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PLANNING FOR AUTOMATIC EXCAVATOR OPERATIONS

A.J. Koivo
School of Electrical Engineering, Purdue University
West Lafayette, IN 47907-1285

ABSTRACT

The operations of an excavator can be automated by making a digital computer control its motion so that the system functions autonomously. In order to realize the automatic operations the nominal (desired) trajectory for the motion of the excavator must be preplanned and stored in the computer. This trajectory should specify the pose (the position and orientation) of the bucket as a function of time in a fixed (Cartesian) coordinate system. Since the pose of the bucket is determined by the configuration of the excavator, the planned trajectory should then be converted into the joint space in order to obtain the nominal trajectories for the joint variables corresponding to the Cartesian space trajectory. This conversion can be accomplished by solving the kinematic equations for the excavator in the backward direction.

The kinematic equations relate the pose of the bucket to the joint variables of the excavator. They can conveniently be written if the local (moving) coordinate frames are first assigned to all links (joints) by following the Denavit-Hartenberg procedure which is a well-known method in robotics. Then after the structural (kinematic) parameters have been determined for the given excavator, the transformation matrices relating any two adjacent coordinate frames can be written. The recursive equations are then utilized to obtain the kinematic equations which relate the joint variable values to the bucket pose. These trigonometric equations can then be solved for the joint variables in terms of the given pose of the bucket. We will present these equations which represent the solution to the inverse kinematic problem for the excavators. These relations are then used to plan the motion of the excavator bucket. The problem of removing soil contained in a sector to a specified depth is then solved. The trajectory for the bucket is planned so that this task will be accomplished.

1. INTRODUCTION

The semi-autonomous or automatic computer control is essential to improve the productivity and the effective utilization of expensive construction machines [1,2]. Many tasks on construction sites can then be performed even in hostile and/or unfavorable conditions, for example, in severe (cold or rainy) weather, or in hazardous, unhealthy, and even poisonous environmental conditions. Moreover, the automatically operated machines can often perform a task faster and with better precision than manually operated machines. In order to automate the operations of these construction machines, the machine motions must be carefully planned. The generation of the nominal trajectory for the bucket of an excavator and thus for the joint variables of the excavator is presented here. The basic approach to be discussed can also be applied to backhoes and various loaders commonly used to transfer ground material in various work areas such as at construction sites, timbering and mining places.

The usual task of an excavator (backhoe) is to free and/or remove surface material (e.g., soil, coal) from its original location and to transfer it to another location by pushing, pulling, and/or lifting it in a bucket. The execution of this task is usually performed by a human operator who controls the motion of the machine manually by using the visual feedback provided through his/her own eyes. In many current applications of excavators (backhoes, loaders), the semi-autonomous or even automatic operation of the machine is desirable and sometimes even necessary; for example, in the transfer of mining products from underground sites or in the removal of poisonous or radioactive wastes and/or explosives. In the semi-autonomous operation, a human tele-operator may guide the motion of the machine over a certain time period so as to make it perform parts of the task and then a computer may control the machine over the rest of the duration of the motion to execute the other parts of the task automatically. In the automatic computer-controlled motion, the bucket of the excavator follows a path specified by the position and the digging angle, i.e., the pose of the bucket, which corresponds to specific values of the angular positions of the joint shafts. The values of these joint variables, in turn, are determined by the lengths (positions) of the hydraulic actuators. The mathematical relations between these variables are described by the kinematic relations of the machines.

The previously reported studies on excavators are mainly qualitative. An entire system for the automatic or semi-automatic operation of an excavator is described in [3]. It includes a hydraulically driven excavator, teleoperation, computer control, which is supplemented with manual override capability, position and force feedback control. Kinematic relations between three joint angles and the position of the bucket in a fixed coordinate system are presented in [3]; specifically, the forward and backward kinematic equations are described using the geometric configuration of the excavator arm in a fixed (world) coordinate system. However, no other details on the system are given. The use of a position and sensor (force and vision) feedback is also discussed qualitatively in [3] without presenting technical details. Similarly, a vision feedback system for an excavator designed for rapid runway repairs is presented in [4] by qualitatively characterizing the system components. This system has successfully been tested in practice, but no technical details are presented in [4]. Although the forces between the soil and a tool (bucket) during the digging operations are studied in [5,6], the kinematic relations between the bucket pose, the joint variables, and the lengths of the actuators are not described in [5,6].

