Procedure Design for Experiments Towards Modeling of the Cutting Force in Excavation of Bulk Media

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ABSTRACT: The work discussed in this paper concerns automation of excavation or automation of loading particulate media by an excavating machine. Based on the analysis of the process, knowledge of the force of cutting/digging is required for feedback purposes. The lack of a reliable and well established model for the force in question dictates the primary work of the development of such a model. This interaction force is a function of a large number of parameters. Up to 32 parameters have been proposed. In addition to the large number of the parameters an analytical formulation of such a model is less likely possible, as can be seen from the past work. The complexity of the matter, therefore, calls for an empirical formulation, based on the results of experiments that must be carried out. This calls for a huge number of tests. It is important to reduce, as much as possible, the number of experiments to be performed. Also, if the material is categorized in a logical manner, various media can be prepared by mixtures of only a finite number of materials. The objective of the present paper is to define a generic function for the mathematical model and a plan for the tests that must be performed on soil type material. This leads to increased efficiency and helps to reduce duplications and unnecessary work before spending time on experiments. Based on this systematic approach the experiments can be arranged in a logical order, and the results can be later plugged in at their proper places.

KEYWORDS: Cutting/digging force, bulk media, model formulation, experiment

1. INTRODUCTION

The work discussed in this paper concerns modelling the force of cutting/digging into a bulk medium. Previous work has shown that, due to the complexity of the process and the large number of parameters, an analytical formulation is quite cumbersome, if possible. In the previous related work (see all the references) as many as 32 parameters hove been considered. As such, a large number of experiments must be carried out. The intention of the paper is not to find the sought relationship at this stage; the focus is a logical decision for the experimental work in order to minimize the number of the necessary tests to be carried out.

The force we are concerned is denoted by f_4 in figure 1. This force of penetrating or cutting into a media is one of the components of the total force to be supplied by bucket (hereafter called tool for generality). The other force components are as defined in [12]. Knowledge of all the force components at any instant during an excavation task becomes necessary for automating an excavation (or loading) process [14]. Also, in other applications associated with bulk material handling, such as in their design is necessary to know the required force for cutting through and penetrating into a medium [5], [11], [13], [17], [18], [20], [23]. The necessary force to be supplied by a tool must overcome all the resisting force from the medium, one of which is the cutting force under consideration. This cutting force (or the force of interaction between a medium and a cutting tool) depends on material (properties of the medium), tool (dimensions, shape and condition such as angle of attack, operation (the way the tool moves with respect to the medium) and environment (gravity, terrain slope and temperature).

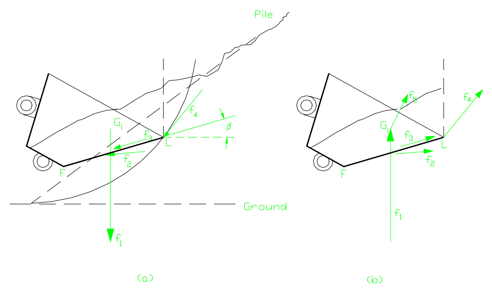

Figure 1. (a) Force components in loading a bucket, (b) force components to be provided

Numerous researchers have worked on modelling the cutting force and many formulations have been proposed. A recent survey [3], however, indicates that these models are not only incompatible, and even sometimes contradictory, most of them are not verified and none of them is universally accepted. In fact, verification of the proposed models, by itself, requires quite a lot of work. These formulations are base on mostly analytical and some experimental work. The previous work can be categorized into the studies towards the properties and the behaviour of soil for agricultural purposes or civil engineering applications [9],[10]. [15], [16], [21], [24], [25], and for material handling and design of the earth moving machinery [1], [2],]6], [7], [8], [22], [26]. In all the cases, because of the complexity of the study and the large number of parameters, each researcher has followed a different approach and has made different assumptions for reducing the number of parameters. Discussing about the previous formulations, their differences or similarities, even in brief, is out of the scope of this paper.

