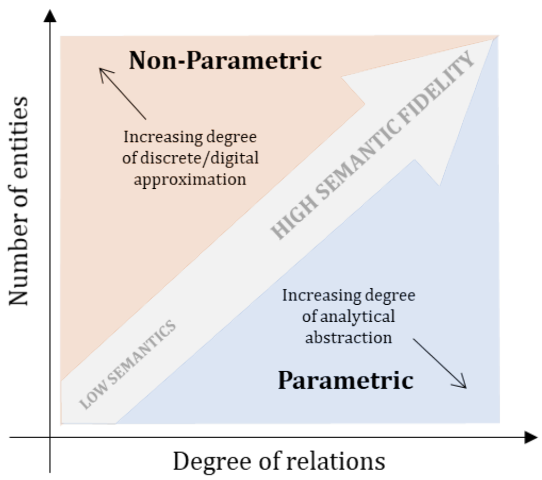

Figure 3. Conceptual model for characterizing the delineation of parametric and non-parametric object representations

3.1 Analogy to Pattern Recognition Techniques

In computer vision, various techniques for pattern recognition can be used to resolutely employ digital approximations of analytical expressions for recognition.

One of the most common methods for shape detection in computer vision is the Hough Transform which detects curves by exploiting the duality between points on a curve and parameters of that curve [25]. It works by first considering the analytical expression of a curve, often in the parametric line form (r, θ), and edge segments of an image given by (x_i, y_i). The transform is then implemented by discretizing the Hough parameter space over finite intervals of the image, or "accumulator cells". This technique takes an inherently parametric mathematical expression and discretely approximates it in a non-parametric manner to extract and transform information from an image (which itself can also be defined as a non-parametric data source).

Gaussian kernels are another example of discrete approximation used in computer vision to perform an analytical transformation on an image (Figure 4). A more general case of this is convolutional kernels and filters, which transform an expression into a discrete nxn matrix and is convoluted over an image. Discrete approximations such as kernels are efficacious in complex algorithms such as Scale Invariant Feature Transform (SIFT) due to the ability to perform analytical operations in a computationally efficient manner.

In the same way that many pattern recognition techniques transform analytical expressions into discrete approximations, this same analogy can be used to delineate between parametric and non-parametric object representations. On one hand, we can postulate a trend that the more abstract an expression or representation is, the fewer entities are required to relate components together. Conversely however, when these relations are broken and discretized, more discrete values are required to perform a similar level of approximation compared to its analytical counterpart.

Analytical Expression   Discrete Approximation

$$h(u,v) = \frac{1}{2pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}} \longrightarrow H[u,v] = \frac{1}{273}\begin{bmatrix} 1 & 4 & 7 & 4 & 1 \\ 4 & 16 & 26 & 16 & 4 \\ 7 & 26 & 41 & 26 & 7 \\ 4 & 16 & 26 & 16 & 4 \\ 1 & 4 & 7 & 4 & 1 \end{bmatrix}$$

Figure 4. Analytical Gaussian Expression and its Digital Approximation (i.e., 5x5 Gaussian Kernel)

3.2 Measuring the Degree of Semantic Fidelity

The final component to the proposed model is characterizing the degree of semantic information encapsulated in an object representation. The greater the number of entities and degree of relations between those entities, the greater the semantic fidelity of a representation. In practice, there is a trade-off that occurs between parametric and non-parametric object representations with respect to the degree of semantic encapsulation. This is perhaps most evident when representing “as-is” objects. While non-parametric object representations are positioned better for obtaining a higher degree of representational accuracy, they cannot be semantically enriched to the same level as parametric object representations. Despite the high level of abstraction in parametric representations and the significant advancements being made to leverage better parametric modelling approaches that maintain representational accuracy of as-is objects, the inability to achieve the same degree of representational accuracy as non-parametric representations restricts its ability to achieve the highest level of semantic fidelity. As such, a notable trade-off occurs between parametric and non-parametric representations with respect to semantics.