An attempt to systematically describe the kinematics of an excavator is presented in [7] by adopting the procedures commonly used in robotics [8]. It outlines a general method for establishing the forward and backward kinematic relations between the pose of the bucket and the angles of the joint shafts for an excavator. However, no relations between the joint shaft angles and the positions (lengths) of the hydraulic actuators are given. Thus, the kinematic relations presented in [7] do not completely describe the configuration of the machine.

The work reported here presents the details of assigning the local and world coordinate systems for an excavator by following the Denavit-Hartenberg (D-H) guidelines [8]. The assignment is systematic and different from that described in [7]. The homogeneous transformation matrices relating any two adjacent coordinate frames in the system are given. The forward and backward kinematic equations between the pose of the bucket, the angular positions of the joint shafts and the lengths of the hydraulic actuators are presented. These relations are then used in the planning of the nominal path for the bucket motion.

2. COORDINATE FRAME ASSIGNMENTS

In order to analyze and plan the motion of the excavator (backhoe or loader) for performing a specific task, it is necessary to define a world coordinate system to describe the pose of the bucket (the end-effector). A fixed rectangular and right hand coordinate system $X_w Y_w Z_w$ is chosen and its origin is placed to an arbitrary point on the ground level in the workspace of the excavator. It is then convenient to define local coordinate frames for all links (joints) by following the Denavit-Hartenberg (D-H) guidelines [8] which are commonly used in robotics. The resulting coordinate frames are shown in Figure 1.

After the coordinate frames have been assigned, an arbitrary point on the excavator can be represented in any of the chosen coordinate systems. In fact, if an arbitrary point in the ith coordinate frame is $p_i = [p_{ix} \;\; p_{iy} \;\; p_{iz} \;\; 1]^T$ where one as the fourth component has been added, for convenience, as a scaling factor, then the same point in the (i−1)st coordinate frame is p_{i-1}. These two representations are related by the homogeneous transformation matrix A^i_{i-1} as follows

$$p_{i-1} = A^i_{i-1} p_i \tag{1}$$

where i = 1,2,3,4. Matrix A^i_{i-1} is conveniently obtained for the chosen coordinate frames by using the structural kinematic parameters of the machine. Their application to the excavator in Figure 1 gives the values shown in Table 1.

Table 1
Structural kinematic parameters

link (joint)	d_i	a_i	α_i	θ_i
1	0	a_1	90	θ_1
2	0	a_2	0	θ_2
3	0	a_3	0	θ_3
4	0	a_4	0	θ_4

The homogeneous transformation matrix in equation (1) is in the general form as follows:

$$A_{i-1}^{i} = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The substitution of the parameter values of Table 1 for i = 1,2,3,4 into equation (2) gives the homogeneous transformation matrices A_0^1, A_1^2, A_2^3 and A_3^4.

3. FORWARD KINEMATIC RELATIONS

For the automatic operation of an excavator, it is desirable to place the bucket to a specified location. It can be accomplished by selecting the lengths of the hydraulic actuators (cylinder-piston), and thus the shaft positions of the joints properly. The mathematical expressions that relate the position and orientation of the bucket to the shaft (joint variable) positions and then to the lengths of the hydraulic actuators are called the kinematic (position) equations. If the lengths of the actuators or the joint variable angles are given, the position and orientation (pose) of the bucket are determined by the *forward* kinematic equations. If the position and orientation of the bucket are specified, the joint variable angles corresponding to this bucket pose and the lengths of the actuators are calculated from the *backward (inverse)* kinematic equations. The forward kinematic equations will next be developed for an excavator (backhoe).

3.1. Equations Relating Joint Shaft Angles to Bucket Pose

The location of the bucket of the excavator in Figures 1 and 2 can be specified by locating the rotational axis of the bucket motion, i.e., by giving the coordinates of point $O_3 = D$, the origin of the third coordinate frame. To obtain the coordinates of point D in the base coordinate system, equation (1) can be applied recursively for i = 1,2 and 3. It follows that the coordinates of point D in the base coordinate system are

$$p_0^D = (A_0^1 A_1^2 A_2^3) p_4^D = A_0^3 p_3^D \quad (3)$$

where vector $p_3^D = [0 \;\; 0 \;\; 0 \;\; 1]^T$ specifies point D in the third coordinate frame and matrix $A_0^3 = A_0^1 A_1^2 A_2^3$ can be calculated on the basis of the expressions given in equation (2). Specifically, equation (3) can be calculated to obtain the coordinates of point D in the world coordinate system:

$$p_0^D = [c_1(a_1+a_2c_2+a_3c_{23}); s_1(a_1+a_2c_2+a_3c_{23}); a_2s_2+a_3s_{23}; 1]^T \quad (4)$$

where $c_{23} = \cos(\theta_2 + \theta_3)$ and $s_{23} = \sin(\theta_2 + \theta_3)$. If the values of the joint variables θ_1, θ_2 and θ_3 are known, the location of the point D in the base coordinate system can be determined by equation (4).