2. PARAMETERS

The following list shows most of the parameters that have been considered in the previous works, without explaining their definition. Some of the parameters are inter-related and not all of them have equal effects. Thus, every researcher has selected and included in the formulation only a few numbers of them. These parameters are:

Tool related: Width, Tool plate thickness, Tip angle, Tip sharpness factor, Blunt edge height, Existence of teeth;

Medium related: Cohesion, Internal friction angle, Density, Elastic modulus, Poisson ratio, Compressive strength, Tensile strength, Water content (moisture), Absolute viscosity, Average particle size, Porosity, Compactness;

Operation related: Tool speed, Cutting angle (rake angle), Tool depth, Surcharge, Tool Acceleration, Failure plain parameters, Type of cutting [27], Cutting index [27], Curvature radius;

Environment related: Gravity constant, Temperature, terrain slope;

Tool-Medium related: External friction angle, Adhesion, Size factor.

There are thirty-two parameters in the above list. As for an example of the formulations suggested, the following model has been developed for bucket filling force based on a large number of experiments and considering the theoretical analysis [27]. Also, it has been cited by some other researchers, implying that it has been accepted as valid (without verification)

$$P = f_4 = 10\, C_0 e^{1.35} (1 + 2.6b)(1 + 0.0075\alpha_c)(1 + 0.03s)\alpha_0 k \qquad (1)$$

where C_0 is a factor corresponding to the type of soil representing its compactness and resistance to cutting (It is based on the number of drops of a drop hammer in penetrating test; not North American standard), e is the depth of cutting in centimetres, b is the width of the bucket in meters, α_c is the angle of cutting in degrees, s is the thickness of the cutting surface of the bucket in centimetres, α_0 is a factor corresponding to a measure of sharpness of the cutting edge (tip angle), k is a factor representing the type of cutting (that is, two open sides, one open side or no open side). Only six parameters are employed in this formula.

In the rest of this paper first a reasonable form of the mathematical model is discussed and then a more important subgroup of the parameters are selected to be included in the model.

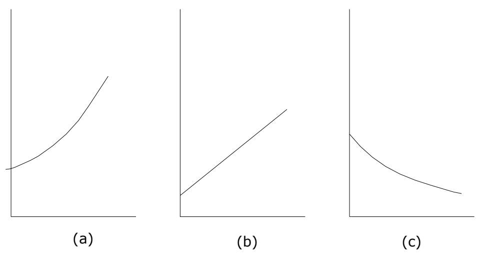

Figure 2.

3. MODELLING ANALYSIS

We consider the general form of a function of several variables, all assumed to have the same degree of importance, without considering the number of variables. It is assumed, also, that no information about the relationship is available. Except for the temperature, whose effect is nonlinear and, thus, is not considered for inclusion in the model, the effect of all other parameters are continuous and none of them has a negative value. Moreover, at least for the range of practical values of these variables, no periodicity can be found. As a result, we may confidently rule out the trigonometric functions. For the sake of simplicity, we want to exclude the exponential and logarithmic functions, also. This is because of two reasons. One is because of the wide scale of variation of these functions. None of the parameters can have either a very wide range of variation or such a drastic effect that necessitates a logarithmic or exponential expression. The second reason is that we may observe the variation of a function with respect to its one variable in one of the forms shown in figure 2 (including the mirror images with respect to the x-axis). With good approximation one may model the first and the third functions as follows (the second curve is a line):

$$y = a\ x^2 + b\ x\ + c$$
$$y = a\sqrt{x} + b\,x + c \qquad (2)$$

This implies that rising and decaying exponential functions for a limited range of variation may be substituted by parabolic functions of the forms in equation 2, respectively. Furthermore, since none of the variables seems to have an effect that requires a third or higher order polynomial function, it seems reasonable to assume only first and second order polynomials. In this sense, the variation of the sought function y in terms of its N variables x_1 to x_N assumes one of the following general forms:

$$y = \alpha_0 + \alpha_1 x_1^* + \beta_1 x_1 + \alpha_2 x_2^* + \beta_2 x_2 + \ldots + \alpha_N x_N^* + \beta_N x_N \qquad (3)$$