4 Demonstrating the Conceptual Model

Previous research has provided information for assessing the trade-offs between geometric schemas (Table 1). Using this breakdown, a simple demonstration can be carried out for the representation of an I-beam element (Figure 5) to show how parametric representations tend to have fewer entities, which are more tightly related. The first digital representation in this figure is a point cloud, which can be obtained by performing sampling reconstruction of existing mathematical descriptions or from reality capture [26]. The other digital representations which are more structured than point clouds are triangular and polygonal (tessellated) meshes. As shown, the point cloud representation has 7296 datapoints (i.e., XYZ points), whereas the triangular mesh contains 1740 datapoints (i.e., triangle vertices), and in the most simplified case, the polygonal mesh structure has 108 datapoints (i.e., polygon vertices). These digital representations are also considered to be non-parametric. As opposed to regular shapes (e.g., rectangular, cylindrical, prismatic, etc.), non-parametric forms cannot be defined parametrically using a shape type and a limited set of parameter values that specifies the object [1]. Figure 5 also depicts two common mathematical representations. Non-Uniform Rational Basis Spline (NURBS) is a common boundary representation, which uses a series of surfaces to completely enclose and represent a given shape. Constructive Solid Geometry (CSG) is a mathematical representation that describes the volume of an object through use of Boolean operations (e.g., addition and subtraction) of simple geometries to create more complex shapes. While CSG has been the preferred method for representing geometry in building information models (BIM) due to its simplistic data structure [1], there are many applications where NURBS are preferred, since it can describe complex geometry more appropriately [21,27]. As shown for the steel beam, the NURBS geometry contains 56 datapoints (i.e., control points), whereas the CSG geometry contains 9 datapoints (3 extrusions built with 3 control points each).

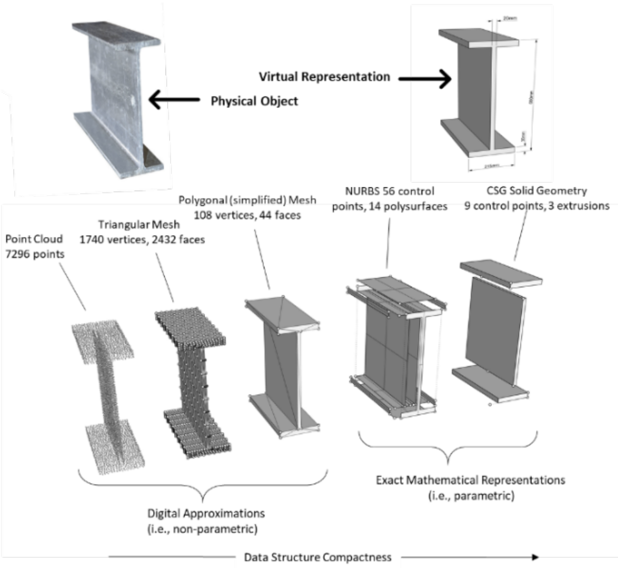

Figure 5. Digital and Mathematical Representations for a Physical Object (Steel Beam Element).

Table 1. Summary of the key trade-offs between geometry representations employed in AEC

Geometry Representation	Geometry Kernel Processing Demand	Continuity	Representational Accuracy (As-is)	Semantic Richness	Exactness for Complex Geometry	Sources
Point clouds	Slow (high computational effort)	Discretized	High	Low	Low	[28]
Mesh	Fast (no interpretation required)	Discretized	Med-High	Low	Med	[28]
BREP	Med (interpretation required)	Exact, continuous	Med	High	High	[17,28]
CSG	Med (interpretation required)	Exact, continuous	Low-Med	High	High	[28]
NURBS	Med (interpretation required)	Exact, continuous	Med-High	High	High	[28]

The general trend for this simple example is that as representation moves from digital to mathematical, it becomes more compact, with fewer entities that are abstractly related. In addition, we can plot each of these geometric descriptors on the conceptual model (Figure 6). One hand the point cloud representation is the most non-parametric with the highest number of entities (which are not related), while the CSG representation is perhaps the most abstract parametric representation. While not directly considered in this example, the Boundary Representation (BRep) geometric schema is another parametric representation which arguably has the highest semantic encapsulation across all geometric schemas. This is because the BRep schema is based on a hierarchical topological structure, with explicit relations between bodies, faces, edges and vertices (refer to [17] for a more detailed breakdown of this schema).