For a given digging task, a fixed world coordinate system $X_wY_wZ_wO_w$ with origin at point O_w is chosen. After the excavator has been driven to a fixed location for executing a task, the origin O_o of the base coordinate system $X_oY_oW_oO_o$ attached to the excavator base structure can be located in the world coordinate system. The position vector of point O_o in the world coordinate system is denoted as $p_w^{ob} = [p_{wx}^{ob} \;\; p_{wy}^{ob} \;\; p_{wz}^{ob} \;\; 1]^T$. Then the homogeneous transformation matrix A_w^0 that relates the base coordinates to the world coordinates is specified as

$$A_w^0 = \begin{bmatrix} 0 & 0 & 1 & p_{wx}^{ob} \\ 1 & 0 & 0 & p_{wy}^{ob} \\ 0 & 1 & 0 & p_{wz}^{ob} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

The location of point D on the rotational axis of the bucket can now also be expressed in the fixed world coordinate system as

$$p_w^D = A_w^0 p_0^D \quad (6)$$

Thus, the position of point D in the world coordinate system can be calculated by combining equations (3) through (6), if the joint variable values θ_1, θ_2 and θ_3 are known.

The coordinates of the center point $N = O_4$ on the bucket edge can also be determined in the base coordinate system by the successive application of equation (1). One obtains

$$p_0^N = A_0^4 p_4^N \tag{7}$$

where $p_4^N = [0 \ \ 0 \ \ 0 \ \ 1]$ represents the origin (O_4) in the fourth $X_4Y_4Z_4O_4$ coordinate frame, p_0^N is the same point expressed in the base coordinate system of the excavator and $A_0^4 = A_0^1A_1^2A_2^3A_3^4$ is the homogeneous transformation matrix that relates the vector of the fourth coordinate frame to a vector expressed in the base coordinate system. Specifically, the A_0^4-matrix can be calculated by using equation (2):

$$A_0^4 = \begin{bmatrix} c_1c_{234} & -c_1s_{234} & s_1 & c_1(a_4c_{234} + a_3c_{23} + a_2c_2 + a_1) \\ s_1c_{234} & -s_1s_{234} & -c_1 & s_1(a_4c_{234} + a_3c_{23} + a_2c_2 + a_1) \\ s_{234} & c_{234} & 0 & a_4s_{234} + a_3s_{23} + a_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

where $c_{234} = \cos(\theta_2 + \theta_3 + \theta_4)$ and $s_{234} = \sin(\theta_2 + \theta_3 + \theta_4)$. It should be noticed by equation (7) that the components of p_0^N, the coordinates of point N in the base coordinate system are given by the fourth column of matrix A_0^4 in equation (8).

When the joint variable values θ_i, $i = 1,...,4$ are known, the position and orientation of the center point O_4 on the edge of the bucket in the base coordinate system can be calculated by equations (7) and (8). These equations represent the first part of the forward kinematic equations for the excavator.

It should be noted that the joint variable θ_1 stays usually constant during the execution of a digging task, i.e., the excavator arm moves on a vertical plane. Moreover, the orientation of the bucket can be specified by the value of the joint variable θ_4 relative to the X_3-axis or by the angle $\theta_2 + \theta_3 + \theta_4$ relative to the X_1-axis. The same information can also be conveyed by the unit vectors of the coordinate frame $X_4Y_4Z_4O_4$ (corresponding to vectors n, s, a, commonly used in robotics), which are expressed in the base coordinate frame.

3.2. Equations Relating Lengths of Hydraulic Actuators to Joint Shaft Angles

The actuators of an excavator (backhoe) consist of the hydraulic cylinders in which the pressures on the pistons are controlled. The lengths of the actuators determine the angles of the joint shafts, and thus the configuration of the excavator and the pose of the bucket. The length of a hydraulic actuator is specified by the line segment between the attachment points; for example, length BE for actuator one (Figure 2). The kinematic equations relating the lengths of the hydraulic actuators to the angles of the joint shafts are next presented.

Actuator one (may not be hydraulic) rotates the base. It determines directly in the base coordinate system the vertical plane (θ_1 = constant) on which the operation of the excavator (or backhoe) takes place.