or

$$y = a_0\left(a_1 x_1^* + b_1 x_1 + 1\right)\left(a_2 x_2^* + b_2 x_2 + 1\right)(\ldots)\left(a_N x_N^* + b_N x_N + 1\right) \qquad (4)$$

where the * stands for 2 or 0.5, based on equation (2). This can be quickly determined from the shape of a curve when data is available. Variation of y can be a combination of equation (3) and (4), too. At this stage, however, consideration of the more difficult case of the third version is premature. Depending on the variation of y with x_i, the coefficients a_i and b_i in the first equation, or α_i and β_i in the second equation, but not both of them simultaneously, can be zero. The task of modelling, thus, is first to determine the pattern for the variation of y with each of the variables and then to determine the values for all the coefficients. That is to say, one cannot simply deduce that the number of required experiments to be performed depends on the number of unknowns.

In general, supposing that in a controlled manner all the variables can be kept constant except one whose values are to be modified as desired, for each individual variable equations (3) and (4) assume the following forms, respectively:

$$y = c\left(ax^* + bx\ + 1\right) \qquad (5)$$

and

$$y = \alpha x^* + \beta x\ + \gamma \qquad (6)$$

where a, b or α, β denote the coefficients for any of the variables in the corresponding equation, and c and γ are relative constants if the values of the variable under consideration, only, are altered and the rest of variables remain unchanged. In this respect, if a series of tests are carried out in which only a parameter x is varied, in both of the above cases there are three unknowns to be found, either *a*, *b* and *c*, or α, β and γ.

Theoretically, if three values for x and y are available, then the values of each unknown coefficient can be found in both of the above two cases. What is important, nevertheless, is that in each case there are 2N+1 unknowns to be determined. It is preferable if all the unknowns can be found simultaneously. Let us first review the solution to the simpler three (nonlinear or linear) equations in three unknowns, nonlinear for equation (5) and linear for equation (6).

3.1. Linear Equations

In the case of system (3), defined by equation (5), having three different sets of values of x's and y's leads to the following system of linear equations:

$$\begin{bmatrix} m_1 & n_1 & 1 \\ m_2 & n_2 & 1 \\ m_3 & n_3 & 1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix} \qquad \ldots\ldots(7)$$

The notation in equation (7) covers both models shown in equations (2), as well as when the function is linear (where one of the unknowns must be zero), m_1 to m_3 and n_1 to n_3 correspond to the variable x and p_1 to p_3 stand for the corresponding values of the function y.

3.2. Nonlinear Equations

In the case of nonlinear equations, having three different sets of values for x and y leads to the following equations:

$$\left(a + m_1 b + n_1\right) c = p_1$$
$$\left(a + m_2 b + n_2\right) c = p_2$$
$$\left(a + m_3 b + n_3\right) c = p_3 \qquad (8)$$

where m_1 to m_3 and n_1 to n_3 correspond to the variable values (here 1/x), and p_1 to p_3 stand for the corresponding values of the function (here, y/x^*, * depending on the model form). Similarly, *a* and *b* stand for the unknown parameters that must be determined. Equations (8) can be solved in the following manner, by subtracting from each other, in order to eliminate *a*:

$$\begin{aligned} &[(m_1 - m_2)b + (n_1 - n_2)]c = p_1 - p_2 \\ &[(m_1 - m_3)b + (n_1 - n_3)]c = p_1 - p_3 \end{aligned} \qquad (9)$$

which, in turn, leads to

$$c = \frac{p_1 - p_2}{[(m_1 - m_2)b + (n_1 - n_2)]} = \frac{p_1 - p_3}{[(m_1 - m_3)b + (n_1 - n_3)]} \qquad (10)$$

The value of *b* and then *c* can be obtained from equation (10), and by substituting in either of the equations in (8), the corresponding value for *a* can be determined.