While this example demonstrates how geometric schemas can be characterized using the proposed model, the purpose of the analysis is not to provide a comprehensive (or exhaustive) classification of all possible schemas. However, certain schemas are better suited for more semantic fidelity than others. For instance, given how NURBS and BRep can be discretized by adding additional control points without changing the initial geometry of an object, these representations potentially have the ability to harness the semantic fidelity requirements of a given application as opposed to those of CSG or non-parametric representations.

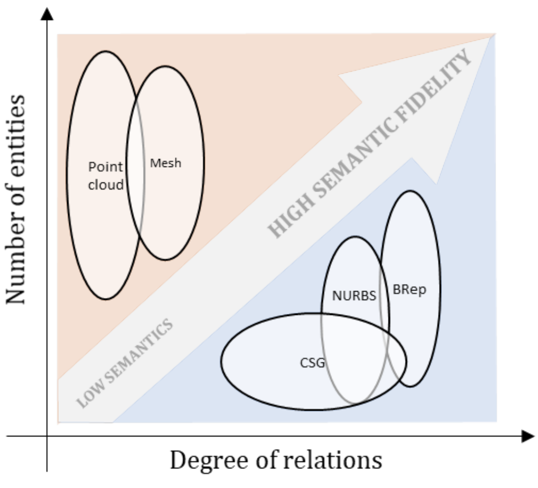

Figure 6. Depicting the Conceptual Model Using Geometric Schemas

5 Conclusions

Within AEC, the choice of object representation form invokes trade-offs with respect to computational efficiency, representational accuracy, and semantic enrichment. Since there remains to be one ubiquitous object representation that can simultaneously meet all of these objectives, the choice of representation will continue to require making trade-offs. Review of existing classifications for object representation reveals that there can be a significant degree of verbosity and ambiguity, further compounding the task of selecting a suitable representation. This paper demonstrates that approaching classification through a parametric vs. non-parametric lens is useful for providing key insight required in selection of a desired object representation. A conceptual model is established by considering two dimensions of an object representation: number of entities used to represent an object and the degree of relations between those entities. These dimensions are fundamental in computer vision for applications such as pattern recognition and shape detection. For instance, the discretization of analytical expressions is a key component to achieving computational efficiency in methods such as the Gaussian kernel, Hough Transform, and Scale Invariant Feature Transform (SIFT) algorithm. In addition to geometric schemas, other attributes can also be described by the proposed model, namely semantic enrichment and associative modelling relationships. These dimensions clearly fit into a parametric vs. non-parametric context within AEC workflows. A functional demonstration using various geometric schemas reveals the trend that parametric representations have a high degree of analytical abstraction, while non-parametric representations have a high degree of digital approximation.

5.1 Future Research Opportunities

In practice, there are numerous workflows which require either parametric or non-parametric representations due to current trade-offs. While this trend is expected to continue, there are opportunities to bridge this gap as evidenced in the following areas of research:

- **'Geometry as a feature'.** Rather than defining objects as an abstraction of geometry, geometry can be viewed as a feature of object. This is already the premise of object-oriented modelling (and is used in IFC, for instance), yet a more concerted focus on this concept can allow for multiple geometric schemas to be used to represent the same object without necessitating the landscape of parametric vs. non-parametric trade-offs.
- **Towards improved geometric schemas.** Current limitations and trade-offs can be addressed by developing new schemas that do not have the limitations of existing schemas. One example is developing a global or 'master' schema that can be modified or updated throughout the lifecycle of AEC projects as required.
- **Developing parametrization methods that preserve fidelity of information.** Rather than converting non-parametric datatypes into a

parametric form at the expense of loss of representational fidelity and semantic information, it is possible to discretize initial parametric forms (e.g., such as NURBS) in such a way that can achieve suitable accuracy while maintaining semantic fidelity.

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