The hydraulic actuator two (Figure 2) causes the rotational motion about joint two. Its length L_{BE}, i.e., the length of segment BE, is related to the joint variable θ_2 by the following expression:

$$L_{BE}^2 = [L_{AB}\sin(\theta_2 + \beta) + L_{AH}]^2 + [L_{AB}\cos(\theta_2 + \beta) - L_{HE}]^2 \tag{9}$$

where β is the constant angle between line segments BA and AC. The subscripts attached to length L refer to endpoints of the line segment whose length is being indicated. L_{AB}, L_{AH} and L_{HE} have specific constant values for a given excavator (backhoe, loader).

If the length L_{BE} of actuator two is known, equation (9) can then be used to determine the joint variable θ_2. Indeed, the trigonometric equation (9) can be solved for θ_2 by the standard method to obtain:

$$\theta_2 = -\beta - \tan^{-1}(L_{AH}/L_{HE}) + \tan^{-1}\{h_1^2/[4L_{AB}^2(L_{AH}^2 + L_{HE}^2) - h_1^4]^{1/2}\} \tag{10}$$

where $h_1^2 = L_{AB}^2 + L_{AH}^2 + L_{HE}^2 - L_{BE}^2$. It is noted that the $\tan^{-1}$-function is used to avoid possible numerical problems which may occur with $\sin^{-1}$- and $\cos^{-1}$-functions. Thus, equation (10) specifies angle θ_2 when the length L_{BE} of actuator two is given.

Actuator three moves the shaft of joint three. To relate the length L_{FI} of this actuator to the angular position of joint three, one observes in Figure 2 that the angles $\underline{/ACI}$ and $\underline{/FCD}$ are constants determined by the structure of the machine. By denoting $\underline{/ACI} = \gamma_1$ and $\underline{/FCD} = \gamma_2$, it follows that $\underline{/ICF} = 2\pi - (\theta_3 - \pi) - \gamma_1 - \gamma_2$. Then the cos-theorem for triangle FIC gives

$$L_{FI}^2 = L_{FC}^2 + L_{CI}^2 - 2L_{FC}L_{CI}\cos(3\pi - \theta_3 - \gamma_1 - \gamma_2) \tag{11}$$

where again the subscripts attached to the length L refer to the endpoints of the line segment. Lengths L_{FC} and L_{CI} are constants which do not change with the configuration of the excavator (backhoe).

Equation (11) can now be solved for the joint variable θ_3 when the length L_{FI} of actuator three is known. By denoting $h_2^2 = L_{FC}^2 + L_{CI}^2 - L_{FI}^2$, one can write

$$\theta_3 = 3\pi - \gamma_1 - \gamma_2 - \tan^{-1}\{h_2^2/[4L_{FC}^2L_{CI}^2 - h_2^4]^{1/2}\} \tag{12}$$

Actuator four causes the bucket to rotate about the axis of joint four. The length of this actuator can be related to the joint variables θ_4 by expressing the cos-theorem for triangle JKL (Figure 2) as follows:

$$L_{JK}^2 = L_{JL}^2 + L_{KL}^2 - 2L_{JL}L_{KL}\cos(\nu_1 - \varepsilon_1) \tag{13}$$

where $\nu_1 = \underline{/JLD}$ is a constant for a given excavator (backhoe), and $\varepsilon_1 = \underline{/KLD}$ is to be determined. Equation (13) can be solved for ε_1 to obtain

$$\varepsilon_1 = \nu_1 - \tan^{-1}[h_3^2/(4L_{JL}^2L_{KL}^2 - h_3^4)^{1/2}] \tag{14}$$

where $h_3^2 = L_{JL}^2 + L_{KL}^2 - L_{JK}^2$. The next step is to relate angle ε_1 to the joint variable θ_4.