4. EXPERIMENTAL WORK ARRANGEMENT

As discussed in the previous section, we have assumed the variation of the force function y in terms of each one of the parameters to be linear or of a quadratic form. We would like to see the shape of the associated curve when necessary. For this reason, and since the results of experiments are not always 100% accurate and reliable, more points are required, even if mathematically only three points are sufficient. This suggests more experiments for the variation of each parameter. For a general discussion, this number is shown by H in the upcoming formulations. However, a value of seven (7) for H seems to be both practical and reasonable. If each parameter x can be varied in a range of values, from its lowest to its highest, the following notation will be used to denote these, not necessarily equally spaced, values:

$x_1, x_2, x_3, x_m, x_{m+1}, \ldots, x_H$

where the m^{th} value is around the middle of the range. For each variable x_i, H experiments are necessary, where the values of x_i are varied between x_1 to x_H, and the values of the other variables are kept constant at their middle range, or m-values. In this way, one experiment with all the parameters at their middle value generates a common point that can be used for plotting all the function curves. Thus, the total number of experiments for N variables is (H-1)N+1. In the particular case of seven point curves, the total number of experiments is 6N+1. This number corresponds to only one curve for each variable.

In what follows for each of the two aforementioned, linear and nonlinear models, the equations to be used for the calculation of parameters are formulated and further discussed.

4.1. Linear Equations

For N variables and H number of points for each curve the linear equation containing all the 2N+1 unknowns assumes the form. The middle value for all parameters is a common point and is considered only once. The total number of equations is, thus, (H-1)N+1.

$$\begin{bmatrix} n_{11} & n_{12} & \cdots & n_{1N} & \vdots & n_{1,N+1} & \cdots\cdots & n_{1,2N} & \vdots & 1 \\ n_{21} & n_{22} & \cdots & n_{2N} & \vdots & n_{2,N+1} & \cdots\cdots & n_{2,2N} & \vdots & 1 \\ n_{31} & n_{32} & \cdots & n_{3N} & \vdots & n_{3,N+1} & \cdots\cdots & n_{3,2N} & \vdots & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \\ n_{(H-1)N+1,1} & \cdots & \cdots & & \vdots & \cdots\cdots & & n_{(H-1)N+1,2N} & \vdots & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \cdots \\ \alpha_N \\ \beta_1 \\ \beta_2 \\ \cdots \\ \beta_N \\ \alpha_0 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ \cdots \\ p_{(H-1)N} \\ p_{(H-1)N+1} \end{bmatrix} \qquad (11)$$

This is an overdetermined system of N(H-1)+1 equations in 2N+1 unknowns. It can be solved by using the pseudo inverse of the matrix on the left side. The advantage of using the pseudo inverse is that it automatically determines the least square error solution for values of unknown parameters. This is desirable. By solving equation (11) the results of all the experiments are simultaneously considered in finding the model coefficients.

4.2. Nonlinear Equations

If the same sort of notation is used for the case of nonlinear equations, the resulting system of equations assumes the form:

$$a_0\left(x_{11}^*a_1+x_{11}b_1+1\right)\left(x_{m2}^*a_2+x_{m2}b_2+1\right)(\cdots)\left(x_{mN}^*a_N+x_{mN}b_N+1\right)=p_1$$
$$a_0\left(x_{21}^*a_1+x_{21}b_1+1\right)\left(x_{m2}^*a_2+x_{m2}b_2+1\right)(\cdots)\left(x_{mN}^*a_N+x_{mN}b_N+1\right)=p_2$$
$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$
$$a_0\left(x_{m1}^*a_1+x_{m1}b_1+1\right)\left(x_{m2}^*a_2+x_{m2}b_2+1\right)(\cdots)\left(x_{mN}^*a_N+x_{mN}b_N+1\right)=p_m$$
$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$
$$a_0\left(x_{H1}^*a_1+x_{H1}b_1+1\right)\left(x_{m2}^*a_2+x_{m2}b_2+1\right)(\cdots)\left(x_{mN}^*a_N+x_{mN}b_N+1\right)=p_H$$
$$a_0\left(x_{m1}^*a_1+x_{m1}b_1+1\right)\left(x_{12}^*a_2+x_{12}b_2+1\right)(\cdots)\left(x_{mN}^*a_N+x_{mN}b_N+1\right)=p_{H+1}$$
$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$
$$a_0\left(x_{m1}^*a_1+x_{m1}b_1+1\right)\left(x_{m2}^*a_2+x_{m2}b_2+1\right)(\cdots)\left(x_{HN}^*a_N+x_{HN}b_N+1\right)=p_{(H-1)N+1} \qquad (12)$$