Since the sum of the angles about the axis of joint four in Figure 2 is 2π, it follows that $\underline{/LDG} = \underline{/LDJ} + \underline{/JDG} = 2\pi - (\theta_4 - \pi) - \nu_2 - \nu_3$ where $\nu_2 = \underline{/CDL}$ and $\nu_3 = \underline{/GDN}$ are constants for a given excavator. By denoting $\underline{/KGD} = \varepsilon_2$ and $\underline{/LKG} = \varepsilon_3$, the sum of the angles in the quadrangle KLDG gives:

$$\varepsilon_1 + \varepsilon_2 = 2\pi - [2\pi - (\theta_4 - \pi) - \nu_2 - \nu_3] - \varepsilon_3 \tag{15}$$

where ε_2 and θ_4 are still is to be determined. The cos-theorem applied to triangles KLD and KDG gives

$$L_{KG}^2 + L_{GD}^2 - 2L_{KG}L_{GD}\cos(\varepsilon_2) = L_{KL}^2 + L_{LD}^2 - 2L_{KL}L_{LD}\cos(\varepsilon_1) \tag{16}$$

Equation (16) can be solved for angle ε_2. If ε_3 is known, it then follows by equation (15) that

$$\theta_4 = \pi - \nu_2 - \nu_3 + \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \tag{17}$$

Thus, equations (10), (12) and (14), (16), (17) relate the lengths of the actuators to the joint variables θ_2, θ_3 and θ_4. They together with equations (7) and (8) represent the forward kinematic equations.

4. BACKWARD (INVERSE) KINEMATIC RELATIONS

The inverse kinematic problem in an excavator (backhoe, loader) is to determine the joint variable values (the first part) and the lengths of the actuators (the second part) which correspond to the specified position and orientation of the bucket given in the base coordinate system. It is solved for the case that the coordinates of point $O_3 = D$ are given in the base coordinate system, i.e., $p_0^D = [p_{ox}^D \;\; p_{oy}^D \;\; p_{oz}^D \;\; 1]^T$ is known, and the corresponding joint variable values θ_1, θ_2 and θ_3 and the lengths L_{BE}, L_{FI} and L_{JK} of the actuators are to be found.

The solution is provided by the backward kinematic equations which are developed next.

4.1. Equations Relating Bucket Pose to Joint Shaft Angles

The first part of the inverse kinematic problem is to determine θ_1, θ_2 and θ_3 that satisfy the kinematic equations, i.e., to find the joint variable values that place point D on the rotational axis of the bucket to the given point. It is assumed that the digging task is performed on the vertical plane containing the line segment O_1O_o joining the origins of the first and zeroth (base) coordinate frames.

In this specific problem it is convenient to first express p_0^D in the first coordinate frame $X_1Y_1Z_1O_1$ as a vector p_1^D. It is obtained directly from equation (3) as

$$p_1^D = A_1^0 p_0^D = A_1^2 A_2^3 p_3^D = A_1^3 p_3^D \tag{18}$$

where p_3^D are the coordinates of point D in the third coordinate frame, i.e., $p_3^D = [0 \;\; 0 \;\; 0 \;\; 1]^T$, $A_1^0 = (A_0^1)^{-1}$ and $A_1^3 = A_1^2 A_2^3$. It follows that

$$p_1^D = \begin{bmatrix} c_1 & s_1 & 0 & -a_1 \\ 0 & 0 & 1 & 0 \\ s_1 & -c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{ox}^D \\ p_{oy}^D \\ p_{oz}^D \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 p_{ox}^D + s_1 p_{oy}^D - a_1 \\ p_{oz}^D \\ s_1 p_{ox}^D - c_1 p_{oy}^D \\ 1 \end{bmatrix} \tag{19}$$

Equation (18) can be solved for θ_1 and θ_2 to obtain

$$\theta_1 = \tan^{-1}(p_{oy}^D / p_{ox}^D) \tag{20}$$

$$\theta_2 = \tan^{-1}\left[\frac{p_{oz}^D}{d}\right] + \tan^{-1}\left[\frac{\{4a_2^2[(p_{oz}^D)^2 + d^2] - [(p_{oz}^D)^2 + d^2 + a_2^2 - a_3^2]^2\}^{1/2}}{(p_{oz}^D)^2 + d^2 + a_2^2 - a_3^2}\right] \tag{21}$$

Having determined θ_1 and θ_2, the third unknown θ_3 is determined by dividing the second equation in (19) by the first one in the same vector equation. It gives

$$\theta_3 = \tan^{-1}\left[\frac{c_2 p_{oz}^D - s_2 d}{s_2 p_{oz}^D + c_2 d - a_2}\right] \tag{22}$$

Equations (20), (21) and (22) determine the joint variable values θ_1, θ_2 and θ_3 of the excavator when the position p_0^D of point D = O_3 is known. The joint angle θ_4 of the bucket relative to the positive x_3-axis is specified when the orientation of the bucket is given.

When the coordinates of point N = O_4 on the center of the edge of the bucket and the orientation angle $\theta_{234} = \theta_2 + \theta_3 + \theta_4$ of the bucket relative to the X_o-axis (or equivalently the X_1-axis) are known, the following expressions are substituted for p_0^D: $p_{ox}^D = p_{ox}^N - a_4 c_{234}$, $p_{oy}^D = p_{oy}^N + a_4 s_{234}$, $p_{oz}^D = p_{oz}^N$ where the bucket orientation θ_{234} is given and p_0^N specifies the location of point N = O_4 in the base coordinate system.

4.2. Equations Relating Joint Shaft Angles to Lengths of Actuators

This part of the inverse kinematic problem is to determine the lengths of the hydraulic actuators, i.e., the line segments between the attachment points of the actuators when the values of the joint angles are given.

The length L_{BE} of actuator two is determined by equation (9), when the joint angle θ_2 and the structural parameters are known.

Similarly, the length of L_{FI} of actuator three can be calculated by equation (11).

The length L_{JK} of actuator four can be calculated by equation (13) if angle ε_1 can first be obtained. By substituting ε_2 from equation (15) into equation (16), the resulting trigonometric equation can be solved for

ε_1 using the standard method. If $\varepsilon_4 = \theta_4 - \pi + \nu_2 + \nu_3 - \varepsilon_3$, then the solution can be written as

$$\varepsilon_1 = \tan^{-1}\{L_{KG}L_{GD}\sin(\varepsilon_4)/[L_{KG}L_{GD}\cos(\varepsilon_4) - L_{KL}L_{LD}]\}$$

$$- \tan^{-1}\{[4L_{KG}^2L_{GD}^2\sin^2(\varepsilon_4) + 4(L_{KG}L_{GD}\cos(\varepsilon_4) - L_{KL}L_{LD})^2 - h_4^4]^{1/2}/h_4^2\} \quad (23)$$

where $h_4^2 = L_{KG}^2 + L_{GD}^2 - L_{KL}^2 - L_{LD}^2$ is a constant. Equation (23) specifies ε_1 in terms of ε_4 and thus the joint shaft angle θ_4, known constants and angle $\varepsilon_3 = \angle LKG$ which is assumed to be available from the measurements. Then, the length L_{JK} of actuator four can be calculated by equation (13).

When the joint shaft angles are known, the lengths L_{BE}, L_{FI}, L_{JK} of the hydraulic actuators can be determined by equations (9), (11), (23) and (13). Thus, the backward (inverse) kinematic equations relating the joint angles to the lengths of the actuators have been established.

The total inverse kinematic relations of the excavator (backhoe, loader) are furnished by equations (20), (21), (22) and (9), (11), (23) and (13).

5. PLANNING OF EXCAVATOR MOTION

In the manual control of an excavator, the operator makes the bucket move so as to execute a specific task. For an automatic operation, a computer controls the motion of the bucket, which requires that the motion of the excavator is planned in advance. The planning can be performed by specifying in the world coordinate system the points in time through which the center point of the excavator bucket is to move. The coordinates of these points are then converted to the values of the joint variables using the inverse kinematic equations. A controller will guide the actuator so as to make the joint variables assume the specified (desired) values at the pre-determined times. As the consequence, the bucket will assume the desired pose (position and orientation) at the specific times.

The preplanning of the trajectory points for the joint variables is straightforward when the bucket moves freely (without touching anything) in a known environment. However, when the excavator is performing a task such as digging, the bucket is subject to environmental effects. For example, the finite volume of the bucket poses a constraint on the digging operation. In fact, the maximum volume of the bucket should be taken into account in the planning when the depth and the length of the soil cut are planned for the automatic excavator operation.

In the following, an excavator is assumed to be at the digging site. It is to perform a digging task whereby soil is removed in a sector to a specified depth. Its workspace contains the soil volume which is to be removed (Figure 3). A world coordinate system, a base coordinate system, and the local coordinate frames have been assigned so that the task to be performed can be specified precisely. The movements of the excavator which are needed for the execution of the entire task may be decomposed into the following stages:

1. Starting from resting position (the initial orientation of the bucket is in a vertical position) the bucket is raised one meter from the ground level.
2. By moving θ_1, the arm is rotated to a position above the place where the digging starts on the X_wY_w-plane (swinging).
3. The bucket is moved to the ground level, i.e., to the plane $Z_w = 0$.
4. The bucket is moved into the ground at the given digging angle and rotated (approximately 5°) to prepare for digging.
5. The bucket is moved so as to perform the digging of a soil cut (sweeping).
6. At the end of the digging motion, the bucket is rotated to keep the soil in the bucket (scooping).
7. The bucket is raised vertically to the height of one meter (raising).
8. The arm is rotated horizontally to the dumping position by changing only θ_1. The bucket will then be approximately one meter over the dumping area (swinging).
9. The bucket soil is dumped by changing the orientation of the bucket (dump).
10. The bucket is rotated to the original pose.