Again there are 2N+1 unknowns (a_0 , a_1 , a_2, ...b_1,… b_N are the unknowns), but (H-1)N+1 equations; that is, the system is overdetermined. These equations are, however, nonlinear and finding a minimum error solution is not straightforward, even if it does exist. A methodology for finding the answer must be developed. One possible approach is to find the approximate values and then using an iterative method find the optimum solution. The approximate values can be found by considering the set of equations for each parameter, separately. Considering, for instance, the first H equations for the first parameter, we obtain

$$\begin{aligned}
&\left(x_{11}^*a_1+x_{11}b_1+1\right)c_1 = p_1\\
&\left(x_{21}^*a_1+x_{21}b_1+1\right)c_1 = p_2\\
&\ldots\ldots\ldots\ldots\ldots\ldots\\
&\left(x_{H1}^*a_1+x_{H1}b_1+1\right)c_1 = p_H
\end{aligned} \qquad (13)$$

where c_1 denotes the corresponding constant that results from the unchanged values of all the other parameters in equation (12). Equation (13) can be changed into a linear system by introducing the new variables:

$$\begin{aligned}
w_1 &= a_1c_1\\
w_2 &= b_1c_1\\
w_3 &= c_1
\end{aligned} \qquad (14)$$

as a result of which the overdetermined system is in the general form

$$\begin{bmatrix} k_{11} & k_{12} & 1\\ k_{21} & k_{22} & 1\\ \ldots & \ldots & \ldots\\ k_{H1} & k_{H2} & 1 \end{bmatrix} \begin{bmatrix} w_1\\ w_2\\ w_3 \end{bmatrix} = \begin{bmatrix} p_1\\ p_2\\ \ldots\\ p_H \end{bmatrix} \qquad (15)$$

Here again, by using the pseudo-inverse of the matrix K in equation (15) the minimum error solution can be found for w_1 to w_3, or accordingly for a_1, b_1 and c_1. The difficulty, however, is that a set of solutions obtained in this way must satisfy the rest of equations. More specifically, a value for w_i corresponds to all other coefficients the product of which with a_0 was called c_1.

If the values obtained in this way for all the coefficients a_0 to b_N are used as a first approximation and an iterative algorithm is developed to refine these values, that can lead to an acceptable set of answers. At this time the data is not available yet and further progress on the matter cannot be made.

5. PARAMETER SELECTION/REDUCTION

Out of the parameters already cited, only the following are selected for inclusion in our model. The parameters with negligible or marginal effects can be ignored and only those parameters with significant role must be taken into account. The reason for selecting or not selecting some of the parameters are discussed below.

5.1. Tool Related Parameters

Tool dimension cannot be neglected since there is a direct relationship between the size of the medium effected and the size of a tool. A bucket width is to be considered. Other parameters of a tool such as tip sharpness factor and blunt edge height and tip angle are relative issues and depend on the size ratio of material particles and the tool. These are less important. The depth of cutting is considered in operation related parameters. The existence of teeth on the cutting edge of a tool is quite important and may be represented by a multiplier (teeth factor). Separate research has to be devoted to finding this factor.

5.2. Medium Related Parameters:

Medium related parameters are the most complicated ones. They are correlated and often uncontrollable. The same material can exhibit very different behavior towards cutting or excavation depending on its past history and other conditions. This is very true especially in the case of the interaction between water and cohesive materials. The most significant parameters are as follow.

<u>Cohesion and adhesion:</u>
Cohesion represents a measure of the internal force between the particles of a substance in bonding to each other, and adhesion is the same property but between the particles of a substance with an external (different) material, like a tool. In this sense, adhesion is indeed a medium/tool property, but for

simplicity we want to represent these two parameters by one factor only. Both of these cause a media to resist to being cut by a tool. Also, both of these depend on the water content. The relationship with water content may be nonlinear, and other conditions may play a role. At this stage, we want to justify the selection of these parameters for inclusion in modelling.