11. The arm is rotated to the next starting position by varying only θ_1. This completes one full digging cycle.
12. Repeat from 4 on until the task is finished.

The stages 2, 5, 6, 7 and 8 correspond to the motions described in [6], which are indicated above by the terms in the parentheses. A side view of the excavator motion is displayed in Figure 4. It is noted that variable θ_1 remains constant during the digging cycle except in stages 2, 8, and 11. Thus, the excavator operates on a plane during the sweeping mode by moving links 2, 3 and 4 (the bucket).

During the stages 4 to 7, the bucket is being filled with soil. When the digging operation is executed, the volume V_s^1 of uncompressed soil in the bucket in the first cycle ($\theta_1 = 0$) is

$$V_s^1 = bz(R_1 - r), \qquad R_2 \leq r \leq R_1 \tag{24}$$

where b is the width of the bucket, R_1 and R_2 are the maximum and minimum radius of the sector in which the soil is being removed, and r is an arbitrary radius specifying the position of the center point on the bucket cutting edge. When the bucket is full of soil, then the bucket volume V_B is equal to V_s^1, i.e.,

$$b(0.5 + 0.25\pi)(a_4/2)^2 = bz(R_1 - r) \tag{25}$$

Equation (25) can be used to specify the depth z of the soil cut and radius r that determines the length $(R_1 - r)$ of the strip.

It will be assumed in the following that depth z is selected so that the soil in an entire strip will fit in the bucket, i.e., the value of z is determined when $r = R_2$.

The second digging cycle is assumed to start by placing the center point (N) of the bucket edge to point A_2 in Figure 3. It is at distance R_1 from the center point of the sector and at angle $\Delta\theta_1$ measured from the centerline of the first strip where $\Delta\theta_1 = b/R_1$. During the execution of the digging cycle, this angle stays constant. Therefore, the width of the strip of land that is to be cut during the motion of the bucket in this cycle is less than the bucket width and varies with radius ρ (see Figure 4). The soil volume of the cut in the second cycle is next determined.

The volume V_s^2 of uncompressed soil being collected into the bucket during the second digging cycle is by Figure 3

$$V_s^2 = \int_{R_1}^{r} z(\rho\Delta\theta_1)(-d\rho) = \frac{b}{R_1}\int_{r}^{R_1} z\rho d\rho \tag{26}$$

where the width w_c of the cut soil slice is $w_c = \rho\Delta\theta_1$ and radius ρ specifies the bucket position from the center (O) of the circle whose arches limit the sector. If the depth z of the digging operation is constant, then equation (26) gives

$$V_s^2 = bz(R_1^2 - r^2)/(2R_1) = 0.5bz(R_1 - r)(1 + r/R_1) \tag{27}$$

Equation (27) can also be obtained directly by approximating the area in which the soil is being removed as a trapezoid with height $(R_1 - r)$ and the lengths of the parallel sides being b and $r\Delta\theta_1$. Again, the volume of the bucket $V_b \geq V_s^2$ or

$$0.5bz(R_1 - r)(1 + r/R_1) \leq b(0.5 + 0.25\pi)(a_4/2) \tag{28}$$

Equation (28) can be used in the planning to specify the depth of the bucket (cutting) edge during digging and the length of $(R_1 - r)$ of the second strip of soil.

The next strip of soil will be removed by repeating the procedure described.

It is convenient for the planning of the excavator motion to specify a sequence of discrete points in the ground through which the center point (N) of the cutting edge of the bucket should move when the execution of the digging operation starts from the outer arc at distance R_1 from point O. These points can be specified along the center line (A_mB_m) of a strip m as (r_i, z_i, θ_i), where $m = 1,2,\ldots,M$, $i = 0,1,\ldots,N$ with $r_o =$

R_1 and $\theta_i = i\Delta\theta_i$. These points are then connected by line segments to form a polygonal path. The digging depth along each line segment is held constant.