Internal and external friction factors:
Friction can exhibit its effect internally or externally; that is, between the separate grains of a substance, or between these grains and an external material such as a tool. In this sense, a common factor for friction sounds reasonable.

Other medium related parameters:
From the numerous medium properties considered, we want to limit ourselves to density, particle size ratio and compactness as well as water content. A measure of size variation, compactness, and medium-tool size ratio must be considered. This latter can be better realized if for instance the insertion of a thin and a thick blade into a pile of sand is considered.

5.3. Operation Related Parameters

For each cutting operation, the angle of attack and the surcharge become more significant than other factors. Surcharge is the additional loose material on the top of the part to be cut. It has its effect on the normal stress by adding its weight. Also, it may be carried with the tool, such as in a bulldozer and grader. The speed of operation may or may not have an effect. This must be found out. The rest of operation related variables listed earlier are embedded in other factors. The depth of cutting has already been counted.

5.4. Tool/Medium Related Parameters

These are adhesion, external friction and tool/ medium size factor. They are already considered since they can fall under tool related parameters or medium related parameters.

5.5. Environmental Effects

The two most important environmental parameters are the gravity constant (in case the excavation is performed in another planet) and the temperature. The slope of terrain is also an environmental effect that affects excavation. Gravity and terrain slope influence the weight component of total force of excavation. They do not have a direct effect on cutting force. Temperature has the most nonlinear influence of changing a soft rock to a hard rock only in the vicinity of freezing point and for the media with noticeable water content. Because of this special effect of temperature, it cannot be treated similar to the other parameters and, thus, it should not be included in the model.

5.6 Other parameters

There are some other factors, too; for example induction of vibrations to a tool and, as Zelenin has represented by a coefficient, if a medium is laterally continuous from one side or both sides are discontinuous. The issues need to be investigates separately, specially when it is obvious that their effect can be reflected by introducing an appropriate factor.

Based on the above discussion, our short list consists of: Angle of attack or cutting angle, Cohesion and adhesion, Compactness, Cutting depth, Density, Friction, Particle size distribution, Tool-medium size factor, Tool width, Water content

6. PRACTICAL ASPECTS

From the above list, changing the tool and the operation related parameters are the easiest ones to manage. For those associated with the media, the ideal situation is if all the properties of a material can be controlled so that they can independently assume any desired value. In practice, however, this is less likely to be possible. It has been shown [23] that certain properties of a mixture of two soils can be mathematically expressed as a function of the properties of the individual components. If this can be true for all the properties in the above short list, and if likewise it can be extended to more than two soils, then change of properties can be achieved by mixing different proportions of a selected number of base materials. In this way, any particular property can be varied within a desired range of values, while the inevitable changes in the other properties could be kept small if proper base materials can be identified.

One set of experiments is not reliable for modelling and any experiment must be repeated a number of times and with various materials for better accuracy. Not all the parameters must be treated the same way. The size of a tool or the density of the material used can be more confidently measured than say the cutting depth or the cohesion. With some different weights for the repeat of experiments, the details of which are omitted here because of space limitation we have arrived at a total number of 495 experiments to be carried out.

7. SUMMARY:

The main objective is to find a mathematical model for the cutting force that a tool encounters during excavation of a bulk material. Since, so many parameters are involved an analytical model has not been possible in the previous work and an empirical relationship must be found. This requires a great number of experiments. The purpose of this paper is first to decide about the general form of such a model and, also, to select only a subset of the parameters to be included in the model. The experimental results then can be fitted into the model. Based on the two possible models, proposed, the generic method of the calculation of the coefficients in the model function was discussed. Ten out of thirty-two parameters were determined to be more significant than others, to be included in the model. According to this work, the formulation of the model, when data is available, can be performed by first a preliminary analysis of the shape of variation of the force with each individual parameter, and then using the data in the generic model function for numeric calculation of the model constants.

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