Equation (26) determines the volume of soil in the bucket gathered during the cutting of a strip after the first one. Let V_s^m be the volume of soil that is filling the excavator bucket on the mth strip, m = 2,...,M, where the first strip is excluded. The integral in equation (26) may now be evaluated along each segmented strip to obtain after the nth segment has been cut:

$$V_s^m(n) = \frac{b}{R_1} \sum_{i=1}^{n} \bar{z}_{i-1}(r_{i-1}^2 - r_i^2)/2 \tag{29}$$

where $n = 1,\dots,N$, $r_o = R_1$ and $V_s^m(n)$ is the volume of soil in the bucket collected over n segmental volumes in the mth strip. Equation (29) may be rewritten as

$$V_s^m(n) = V_s^m(n-1) + 0.5b\bar{z}_{n-1}(r_{n-1}^2 - r_n^2)/R_1 \tag{30}$$

where $V_s^m(0) = 0$. It is noted that the digging depth in each segment is constant but it can vary from one segment to another. Equation (30) represents a recursive relation for calculating the volume of soil collected in the bucket. It is expressed in a computationally efficient form.

The condition that $V_s^m(n) \leq V_b$ can then be used to calculate the length of the soil strip if the depth of the cut is specified. For example, this condition with equation (30) leads to equation (28), which can be solved for $r = r_1$ when $z = \bar{z}_o$:

$$r_1 \leq [R_1^2 - 2V_bR_1/(b\bar{z}_o)]^{1/2} \tag{31}$$

The equality in (31) expresses the maximum length of the strip of uncompressed soil cut at depth $\bar{z}_o$ that will fill the bucket.

6. CONCLUSIONS

For the automatic operation of an excavator, the nominal trajectory (path) for the bucket is specified so that the soil in a sector to a specified depth will be removed. The kinematic equations of the excavator are presented and used to convert the nominal motion of the bucket to the nominal motions of the joint variables. It can also be expressed as the nominal motions of the hydraulic actuators. The volume of the bucket determines the depth for the digging and the length of the strip for the bucket motion. The trajectories thus determined can then be utilized by a controller which can be implemented on a digital computer. It will then automatically operate the excavator to perform the specified task.

7. ACKNOWLEDGEMENTS

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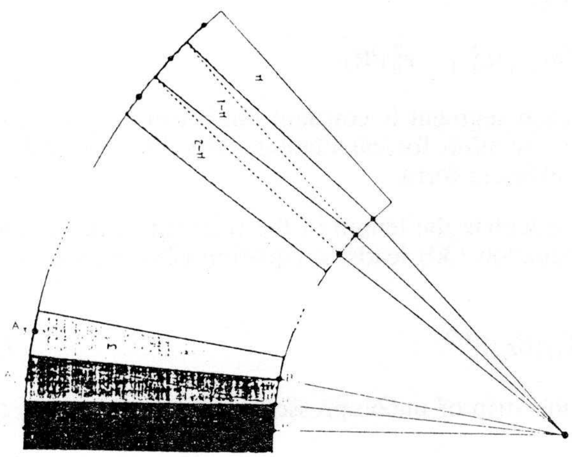
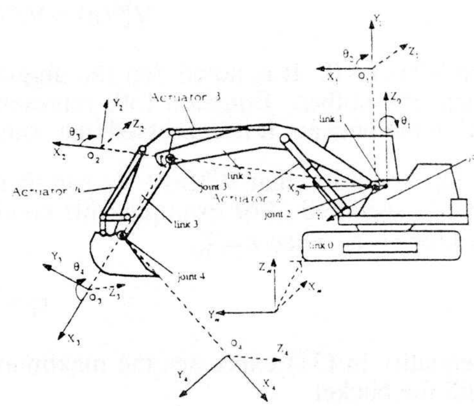


Figure 3



Figure

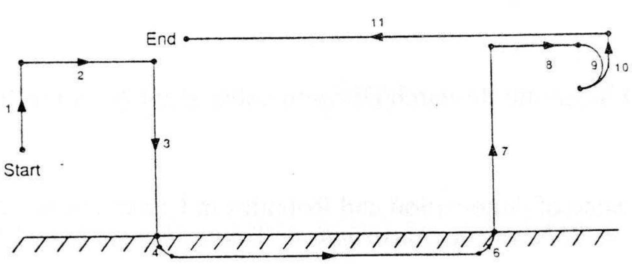


Figure 4

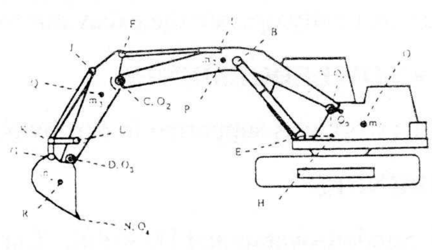


Figure 2